PRINCIPLES OF DIRECT-CURRENT ELECTRICAL ENGINEERING

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WITH 295 ILLUSTRATIONS

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PREFACE

In lecturing on Electrical Engineering to Second-Year Students the author has realised the want in recent years of a really suitable text-book—one which he could recommend as adequately bridging the gap between the several elementary manuals and the many excellent works on specialised branches of the subject.

This treatise has been written primarily for the intermediate class-work of Universities and Technical Colleges, the basis being a series of the author's Lectures to Second-Year Students, with a selection of problems systematically arranged in the order of treatment of the subject.

A knowledge of the elementary principles of electrical physics and mathematics has been assumed. In writing the book it has been an aim to give a clear and concise account of the fundamental principles of direct-current electrical engineering. The work deals essentially with PRINCIPLES, and whenever possible the methods of applying them are illustrated with numerical and descriptive examples. Only those theories and methods of solving problems have been discussed which experience in design and operation has shown to be of practical utility.

The descriptive work has been confined to apparatus and machines associated with the best practice in present-day electrical engineering. Useless catalogue illustrations have been rigidly excluded, the drawings, with a few exceptions, having been specially prepared. In several cases, for the sake of clearness, coloured diagrams have been adopted; the distinct advantages of which in dealing with armature windings, switchboard connections, etc., need no comment.

The author is specially indebted to his friend Charles W. D. Newman, A.M.I.E.E., for the unwearied assistance given in reading over the original MS., and in making numerous and valuable suggestions. His thanks are also due to Dr. W. Mansergh Varley and T. K. Evans, B.A., for looking over proofs; and to John T. Wight, who prepared most of the drawings for Chapters VIII. to XI.

J. R. B.

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PUBLISHER'S NOTE TO FIFTH IMPRESSION

THE rapid sale of the last edition has rendered a further reprint necessary much sooner than was anticipated. Although no extensive revision has been attempted, the opportunity has been taken to add certain information relating to such recent innovations as the Gasfilled Lamp, and the new Table of Standard Sizes of Copper Conductors which has recently been adopted. It is hoped that these additions will add to the utility of the work.

JANUARY, 1922.

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PRINCIPLES OF DIRECT-CURRENT ELECTRICAL ENGINEERING

CHAPTER I

UNITS EMPLOYED IN ELECTRICAL ENGINEERING

In commencing a study of the subject of Electrical Engineering it is essential to thoroughly understand how the units employed in practice are derived from the fundamental units. This chapter will be devoted to a brief résumé of the derivations.

A scientific treatment of such a subject as Electrical Engineering is only possible when there is some means of making definite statements as to quantity. This is done by stating how many arbitrarily chosen units are required to make up any quantity in question.

The ideas of length and time are regarded as primary—*length*, the conception of an interval of space; *time*, the conception of a duration of a period. Length and time being primary, are the first two fundamental quantities; *mass* is usually considered third, and is the amount of matter in a body.

The system of units adopted for engineering work is based on those of length, mass, and time. Starting with the fundamental units, it will be shown how (1) the Mechanical, and (2) the Electrical units used in practical engineering are derived, and also the relationship between the two sets.

FUNDAMENTAL UNITS

Length.—In Britain the unit of length is the yard, and in France, the metre. Both these units are selected arbitrarily for their convenience, though the original idea of the metre was derived from a connection with the supposed dimensions of the earth. For scientific purposes the hundredth part of the metre, *i.e.* the centimetre, is taken as the unit of length.

1 inch = 2.54 centimetres.
1 foot = 30.48 centimetres.
3.28 feet = 1 metre.
0.621 mile = 1 kilometre.

Mass.—The unit of mass is fixed arbitrarily, the British and French units being respectively the pound and the kilogramme. The kilogramme originally represented the weight of a cubic decimetre of water at 4° C., *i.e.* its point of maximum density. More recent investigations have, however, shown that this is not exactly fulfilled. A smaller unit is more convenient in the laboratory, so the thousandth part of the kilogramme, *i.e.* the gramme, has been chosen as the unit of mass. For all practical purposes the gramme is taken as the mass of a cubic centimetre of water at 4° C.

1 lb. = 453 grammes.
1 ton $\approx$ 1000 kilogrammes.

Time.—The unit of time is the $\frac{1}{86400}$ part of a mean solar day, and is called the second.

The system of units based on the centimetre, the gramme, and the second is in universal use for scientific purposes, and is referred to as the *C.G.S.* system. All other dynamical units are derived from these three fundamental ones of length, mass, and time. As the dynamical units, so derived, are, with a few exceptions, generally inconveniently small for practical work, the practical units are multiples of the absolute units.

MECHANICAL UNITS

Area.—The unit of area is the square centimetre.

1 square inch = 6.45 square centimetres.

Volume.—The unit of volume is the cubic centimetre.

1 cubic inch = 16.4 cubic centimetres.

Velocity is the rate of change of position. It involves the idea of direction as well as that of magnitude; thus the velocity of a body may be changed by altering the rate at which it moves, or by altering the direction in which it is moving.

The unit of velocity is the velocity of a body which moves through unit distance in unit time, *i.e.* a velocity of 1 centimetre per second.

Velocity in centimetres per second

$$= \frac{\text{space (centimetres) traversed}}{\text{time (seconds)}}.$$

Momentum is the quantity of motion in a body, and is measured by the product of mass and velocity.

The unit of momentum is the momentum of a body 1 gramme in weight moving at the rate of 1 centimetre per second.

Acceleration is the rate at which velocity changes, *i.e.* increases or decreases. Unit acceleration is the change in velocity of a body at the rate of 1 centimetre per second for every second the motion continues, and is usually expressed in centimetres per second per second, or centimetres per second^2.

The acceleration due to gravity (denoted by "g") is

981 centimetres per second^2

or 32.2 feet per second^2.

Force is that which tends to alter a body's state of rest or uniform motion in a straight line, and is measured by the rate of change of momentum which it produces.

The unit of force is that force which, acting for 1 second on a mass of 1 gramme, gives it a velocity of 1 centimetre per second. This unit is termed the *dyne*. One gramme weight exerts a force downwards of 981 dynes due to the action of gravity.

1 dyne = weight of $\frac{1}{981}$ gramme.

1 poundal = ,, ,, $\frac{1}{32.2}$ lb.

1 ,, = 13,825 dynes.

1 lb. weight = 32.2 poundals = 445,000 dynes.

1 kilogramme weight $\approx$ 1,000,000 dynes.

Work is the product of a force and the distance through which it acts. The unit of work is the work done in overcoming unit force through unit distance, *i.e.* the work done when 1 dyne acts through a distance of 1 centimetre. This is called the *erg*.

Since the weight of 1 gramme is 981 dynes, the work done in raising 1 gramme through a height of 1 centimetre against the force of gravity is 981 ergs. This unit is exceedingly small, and the *joule*, equal to 10^7 ergs, is used as the practical unit of work in the C.G.S. system.

This unit of work is, however, very seldom used by British engineers, the unit employed being the *foot-pound* (ft.-lb.). The foot-pound is the work done when a body moves through 1 foot against a force equal to the weight of 1 pound.

Work = force × distance.
1 ft.-lb. = 13,825 gramme-centimetres.
= 1.356×10^7 ergs.
0.7373 ft.-lb. = 10^7 ergs = 1 joule.
7.233 ft.-lbs. = 1 kilogramme-metre.

Power is the rate at which work is done. This clearly introduces the idea of work done with reference to the time in which it is accomplished, whereas work itself has no reference to time. It is important to be able to distinguish between power, *i.e.* the rate of doing work, and the amount of work done.

The absolute unit of power is the *erg per second*, but the practical unit is the *foot-pound per minute*. The unit of power generally used by engineers is an arbitrary one established by James Watt, namely, a horse-power which is 33,000 ft.-lbs. of work per minute, or 550 ft.-lbs. per second.

$$\text{Horse-power (written H.P.)} = \frac{\text{ft.-lbs. of work done}}{\text{time in minutes} \times 33{,}000}.$$

1 H.P. = 7.46×10^9 ergs per second.
= 76 kilogramme-metres per second.
= 33,000 ft.-lbs. per minute.
= 550 ft.-lbs. per second.

Energy is the capacity for doing work. There are two kinds of energy—*Kinetic Energy* and *Potential Energy.*

Kinetic energy is possessed by all moving matter, and is the work a body can do in virtue of its motion. If a body of mass m units is moving with a velocity of v units, then the energy or units of work stored up in the mass

$$=\tfrac{1}{2}\,mv^2.$$

Potential energy is the work a body can do in virtue of its position in space. Thus when a body of mass m units is raised through a distance s units, the units of work stored up in it

$$=ms.$$

ELECTRICAL UNITS

The electrical units will now be considered. There are two systems derived from the fundamental units, one set being based upon the force exerted between two quantities of electricity, and the other upon the force exerted between two magnetic poles. The former set are called the electro-static and the latter the electro-magnetic units. The commercial units are derived from the latter system.

Unit Magnetic Pole.—Consider two magnet poles of which the strengths are m_1 and m_2, placed a certain distance apart *in air*. The mutual attraction or repulsion between the two magnets is proportional to the product of the pole strengths and inversely proportional to the square of their distance apart. This force may be written:

$$f=k\frac{m_1m_2}{d^2}$$

where k is a constant depending in value upon the units in which f, m, and d are measured. If the poles are equal in strength then $f=k\frac{m^2}{d^2}$, and the unit pole is so chosen that $m=1$ when f and d are both unity: the value of k will thus also be unity, and the above formula may be written:

$$f=\frac{m^2}{d^2}.$$

The unit magnetic pole is that which, when placed at a distance of 1 centimetre from a similar pole, is repelled with a force of 1 dyne (the medium between the poles being air).

Unit Strength of Magnetic Field.—A magnetic pole, situated in a magnetic field, experiences a definite force acting in a definite direction: the magnitude of the force is proportional to the pole strength (m) of the magnet and the strength (H) of the field, or

$$f=km\mathrm{H}.$$

The units in which f, m, and H are measured are so chosen that $k=1$; in which case $f=m\mathrm{H}$. Hence magnetic field has unit strength when a unit magnetic ole situated in the field is acted on by the force of dyne. Unit magnetic field exists 1 centimetre from unit pole.

Unit Potential-Difference, or Electromotive force (E.M.F.), is set up between the ends of a conductor, which moves through a magnetic field at such a rate that one line of force is cut per second. The practical unit of pressure is 100,000,000 times the absolute unit, and is called the *volt*.

$$1 \text{ volt} = 10^8 \text{ absolute units.}$$

$$1 \text{ micro-volt} = \tfrac{1}{1000000} = \frac{1}{10^6} \text{ of a volt.}$$

From the definition of E.M.F. it will be seen that when a conductor of length l centimetres cuts a magnetic field having a density of H lines of force per square centimetre at the rate of v centimetres per second, the electromotive force induced in the conductor is expressed by the equation

$$\mathrm{E}\ (\text{volts}) = \mathrm{H}\times l\times v\times 10^{-8}.$$

Unit Current is that current which flows in a conductor of unit length bent so as to form an arc of a circle 1 centimetre in radius, and acts upon unit pole at the centre of the circle with a force of 1 dyne. The practical unit of current is $\frac{1}{10}$ of the absolute unit, and is called the *ampère.*

$$1 \text{ ampère} = 10^{-1} \text{ absolute units.}$$

$$1 \text{ milliampère} = \frac{1}{1000} \text{ ampère.}$$

Unit Resistance is the resistance of a conductor in which unit current is produced by unit potential difference between its ends. By Ohm's law*, the resistance of a conductor

$$= \frac{\text{fall of pressure in conductor}}{\text{current flowing in conductor}}$$

$$= \frac{\text{volts}}{\text{ampères}}.$$

But 1 volt = 10^8 absolute units of pressure,
and 1 ampère = 10^{-1} ,, ,, current;
therefore the practical unit of resistance, named the *ohm,* $= \frac{10^8}{10^{-1}} = 10^9$ absolute units. The ohm is often denoted by the symbol ω.

$$1 \text{ microhm} = \frac{1}{10^6} \text{ohm} = 10^{-6} \text{ ohm.}$$

10^6 ohms = 1 megohm; a term employed in specifying the insulation resistance of cables or electrical machinery, and denoted by the symbol Ω.

Unit Quantity of electricity is that which passes round a circuit in 1 second when a current of 1 absolute unit flows. The ampère being $\frac{1}{10}$ of this current, the practical unit of quantity is likewise a $\frac{1}{10}$ of the absolute unit, and is called the *coulomb.*

$$1 \text{ ampère-hour} = 3600 \text{ coulombs.}$$

Unit Force is that force exerted on a conductor of unit length and carrying unit current when it is placed in, but at right angles to, a field of unit intensity.

---

* See p. 12.

Hence if H denote the strength of field, and i the current in absolute units, then the force acting on a centimetres is given by

$$\text{Force (dynes)} = H \times l \times i.$$

ower.—Power has been defined as the rate ork is done, and is measured by the number vork done per second. If in t seconds a lectricity Q is conducted against a potential , the work done per second $= \dfrac{QE}{t}$.

But $Q/t =$ current, hence
$$\text{Power} = C \times E.$$

G.S. units power is measured in ergs per

Vatt is the practical unit of power and

$0^8 \times 10^{-1} = 10^7$ absolute units
0^7 ergs per second = 1 joule per second.

product of electromotive force in volts, and ampères, gives the power in watts.

$$W = E \times C.$$

As the value of the watt is very small and inconvenient to denote the power of large dynamos, the practical unit of power is 1000 watts, and is called the *Kilowatt*, usually written k.w.

$$\text{Kilowatts} = \frac{E \times C}{1000}.$$

Now 1 H.P. $= 7.46 \times 10^9$ ergs per second, and 1 watt $= 10^7$ ergs per second.

Therefore 1 H.P. $= \dfrac{7.46 \times 10^9}{10^7} = 746$ watts, and

1 k.w. $= \dfrac{1000}{746}$ H.P. $= 1.34$ H.P. $\approx 1\frac{1}{3}$ H.P.

This is an important result, as it forms a connecting link between the mechanical and electrical units.

The French horse-power (*Force de cheval*) = 736 watts.

Unit Energy is expended by 1 watt in 1 second, or by a coulomb flowing through a conductor in which there is a difference of potential of 1 volt. The watt-hour is a more convenient unit, and is the energy expended in 1 hour by 1 ampère flowing along a conductor under 1 volt pressure.

The *Kilowatt-hour* or *Board of Trade Unit* is the legal unit of electrical energy, fixed by the Board of Trade for public supply purposes, and is the quantity of energy supplied in 1 hour by a current at such a pressure that the product of volts, ampères, and hours equals 1000.

The Board of Trade Unit, written B.T.U., equals

$$\frac{\text{ampères} \times \text{volts} \times \text{hours}}{1000}.$$

1 B.T.U. = 1 k.w. hour = 1.34 H.P. hours.

Unit Capacity is that of a condenser which will be at unit difference of potential when charged with unit quantity. The practical unit is the *Farad*, and equals 10^{-9} absolute units.

In order to thoroughly familiarise the student with the above units a few examples are worked out, showing how the knowledge already acquired may be applied to practical problems. After carefully studying each of these examples there should be no difficulty in working through the problems on this chapter at the end of the book.

Example.—A weight of 2.5 lbs. is raised vertically through a distance of 20 feet in 30 seconds. Calculate

(1) Force exerted in dynes,
(2) Work done in joules,
(3) Power in watts to do this work.

(1) Force in dynes = weight in grammes × 981
$= 2.5 \times 453 \times 981$
$= 1,112,000.$

(2) Work done in joules—

$$= \frac{\text{work in ergs}}{10^7}$$

$$= \frac{\text{force in dynes} \times \text{distance in centimetres}}{10^7}$$

$$= \frac{1,112,000 \times 20 \times 12 \times 2.54}{10^7}$$

$$= 67.8.$$

(3) Power in watts = joules per second

$$= \frac{67.8}{30}$$

$$= 2.36.$$

Example.—A hoist raises 15 cwts. through 400 feet. Calculate the work done.

$$\text{Work done} = \text{weight in lbs.} \times \text{distance in feet}$$
$$= 15 \times 112 \times 400$$
$$= 672,000 \text{ ft.-lbs.}$$

If the work has to be done in 2 minutes find the H.P. of the motor required to drive the hoist, assuming the gearing has an efficiency of 75 per cent.

H.P. to raise the weight—

$$= \frac{\text{ft.-lbs. of work done}}{\text{time in minutes} \times 33,000}$$

$$= \frac{672,000}{2 \times 33000} = 10 \text{ H.P.}$$

Allowing 75 per cent. for efficiency of gearing,

$$\text{H.P. of motor} = 10 \times \frac{100}{75} = 13.4.$$

The motor has an efficiency of 87 per cent. and is connected to a 220 volt supply. Calculate the current taken from the mains at full load.

$$\text{Watts output at motor pulley} = 13.4 \times 746.$$

Watt input from electrical supply—

$$= 13.4 \times 746 \times \frac{100}{87}.$$

$$\text{Current} = \frac{\text{watts}}{\text{volts}} = \frac{13.4 \times 746 \times 100}{220 \times 87}$$

$$= 52 \text{ ampères.}$$

Example.—A dynamo gives an output of 100 ampères at 500 volts. Find the H.P. of engine to drive it, assuming the dynamo to have an efficiency of 90 per cent.

$$\text{Watts output} = 100 \times 500 = 500{,}000.$$

$$\text{Watts input} = 50{,}000 \times \frac{100}{90} = 55{,}500.$$

$$\text{H.P. of engine} = \frac{55{,}500}{746} = 75.$$

If the above dynamo gives the full load for 6 hours, find the number of Board of Trade units generated.

$$\begin{aligned}\text{Board of Trade units} &= \frac{\text{volts} \times \text{ampères} \times \text{hours}}{1000}\\ &= \frac{500 \times 100 \times 6}{1000}\\ &= 300.\end{aligned}$$

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CHAPTER II

FUNDAMENTAL PRINCIPLES

RESISTANCE

OHM'S LAW.—When a current flows through a conductor a definite resistance is offered by it to the passage of the current, and consequently there must be a drop or loss of pressure in that conductor the value of which depends upon the resistance offered. The relation between the electromotive force E and the current C was first enunciated by Ohm in 1827 and is called *Ohm's Law.* Ohm found by experiment that the ratio of E to C was constant so long as the physical state (temperature, etc.) of the conductor remained the same. The constant ratio between the electromotive force and the current is called the resistance of the conductor, and representing it by R, Ohm's law may be stated algebraically as follows:

$$E/C = R.$$

In words, Ohm's law states that the resistance of a given conductor, when its physical state is maintained constant, is the ratio of fall of E.M.F. in a conductor to the current flowing through it. It is of importance to observe that the quantity R defined by the above equation is independent of the strength of the current flowing through the conductor. The value of R will be investigated in this chapter, and depends upon the sectional area, length of the conductor, the material of which it consists, and upon the physical state of the material. The most searching investigations have been made as to the truth of this law when currents are passed through metals or electrolytes; these have all failed to discover any exceptions to it. The law, however, does not hold when currents are passed through gases.

Example.—A circuit consists of 50 16-c.p. lamps in parallel, each lamp taking 4 watts per candle. If each lead has a resistance of 0.25 of an ohm, find the voltage of the dynamo so that the pressure at the lamps may be 200 volts.

Figure 1 represents the circuit.

Since each lamp gives 16 c.p. and takes 4 watts per candle, the watts absorbed by 50 lamps = 4 × 16 × 50.

By definition of power, watts = E × C.

Therefore current taken by the lamps

$$= \text{watts}/E$$

x=

$$= \frac{4 \times 16 \times 50}{200} = 16 \text{ ampères.}$$

This is also the value of the current flowing in the leads. The next part of the problem is to obtain the voltage drop in the leads.

By Ohm's law E = CR, R in this case being the resistance of both leads and = 2 × 0.25 = 0.5 of an ohm. Therefore the voltage drop in the leads = 16 × 0.5 = 8 volts. Now the pressure at the lamps = 200 volts, hence the voltage of the dynamo = 200 + 8 = 208 volts.

Specific Resistance.—Materials used in electrical engineering may be divided into two classes—(*a*) those which are good conductors of electricity, and (*b*) those which are bad conductors; the former are simply called *conductors* and the latter *insulators*. Conductors provide paths along which electricity can flow, and insulators are used for preventing it from flowing along any other path simultaneously. As there are no perfect conductors, all offering some resistance to the flow of electricity, similarly

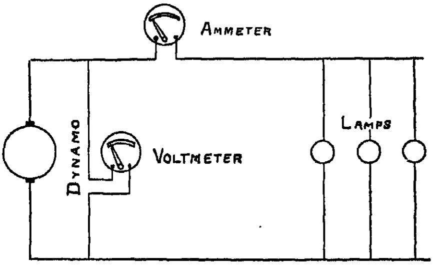

Fig. 1.

there are no perfect insulators, the best insulators known having some conductivity.

The resistance of metallic conductors will now be considered. The starting-point in the calculation of resistance is the knowledge of the resistance offered between two opposite faces of a material, having unit length and unit area of cross-section. This is known as *Specific Resistance*, and varies for different materials. Taking the centimetre as the unit of length and the square centimetre as the unit of area, the specific resistance of a material may be defined as the resistance offered by a wire of that material having a length of one centimetre and a cross-sectional area of one square centimetre, or, as the resistance between the opposite faces of a centimetre cube of the material. Specific resistance is sometimes defined in terms of the inch and square inch, but the former method is the more often used. Table I. gives the specific resistance of some of the conductors employed in electrical engineering, the resistance being given in microhms per centimetre cube. The student is here cautioned against the practice of defining the specific resistance as so many microhms per cubic centimetre or cubic inch. This would imply that the resistance varies as the volume, which, as will be seen directly, is quite erroneous.

From the table it will be seen that silver is the best conductor, being followed closely by copper. The latter substance is in most general use as a conductor of electricity, and its specific resistance should be committed to memory, as it is continually required in calculations. The specific resistance of liquids is very variable, its value depending upon the density. Column 2 gives the relative resistance of conductors, that of annealed copper being taken as unity.

Calculation of Resistance.—For a conductor of a given material under constant conditions of temperature, the resistance is directly proportional to the length and inversely proportional to the area of cross-section. Hence if l be the length, a the area of cross-section, and ρ the

TABLE I

TABLE OF SPECIFIC AND RELATIVE RESISTANCES OF CONDUCTORS

Substance.	Specific Resistance in microhms per centimetre cube at 0° C.	Relative Resistance.
Metals—		
Silver (annealed)	1.468	0.940
Copper (annealed)	1.561	1.0
Copper (hard drawn)	1.592	1.020
Aluminium (99 per cent. pure)	2.563	1.642
Aluminium (97.5 per cent. pure)	2.665	1.707
Zinc	5.751	3.684
Iron (pure)	9.065	5.806
Iron (Telegraph)	14.910	9.552
Platinum	10.917	6.995
Nickel	12.323	7.893
Tin	13.048	8.358
Tantalum	16.000	10.25
Lead	20.380	13.050
Mercury	94.070	60.26
Alloys—		
German silver ($Cu+Zn+Ni$)	30.01	19.21
Manganin ($84\ Cu+12\ Mn+4\ Ni$)	42.92	27.50
Platinoid (German silver + 1 per cent. Tungsten)	41.73	26.73
Carbon (glow lamp)	40×10^2	2,500
„ (arc lamp)	65×10^2	4,200
Liquids (at 15° to 20° C.)—		
Water (ordinary distilled)	$72{,}000 \times 10^3$	
Sulphuric acid (10 per cent.)	850×10^3	
Copper sulphate (saturated)	$25{,}000 \times 10^3$	

specific resistance of the material, the resistance R is given by the equation

$$R = \frac{\rho \times l}{a}.$$

If ρ be in ohms per centimetre cube, then l and a must be in centimetres and square centimetres respectively, and the resistance R will be in ohms. The same expression will hold good if l be in inches, a in square inches, and ρ in ohms per inch cube. Since 1 inch = 2.54 centimetres, and 1 square inch = 6.45 square centimetres, the specific resistance in ohms per inch cube

$$= \frac{\text{specific resistance in ohms per centimetre cube} \times 2.54}{6.45}.$$

Therefore the value of ρ for copper in ohms per inch cube at 0° C.

$$= \frac{1.592 \times 10^{-6} \times 2.54}{6.45} = .63 \times 10^{-6}.$$

Electrical conductors are generally made of circular cross-section. This is merely a matter of convenience, as the shape of the cross-section does not affect the conductivity. For a conductor of circular cross-section the area $a = \frac{\pi d^2}{4}$ where d is the diameter.

Then $R = \frac{\rho \times l \times 4}{\pi d^2}$

or $d = \sqrt{\frac{\rho \times l \times 4}{\pi R}}$ where R and ρ are in ohms and d and l in centimetres.

Example.—Find the resistance of a copper wire 2000 yards long if it has a cross-sectional area of 0.15 square inch.

Whether the specific resistance is expressed in ohms per centimetre cube or in ohms per inch cube is immaterial, the final result in both cases being the same. Thus (1) taking $\rho = 1.592 \times 10^{-6}$, say 1.6×10^{-6} ohms per centimetre cube,

$$R = \frac{\rho \times l}{a}$$

l = 2000 yards = 2000 × 36 × 2.54 centimetres

a = 0.15 square inch = 0.15 × 6.45 square centimetres

$$\text{Therefore } R = \frac{1.6 \times 10^{-6} \times 2000 \times 36 \times 2.54}{0.15 \times 6.45}$$

$$= 0.3 \text{ of an ohm.}$$

(2) Taking $\rho = 0.63 \times 10^{-6}$ ohms per inch cube, l and a must be reduced to inches and square inches.

l = 2000 yards = 2000 × 36 inches

a = 0.15 square inch

$$\text{Therefore } R = \frac{0.63 \times 10^{-6} \times 2000 \times 3}{0.15}$$

$$= 0.3 \text{ of an ohm.}$$

Example. — Find the area of cross-section and diameter of a copper wire to have a resistance of 0.13 ohms per kilometre.

$$\text{Since } R = \frac{\rho \times l}{a}$$

$$a = \frac{\rho \times l}{R}$$

Taking $\rho = 1.6 \times 10^{-6}$ ohms per centimetre cube
then $l = 10^5$ centimetres.

$$\text{Therefore } a = \frac{1.6 \times 10^{-6} \times 10^5}{0.13}$$

$$= 1.22 \text{ square centimetres.}$$

$$\text{Now area} = \frac{\pi d^2}{4} \quad \therefore \; d = \sqrt{\frac{4a}{\pi}}$$

$$d = \sqrt{\frac{4 \times 1.22}{\pi}} = 1.25 \text{ centimetres.}$$

Diameter of wire = 1.25 centimetres.

Example.—Find the area of cross-section of a cable 1000 metres long, to transmit 500 ampères so that the total drop in pressure along the cable may not exceed 25 volts. Take $\rho = 1.6 \times 10^{-6}$ ohms per centimetre cube.

By Ohm's law the resistance of the cable

$$= \frac{\text{drop in volts}}{\text{current flowing}} = \frac{25}{500} = 0.05 \text{ ohm.}$$

$$\text{Now } R = \frac{\rho \times l}{a} \quad \therefore \quad a = \frac{\rho \times l.}{R}$$

$$l = 10^5 \text{ centimetres.}$$

$$\therefore \text{ area } = \frac{1.6 \times 10^{-6} \times 10^5}{0.05} = 3.2 \text{ square centimetres.}$$

Variation of Resistance with Temperature. — In stating the specific resistance of a material the temperature at which it was obtained is also given, the reason for this being that all metals increase in resistance with rise of temperature. The variation in specific resistance with temperature for some metals, as determined by Professors Fleming and Dewar, is shown in Figure 2. The specific resistance was measured for temperatures between −200°C. and +200° C. If the curves be produced towards the origin, as shown by the dotted lines, they all tend to cut the axis of abscissae at a point corresponding to $-273°$ C. From these curves it would appear that at absolute zero of temperature the resistance of all pure metals would be zero. This leads up to the ***temperature coefficient of resistance***, which is the increase in resistance of a material having an original resistance of 1 ohm when the temperature has increased 1° C. and is denoted by the Greek letter α. If R_0 be the resistance of a metal at 0° C., then if the temperature increases to $t°$ C., the increase in resistance $= R_0 \alpha t$. So that the total resistance R_t is given by the equation

$$\begin{aligned} R_t &= R_0 + R_0 \alpha t \\ &= R_0(1 + \alpha t). \end{aligned}$$

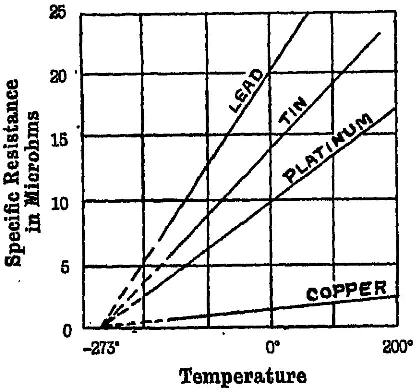

FIG. 2.

Since the resistance at 0° C. is seldom determined, the last equation may be written in another form. If R_1 be the resistance at some temperature t_1, and R_2 the resistance at a higher temperature t_2

$$\text{then } R_2 = R_1\{1 + \alpha(t_2 - t_1)\}.$$

For all pure metals, the temperature coefficient α has almost the same value, namely, about 0.004. Table II. gives the temperature coefficient of various conductors, the values holding good when the temperatures are between the limits of 0° and 100° C.

It is important to notice that carbon has a negative temperature coefficient. This phenomenon is also shown by most non-metals. In the case of electrolytes the specific resistance falls rapidly with increase of temperature, and the coefficient ranges from 0.2 per cent. to 2 per cent. per degree centigrade. This extremely important effect of temperature on the resistance of

TABLE II

TEMPERATURE COEFFICIENT OF VARIOUS CONDUCTORS

Conductor.	Temperature coefficient per ° C.
Silver	0.004
Copper	0.00428
Aluminium	0.00423
Zinc	0.00406
Iron	0.00625
Platinum	0.00367
Nickel	0.00622
Tantalum	0.003
Mercury	0.00072
German silver	0.000273
Platinoid	0.000310
Manganin	0.000001
Electrolytes	Negative temp. coefficient
Carbon	−0.00052

substances causes some substances to be more suitable for particular purposes than others.

Specific Resistance and Temperature Coefficient of Alloys.—As far as their electrical properties depend on their constituent metals, alloys can be put into two classes. Alloys containing lead, tin, or zinc have a specific resistance which can be calculated from that of the constituent metals, knowing the proportions in which each is present. Thus an alloy having equal quantities of lead and tin, has a specific resistance equal to the mean of the specific resistance of the constituent metals.

In the case of most other metals the specific resistance of the alloy is much greater than that calculated in this manner. Not only is the specific resistance of such an alloy greater than that of the constituents, but the temperature coefficient is less than that of the constituents. For instance, the alloy German silver, having a specific resistance of 30.01 microhms per centimetre cube, and a temperature coefficient 0.000273, consists of the elementary metals copper, zinc, and nickel, the specific resistances of which are respectively 1.592, 5.751, and 12.323 microhms per centimetre cube, and the temperature coefficients 0.00428, 0.00406, and 0.00622 respectively.

This is a very important property of alloys from the point of view of the construction of standard resistances and shunts, for the smaller the temperature coefficient of the material used, the less its susceptibility to temperature variations. It is found that by using manganese as a constituent of an alloy it is possible to prepare a material of which the temperature coefficient at ordinary temperatures is either zero or very nearly so. Manganin is one of these alloys.

Example.—The field magnet coils of a shunt-wound dynamo have a resistance of 160 ohms at 15° C.; what will be their resistance at 35° C.? The temperature coefficient of copper = 0.0043.

$$R_2 = R_1\{1 + \alpha(t_2 - t_1)\}$$

then substituting

$$\begin{aligned} R_2 &= 160\{1 + 0.0043(35 - 15)\} \\ &= 160\{1 + 0.0043 \times 20\} \\ &= 160 \times 1.086 \\ &= 174 \text{ ohms.} \end{aligned}$$

Resistance at 35° C. = 174 ohms.

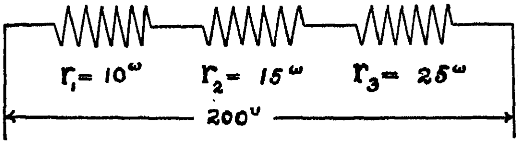

FIG. 3

Resistance of Systems of Conductors—(1) *In Series.*—If several conductors be joined in series, as shown in Figure 3, the resistance R of the arrangement is equal to the sum of the resistances of the separate conductors. If $r_1\ r_2\ r_3$ . . . denote the resistance of each conductor, then $R = r_1 + r_2 + r_3 + \ldots$ In Figure 3 there are three conductors, so that $R = r_1 + r_2 + r_3$.

(2) *In Parallel.*—When a number of conductors are connected in parallel, as shown in Figure 4, the reciprocal of the total resistance R is equal to the sum of the

reciprocals of the resistances of the separate conductors. If $r_1\, r_2\, r_3$ . . . be the resistance of each conductor respectively, then $\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\ldots$ In Figure 4 there are three conductors, so that $\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$.

When there are two circuits of resistance, r_1 and r_2 in parallel, $\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}$ or $R=\frac{r_1 r_2}{r_1+r_2}$.

If there be n circuits in parallel, each of the same resistance, then

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+$$

$$\ldots \frac{1}{r^n}=\frac{n}{r}$$

$$\text{or } R=\frac{r}{n}.$$

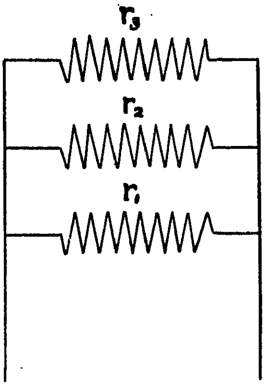

FIG. 4.

Example.—Three coils, having a resistance of 10, 15, and 25 ohms respectively, are connected in series across 200 volts. What is the current flowing through the coils, and the voltage drop in each coil?

Figure 3 gives the diagram of connections for this circuit.

By Ohm's law $C=\frac{E}{R}$.

Now $E=200$ volts, and $R=r_1+r_2+r_3=10+15+25$
$=50$ ohms.

Then the current flowing $=\frac{200}{50}=4$ ampères.

Volts drop in $r_1=Cr_1=4\times 10=$ 40 volts
" " " $r_2=Cr_2=4\times 15=$ 60 "
" " " $r_3=Cr_3=4\times 25=$ 100 "
Total volts = 200.

Example.—In the previous example, if the three coils be placed in parallel, what will be the combined resistance?

$$\text{Now } \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{10} + \frac{1}{15} + \frac{1}{25} = \frac{31}{150}$$

$$\therefore\ R = \frac{150}{31} = 4.8 \text{ ohms.}$$

The resistance of the 3 coils in parallel is 4.8 ohms.

EFFECTS OF ELECTRIC CURRENTS

Heating Effect of the Electric Current.—The conversion of energy from an electrical form into that of heat takes place when a current flows through a conductor, the incandescent lamp being a particular example. The power, in watts, absorbed by a circuit is the product of the current in ampères and the fall of pressure in volts in that circuit. This may be expressed by the equation $W = EC = C^2R$, since $E = CR$. The energy in joules absorbed by this circuit in t seconds is equal to $ECt = C^2Rt$, since a joule equals a watt per second. If no mechanical work be done then all this energy is transformed into heat, and in doing this a certain amount of energy is expended.

There is a relation between the energy expended and the heat produced. This was first investigated by Joule in 1843. The method consisted in measuring the amount of mechanical work expended in stirring a known weight of water in a calorimeter, and in measuring the rise in temperature of the water. The relation is such that if 4.2×10^7 ergs, *i.e.* 4.2 joules, of work are done, a quantity of heat is developed sufficient to raise 1 gramme of water 1° C. This is known as the *calorie*, or the French unit of heat. (The British heat unit is the quantity of heat required to raise 1 lb. of water 1° F., and is equivalent to about 780 ft.-lbs. of work.)

Now C^2Rt is also a measure of joules. Hence the quantity of heat H developed in t seconds in a circuit of which the resistance is R ohms and current C ampères, is given by

$$H = \frac{C^2Rt}{4.2} \text{ calories}$$
$$= 0.24\ C^2Rt \text{ calories.}$$

This is known as *Joule's Law*, and the heating effect as the Joule effect. It may be written thus. The heat generated in a simple circuit is proportional to the product of the square of the current into the resistance and the time during which the current continues to flow.

It is important to remember that the heat developed is proportional to C^2, as this principle is applied to several pieces of apparatus and measuring instruments used in electrical engineering.

The heating effect of the electric current has now been utilised for heating and cooking purposes. Electric radiators are of the luminous and non-luminous types. A luminous radiator is merely a large lamp, and, as will be seen in the chapter on incandescent lamps, the energy transformed into heat is about 95 or 96 per cent., the remaining energy, between 4 and 5 per cent., being transformed into light. In the non-luminous radiator the current is passed through resistance wires embedded in cement or other suitable insulating material.

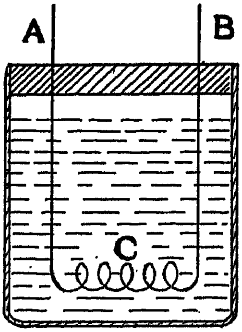

FIG. 5.

Figure 5 shows an arrangement for heating a liquid. The ends, A and B, of an open coil C of a high resistance wire are connected to a source of electrical supply, and the coil is immersed in the liquid contained in a vessel. The heat generated raises the temperature of the liquid; some of the heat, however, is radiated from the sides of the vessel to the surrounding air, and consequently the efficiency is reduced to between 65 and 85 per cent. The time taken to raise the temperature to a definite value will depend upon the power absorbed, and as an exercise in calculation some examples will be worked.

Example.—An electric heater has an efficiency of 85 per cent., and works on a 230-volt circuit, taking 1.5 ampères. How long will it take to raise the temperature of 906 grammes of water to the boiling point if the initial temperature be 7° C.?

Calories in the water at boiling point

$$= \text{mass in grammes} \times \text{temperature rise in } ^\circ\text{C.}$$

$$= 906 \times 93.$$

Power supplied in calories $= 0.24\ ECt$

$$= 0.24 \times 230 \times 1.5 \times t.$$

$$\text{Now } 906 \times 93 \times \frac{100}{85} = 0.24 \times 230 \times 1.5 \times t$$

∴ time t taken to boil the water

$$= \frac{906 \times 93 \times 100}{230 \times 1.5 \times 85} = 1200 \text{ seconds}$$

$$= 20 \text{ minutes.}$$

Example.—If a water-heating vessel absorbs 1.5 ampères at 260 volts for 10 minutes in order to heat 0.5 kilogramme of water from 15° C. to 100° C., what is the efficiency of the apparatus?

$$\text{Efficiency} = \frac{\text{calories in water at end of 10 minutes}}{\text{calories input}}.$$

Calories in water at end of 10 minutes $= \text{mass} \times \text{temp. rise}$

$$= 500 \times (100 - 15) = 500 \times 85.$$

Calories input $= 0.24\ E \times C \times t$

$$= 0.24 \times 260 \times 1.5 \times 10 \times 60$$

∴ efficiency of apparatus

$$= \frac{500 \times 85}{0.24 \times 260 \times 1.5 \times 10 \times 60}$$

$$= 75 \text{ per cent.}$$

Chemical Effect of the Electric Current.—It has been shown that, when an electric current passes through a metallic conductor, a certain quantity of heat is developed in the conductor, but, after the conductor has become heated due to the passage of the current, in general there will be no chemical or physical change. Besides metals, some liquids known as electrolytes conduct electricity, and the phenomena accompanying the passage of a current through these liquids will now be considered. Almost all liquids are compounds of two or more elements. Water, for instance, is a compound of the two elements hydrogen and oxygen in the proportion, by volume, of two of hydrogen to one of oxygen. Water is therefore denoted symbolically H_2O. Again, copper sulphate, composed of 1 part of copper (Cu), 1 part of sulphur (S), and 4 parts of oxygen (O_4), is symbolically expressed by $CuSO_4$.

When a current is passed through an electrolyte by inserting two plates into the solution (the plate connected to the positive pole of the supply mains being known as the anode and the other connected to the negative main, the cathode), decomposition of the electrolyte takes place. The products of decomposition, whether they be elements or compounds, will be liberated either at the anode or at the cathode. The elements or groups of elements liberated are called *ions*: the ion liberated at the anode is termed the anion, and that liberated at the cathode is termed the cation. The ions, such as the elements chlorine, iodine, and a number of acidic groups or salt radicals, such as SO_4, which appear at the anode, are negatively electrified or convey negative electricity. The ions, such as hydrogen and the metals which travel to the cathode, are positively electrified or convey positive electricity.

Faraday deduced the laws which govern electrolysis, and hence they are known as *Faraday's Laws*, and are as follows:

(1) The mass of any ion liberated is proportional to the quantity of electricity that has passed through.

(2) The mass of any ion liberated by a given quantity of electricity is proportional to the chemical equivalent of the ion.

The Chemical Equivalent of an element is the weight of it which will combine with or replace 1 part by weight of hydrogen, and is numerically equal to the ratio of the atomic weight of the element to that of hydrogen divided by the valency: the valency of an element being the number of hydrogen atoms which will combine with, or are replaced by, one atom of the element.

The Electro-Chemical equivalent of an element is the weight in grammes which is deposited by the passage of unit quantity of electricity, *i.e.* one coulomb. Investigations show that, when a current of one ampère passes through a solution of silver nitrate for one second, the weight of silver deposited is 0.001118 grammes. The chemical equivalents of silver and hydrogen are 107.66 and 1 respectively, hence the electro-chemical equivalent of hydrogen is $0.001118 \times 1/107.66 = 0.00001038$. As another example, in the case of copper as a cuprous salt, the chemical equivalent of copper is 63.0, hence the electro-chemical equivalent of copper is $0.00001038 \times \frac{63.0}{1} = 0.0006539$. In a similar manner the electro-chemical equivalent of any other element may be determined.

Table III. gives for a number of elements their atomic weights, valency, chemical equivalent, and electro-chemical equivalent; the latter being the weight in grammes liberated by one coulomb. Some elements, such as mercury, have different chemical equivalents in different compounds, so that the contituents of the compound must be known. The last column gives the approximate grammes deposited in one hour by one ampère.

Now it has been shown that the weight of an element liberated per second by one ampère is given by the electro-chemical equivalent usually denoted by the letter e, so that if C be the current flowing for t seconds, then the total weight W (in grammes) deposited is expressed by $W = e \times C \times t$.

This is a very important relation, as it forms a means of measuring a current in ampères when that current is known to deposit a certain weight in a known time. Thus

$$C = \frac{W}{e\,t}.$$

The apparatus used for this purpose is known as a

TABLE III

TABLE OF ELECTRO-CHEMICAL EQIVALENTS

Element.	Symbol.	Atomic weight.	Valency.	Chemical equivalent.	Electro-chemical equivalent (grammes per coulomb).	Approximate grammes deposited in one hour by one ampere.
Electro-positive—						
Copper (cuprous) .	Cu	63.0	1	63.0	0.0006539	2.3665
Hydrogen .	H	1.0	1	1.0	0.00001038	0.037368
Mercury (mercurous) .	Hg	199.8	1	199.8	0.002075	7·470
Potassium .	K	39·04	1	39·04	0.0004054	1·4594
Silver . .	Ag	107.66	1	107.66	0.001118	4.0250
Sodium . .	Na	22.99	1	22.99	0.0002387	0.8593
Copper (cupric)	Cu	63.0	2	31.5	0.0003271	1.1832
Iron (ferrous) .	Fe	55·9	2	27·95	0.0002902	1.0436
Lead . .	Pb	206.4	2	103.2	0.001072	3·857
Mercury (mercuric) .	Hg	199.8	2	99·9	0.001037	3·733
Nickel . .	Ni	58.6	2	29·3	0.0003043	1.0993
Zinc. . .	Zn	64·9	2	32.45	0.0003370	1.2113
Aluminium .	Al	27·3	3	9·1	0.00009450	0·3355
Electro-negative—						
Chlorine . .	Cl	35·37	1	35·37	0.0003673	1.3212
Iodine . .	I	126.53	1	126.53	0.001314	4·7304
Oxygen . .	O	15·96	2	7·98	0.00004850	0.29808

voltameter, a sketch of which is shown in Figure 6. When the electrolyte is copper sulphate the two plates A and B are of copper and form the electrodes. If A be the positive plate, *i.e.* the plate by which the current enters the electrolyte, then the current flows through in the direction of the arrow, and copper will be deposited on the plate B. The solution used is a 15 per cent. solution of copper sulphate, to which about 2 per cent. of sulphuric acid is added to prevent the formation of basic sulphate. The cathode B should be sufficiently large to allow 50 square centimetres of surface per ampère.

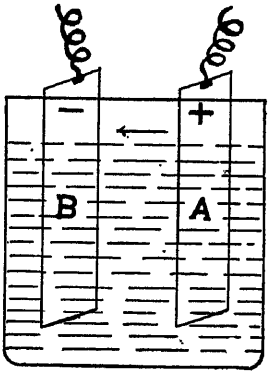

FIG. 6.

The principles of electrolysis are applied to electro plating where the metal to be deposited in the form of a suitable salt in aqueous solution forms the electrolyte. The object to be plated is made the cathode, that is, it is suspended in the liquid and is connected to the negative pole of the dynamo or battery. The anode consists of a strip of the metal to be deposited.

These principles have also been employed in the separation of the elements such as aluminium, sodium, and potassium.

Example.—What weight of copper will be deposited in a copper voltameter by a current of 10 ampères flowing for 1 hour?

From Table III. the electro-chemical equivalent of cupric copper = 0.0003271.

$$\begin{aligned} \text{The weight deposited} &= e \times C \times t \\ &= 0.0003271 \times 10 \times 1 \times 3600 \\ &= 11.8 \text{ grammes.} \end{aligned}$$

Example.—Taking the electro-chemical equivalent of silver as 0.001118, calculate the current required to deposit a coating of silver 0.1 millimetre thick to a surface of 1000 square centimetres in one hour.

Density of silver = 10.5.

Volume of silver deposited = 1000 × 0.01 = 10 cubic centimetres.

Mass of silver deposited = 10 × 10.5 = 105 grammes.

1 coulomb deposited 0.001118 grammes

$$\therefore \text{ the current required} = \frac{W}{e\,t}$$

$$= \frac{105}{0.001118 \times 3600} = 26 \text{ ampères.}$$

Example.—An ammeter was connected in series with a silver voltameter and a current sent through: at the end of 30 minutes a weight of 205 grammes was deposited. The current was maintained steady during the test and indicated 99 ampères. Did the ammeter read correctly; if not, what was the error?

The current passing through the ammeter and voltameter

$$= \frac{W}{e \times t} = \frac{205}{0.001118 \times 1800} = 101 \text{ ampères.}$$

Ammeter read low by 2 per cent.

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CHAPTER III

ELECTRO-MAGNETISM AND MAGNETISATION OF IRON

When a current flows through a circuit, magnetic lines of force are invariably set up around the circuit, causing it to act magnetically on any body in the vicinity. This is known as the magnetic effect of an electric current, but the reason why an electric current is accompanied by a magnetic field is a matter of hypothesis. This phenomenon is known as electro-magnetism, and forms the foundation of the science of electrical engineering. It is therefore of great importance, and before progress can be made in the study of electrical engineering the principles of electro-magnetism must be thoroughly understood, As an introduction some of the terms employed will be briefly defined.

Permeability.—As mentioned in the preceding chapter certain substances are better conductors of electricity than others, so, also, there are good and bad conductors of magnetic lines of force. *Permeability* is the term employed to express the relative magnetic conductance of materials. The permeability of air is taken as unity, and the relative permeability of other substances is usually expressed symbolically by the Greek letter μ. The permeability of non-magnetic materials is a little less than 1, but for practical purposes may be taken also as unity. For magnetic materials, *e.g.* iron, steel, nickel, and cobalt, the value of μ is much greater than 1, attaining a value as high as 900 for steel and 3600 in the case of soft iron.

Intensity of Magnetisation.—In a long bar magnet of length l, area of cross-section a, and ends having a pole strength m, the intensity of magnetisation I is the ratio of the magnetic moment of the magnet to its volume. The magnetic moment of a magnet is the product of its pole strength into the length of the magnet. Therefore the intensity of magnetisation is given by

$$\mathrm{I}=\frac{\text{magnetic moment}}{\text{volume of magnet}}=\frac{ml}{al}=\frac{m}{a}.$$

In other words, the intensity of magnetisation is the pole strength per unit area.

Lines of Force per Unit Magnetic Pole.—Suppose a magnetic pole of strength m units be isolated from all magnetic fields, and that the pole is the centre of an infinitely thin spherical surface of radius r, then the number of lines of force cutting unit area of the spherical surface $=\frac{m}{r^2}$. Now the area of a sphere $=4\pi r^2$, therefore the total number of lines of force cutting the sphere $=4\pi r^2\times\frac{m}{r^2}=4\pi m$.

If $m=1$, the number of lines $=4\pi$, that is unit magnetic pole has 4π lines of force emanating from it.

Magnetic Induction.—If a long thin rod of iron be placed in a magnetic field of intensity H in air with its length parallel to the lines of force of the field, then the total number of lines of force cutting unit area of the bar is given by

$$\mathrm{B}=\mathrm{H}+4\pi\frac{m}{a}=\mathrm{H}+4\pi\mathrm{I}$$

where $m=$ the pole strength developed at the end of the iron, a the area of cross-section of the iron, and I the intensity of magnetisation. The ratio $\frac{\mathrm{B}}{\mathrm{H}}$ gives the permeability μ of the iron, *i.e.* $\frac{\mathrm{B}}{\mathrm{H}}=\mu$—one of the fundamental equations of magnetism. H is the magnetising force and may be looked upon as a stress, while B, the number of lines of force passing through unit area is the resultant strain. B is termed the *Induction density* in lines per square centimetre or square inch, and may be as high as 16,000 per square centimetre in cast steel or wrought iron, but seldom exceeds 9000 per square centimetre for cast iron. The total number of lines of force N passing through a magnet having an area of cross-section a is given by $N = Ba$,

$$\text{so that } B = \frac{N}{a},$$

N being termed the *total magnetic flux.* For instance, if the total number of lines of force passing through a magnet be 70,000, and the area of cross-section 7 square centimetres, then the induction density is given by

$$B = \frac{N}{a} = \frac{70{,}000}{7} = 10{,}000 \text{ lines per square centimetre.}$$

MAGNETIC FIELD PRODUCED BY AN ELECTRIC CURRENT

Magnetic Field surrounding a Conductor.—When a direct current traverses a straight conductor, the conductor is surrounded along its entire length by concentric circles of lines of force, as shown in Figure 7, in which AB is the conductor. The direction of the lines of force depends upon the direction in which the current flows. Referring to the figure, if the current flow from A to B, the direction of the lines of force is as indicated by the arrow heads, but if the current flow from B to

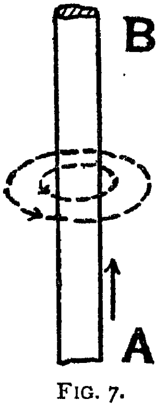

FIG. 7.

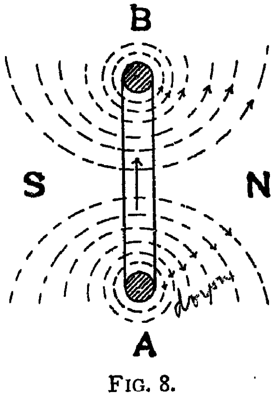

FIG. 8.

A, then the direction of the lines of force will also be reversed. This is the starting-point in the theory of electro-magnetism.

The magnetic effect, when a wire carrying current is bent into a circular loop, is shown in Figure 8, which is a vertical section. The direction of the current in the part of the loop shown is indicated by the arrow, and the field by the dotted lines, the direction being indicated by the arrow-heads. At A the lines of force are in a clock-wise direction, but in a counter-clock direction at B. As all the lines of force enter into one face of the loop and leave at the other, the loop may be considered a magnet having north and south poles, as indicated by N and S. If C be the current in ampères flowing in a loop of radius r centimetres, then the magnetic force H at the centre of the loop

$$= \frac{2\pi C}{10r} \text{ C.G.S. units.}$$

Magnetic Field of a Solenoid.—Consider several loops of wire side by side in a straight line carrying current. This is best effected by coiling up a length of insulated wire into a number of turns. Such an arrangement is called a solenoid, and Figure 9 shows a vertical section through one consisting of 3 turns. The arrows indicate the direction in which the current is flowing, and lines of force are set up around each conductor as shown by the dotted lines. Inside the solenoid all the lines of force due to each coil have a common direction, and the resultant field is in the direction SN ; outside

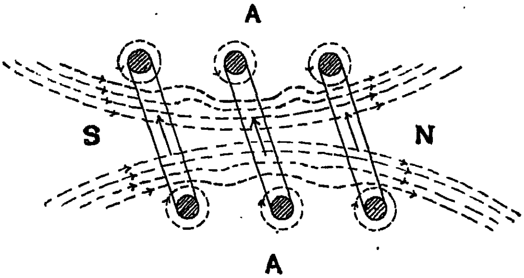

FIG. 9.

the solenoid the direction is NAS. Thus the lines of force flow through the solenoid from S to N, and emanating from the end N, complete their paths outside the coil and re-enter again at the end S. There is a crowding of the lines of force inside the solenoid and a spreading outside, as indicated in Figures 9 and 10.

Magnetising Force.—Referring to Figure 10, let l be the length of the solenoid in centimetres, T the number of turns, and C the current in ampères. If the coil be wound uniformly on a cylinder of non-magnetic material the magnetic force inside the cylinder parallel to the axis is given by the equation

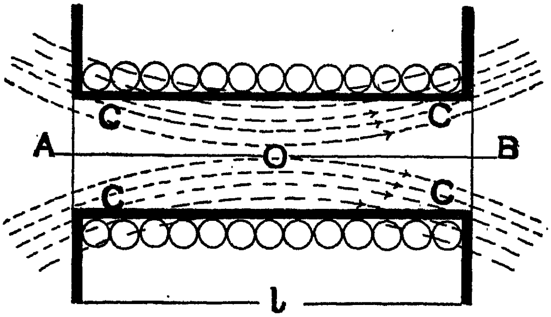

FIG. 10.—Magnetic field due to a solenoid.

$$H = 4\pi n i \text{ C.G.S. units}^*$$

where n is the turns of wire per unit length of the cylinder and i is the current flowing in absolute units.

In this case $i = \frac{C}{10}$ and $n = \frac{T}{l}$ so that

$$H = \frac{4\pi CT}{10\,l}$$

but $\frac{4\pi}{10} = 1.25$, therefore $H = 1.25\,\frac{CT}{l}$.

CT is usually considered as one term and is referred to as the *ampère-turns*, and for a given solenoid H is a constant for all combinations of C and T which make CT a constant. The magnetising force H will be the same for two coils having the same dimensions, but one having 100 turns and carrying 10 ampères, while the other has only 50 turns but carries 20 ampères, the product being 1000 in both cases. The above formula is theoretically correct only for solenoids which are very long compared with their diameter, but in practical applications it is sufficiently accurate for any solenoid. The magnetic field inside a solenoid is strongest at its centre O. Since, however, there is no iron to guide the lines of force, they leak out sideways at or near the ends. The field is therefore more dense at the sides CC than near the axis AB, and consequently the field is stronger at the sides CC than at AB. This latter property of the solenoid is made use of to effect the movement of the needle in a class of indicating instrument of the moving soft-iron type.

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* For proof see J. J. Thomson's *Electricity and Magnetism*, 3rd edition, chap. x. pp. 340–345.

The interior of the solenoid being of air, the value of μ is unity; therefore since $\frac{B}{H} = \mu$, $B = H$. That is, inside a coreless solenoid the value of H is always the number of lines of force per square centimetre.

THE MAGNETIC CIRCUIT

Lines of force are continuous and unbroken throughout their entire length, and for the sake of convenience are treated as if they have a material existence and flow in definite numbers along definite paths. The paths followed by lines of force are called magnetic circuits, for the same reason that the path taken by an electric current is called the electric circuit. Iron, steel, nickel, and cobalt are, as already stated, the best conductors of magnetic lines of force, but for the magnetic circuits in practical machines and instruments, iron and steel alone are used, being cheaper and more easily worked than the other metals named.

There is a remarkable similarity between the electric and magnetic circuits, although the latter is not so well defined as the former. Leakage from the main circuit takes place in each; in one case there is a leakage of lines of force, and in the other a leakage of current.

The laws of leakage are the same, namely, that leakage takes place across the paths of least resistance. The total flux generated in a magnetic substance depends both on the magnetising force and the permeability of the material.

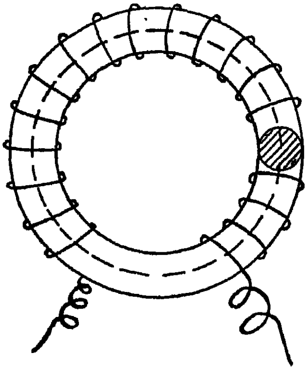

FIG. 11.

The simplest case of the magnetic circuit is an iron ring of circular cross-section, as shown in Figure 11. Suppose the ring to have a mean length of l centimetres, an area of cross-section of a square centimetres, and be wound with a magnetising coil of T turns. Then, when a current of C ampères flows through the coil, the magnetising force is given by

$$H = 1.25\,\frac{CT}{l}.$$

The magnetic flux through the iron $= N = Ba$, but $B = \mu H$, therefore $N = \mu H a$

$$= \frac{\mu \,.\, 1.25\ CTa}{l} = \frac{1.25\ .\ CT}{\frac{l}{a}\,.\,\frac{1}{\mu}}.$$

This expression is analogous to the expression for Ohm's law in the electric circuit.

By Ohm's law, current $= \dfrac{\text{electromotive force (E.M.F.)}}{\text{resistance}}$

and from the previous equation

number of lines of force $= \dfrac{\text{magnetomotive force (M.M.F.)}}{\text{reluctance}}$

Where magnetomotive force $= 1.25$ CT
Number of lines of force $= N$
Reluctance $= \dfrac{l}{a}\,.\,\dfrac{1}{\mu}.$

The equation for reluctance is also comparable with the equation for the resistance of a wire. In the case of the resistance of a wire

$$\text{Resistance} = \frac{\rho \times l}{a} = \frac{\text{specific resistance} \times \text{length}}{\text{area of cross-section}}.$$

In the magnetic circuit

$$\text{Reluctance} = \frac{l}{a} \cdot \frac{1}{\mu}.$$

The reciprocal $\frac{1}{\mu}$ is termed the reluctivity of the magnetic substance

$$\therefore \text{ reluctance} = \frac{\text{reluctivity} \times \text{length}}{\text{area of cross-section}}.$$

Reluctivity might, therefore, be defined as the magnetic resistance offered to magnetic flux between two opposite faces of a unit cube of the substance, but this definition is never employed, as the reluctivity of a substance varies with the flux density B, whereas the specific resistance of a conductor is independent of the current flowing through it.

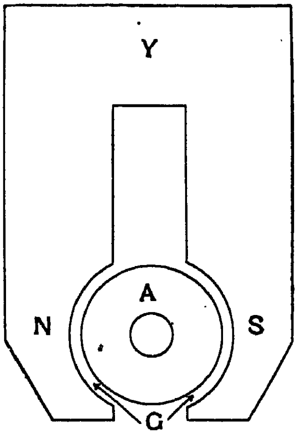

FIG. 12.—Magnetic circuit of a two-pole dynamo.

In practical cases the magnetic circuit generally consists of a number of paths in series, each part having a different reluctance. Figure 12 represents the magnetic circuit of a two-pole dynamo, in which there are indicated the following parts —(1) the armature A; (2) the air-gap G; (3) the poles N and S; and (4) the yoke Y. Since each part of the circuit is in series, the total reluctance is obtained by adding together the reluctances of the different portions. Then if

$l_1\ l_2\ l_3$ . . . be the lengths of respective parts,
$a_1\ a_2\ a_3$ . . . be the areas of respective parts,
and $\mu_1\ \mu_2\ \mu_3$ . . . be the permeabilities of respective parts,

the general equation for the total reluctance is given by

$$\text{Reluctance} = \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} + \quad . \quad . \quad .$$

$$\text{Therefore N} = \frac{1.25 \text{ CT}}{\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} + \quad . \quad . \quad . \right)}$$

This formula was introduced by the late Dr. John Hopkinson, and is applied in a modified form to calculations relating to the magnetic circuit of dynamos and electro-magnetic mechanism. In practical problems the ampère-turns CT is generally the unknown, and for determining its value the equation may be written as follows:

$$\text{CT} = \frac{\text{N} \left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} + \quad . \quad . \quad . \right)}{1.25}$$

$$\text{or} \quad \text{CT} = 0.8\left(\frac{\text{N}l_1}{a_1\mu_1} + \frac{\text{N}l_2}{a_2\mu_2} + \frac{\text{N}l_3}{a_3\mu_3} + \quad . \quad . \quad . \right)$$

$$\text{or} \quad \text{CT} = 0.8\left(\frac{\text{B}_1 l_1}{\mu_1} + \frac{\text{B}_2 l_2}{\mu_2} + \frac{\text{B}_3 l_3}{\mu_3} + \quad . \quad . \quad . \right)$$

where B_1, B_2, B_3 . . . are the induction densities of each part. If the symbol Σ denote the sum of all such terms as $\frac{\text{B}l}{\mu}$, the above equation may be written in the form

$$\text{CT} = 0.8 \, \Sigma \, \frac{\text{B}l}{\mu}.$$

Magnetic Leakage.—In the electric circuit it is comparatively easy to confine the current to a definite path, because some materials have practically infinite resistance. In the magnetic circuit, however, air and the other so-called non-magnetic materials have a permeability which, at the very best, is as high as $\frac{1}{2500}$ that of iron. There is thus no possibility of magnetically insulating any circuit to anything like the extent attainable in the electric circuit; consequently, leakage lines must be allowed for. This leakage of lines attains to appreciable dimensions in dynamos and other apparatus where the magnetic circuit is composite in character. Suppose a flux of N_a lines is required to pass through the armature of a dynamo; since the lines of force have to cross an air-gap, a considerable amount of leakage will take place. A flux equal to N_m lines must therefore be generated in the magnet, so that $N_m = N_a +$ leakage lines. Dr. Hopkinson presented this fact by stating that N_m lines through the magnet were v times the lines through the armature, *i.e.* $N_m = vN_a$. The ratio N_m/N_a is termed the *coefficient of magnetic leakage.* Hence

$$v = \frac{\text{total flux generated}}{\text{useful flux}}.$$

The value of v varies between 1.1 and 1.5, depending upon the permeability of the leakage paths.

Example.—A solenoid 20.5 centimetres long is wound on a brass former and has an internal diameter of 10 centimetres. It is wound to the depth of 2.5 centimetres with 2500 turns of wire, and carries 1 ampère. Find (1) the magnetising force, and (2) the flux at the centre of the solenoid.

(1) The magnetising force $H = \frac{1.25\ CT}{l}$.

Now $l = 20.5$ centimetres.

$C = 1$ ampère $\quad T = 2500$

$$\therefore H = \frac{1.25 \times 1 \times 2500}{20.5} = 153.$$

(2) Since the coil has no iron core

$$B = H$$

and the flux at the centre of the solenoid $= N = Ba$.

Now $a = \frac{\pi d^2}{4}$ where d is the mean diameter of the solenoid and $= \frac{10+15}{2} = 12.5$ centimetres;

thus $a = \frac{\pi \times 12.5^2}{4} = 126$ square centimetres.

$$\therefore N = B \times a = 153 \times 126$$
$$= 19{,}300 \text{ lines.}$$

Example.—A cast-iron ring has a cross-section of 4 square centimetres and a length of 20 centimetres. Calculate the ampère-turns required to drive a flux of 32,000 through the iron, permeability of the iron being 100. Figure 11 shows a diagram of such a circuit.

Starting with the fundamental equation

$$H = \frac{1.25\,CT}{l}$$

$$\text{the ampère-turns} = CT = \frac{Hl}{1.25} = 0.8Hl$$

$$= 0.8\,\frac{Bl}{\mu} = 0.8\,\frac{Nl}{a\mu}$$

$$= \frac{0.8 \times 32{,}000 \times 20}{4 \times 100} = 1280.$$

Example.—In the previous example find the additional ampère-turns if an air-gap 0.5 centimetre wide be cut in the iron ring. The coefficient of leakage v may be taken as 1.2.

From the above the ampère-turns $= CT = \frac{0.8Nl}{a\mu}$.

In the air-gap $N = \frac{32{,}000}{v} = \frac{32{,}000}{1.2}$

$u = 1$, $l = 0.5$, and $a = 4$, the same as the iron, this being on the assumption that no spreading of the lines of force takes place in the air-gap. Figure 13 shows the ring with the air-gap introduced, the leakage lines also being indicated.

The additional ampère-turns to drive the flux through

$$\text{the air-gap} = \frac{0.8 \times 32{,}000 \times 0.5}{4 \times 1 \times 1.2} = 2660.$$ — 1280 × 5
20

This example shows very clearly the effect of introducing a small air-gap into the magnetic circuit, namely, that the ampère-turns required to drive the flux through the magnetic circuit must be considerably increased, so that to be economical the air-gap in all magnetic circuits should be as small as possible.

Example.—An iron ring 3 square centimetres in cross-section and 50 centimetres in mean circumference is wound with a coil of 100 turns. It is found that a current of 8 ampères produces a flux of 40,000 lines.

Find the permeability of the iron.

Now (1) $H = 1.25 \dfrac{CT}{l}$

and (2) $H = \dfrac{B}{\mu} = \dfrac{N}{a\mu}$

therefore $\dfrac{N}{a\mu} = 1.25 \dfrac{CT}{l}$

then the permeability $\mu = \dfrac{N}{a} \cdot \dfrac{l}{1.25\ CT}$

$$= \frac{40{,}000 \times 50}{3 \times 1.25 \times 8 \times 100}$$

$$= 660.$$

MAGNETISATION CURVES

The manner in which the induction density B of a magnetic material varies with the magnetising force H will now be examined. Unfortunately, these quantities are not connected by any simple relation, but the most convenient mode of studying them is to plot corresponding values of B and H on squared paper, taking values of H as abscissæ and values of B as ordinates. The curve obtained by joining up the various points is known as the magnetisation or B-H curve. Methods of measuring B and H are described on pages 46 to 52, but for the present it will be assumed that both quantities have been obtained. Figure 14 represents the

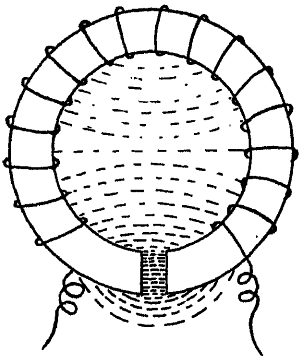

FIG. 13.

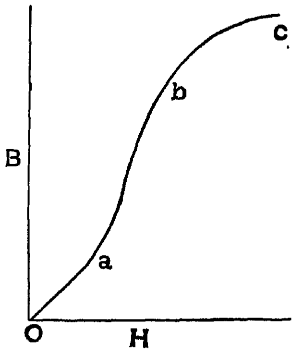

FIG. 14.

characteristic form of curve obtained with soft iron or steel. In the first part of the curve, between O and *a*, the induction B gradually increases as the magnetising force increases; between *a* and *b* a small increase in H causes B to increase very rapidly. After the point *c* has been reached the curve becomes almost flat, showing that a large increase in H produces a very small increase in B. The part *b c* is known as the knee of the curve, and at the point *c* the iron is said to be practically saturated. Any further increase in the induction above this is due to the coil alone.

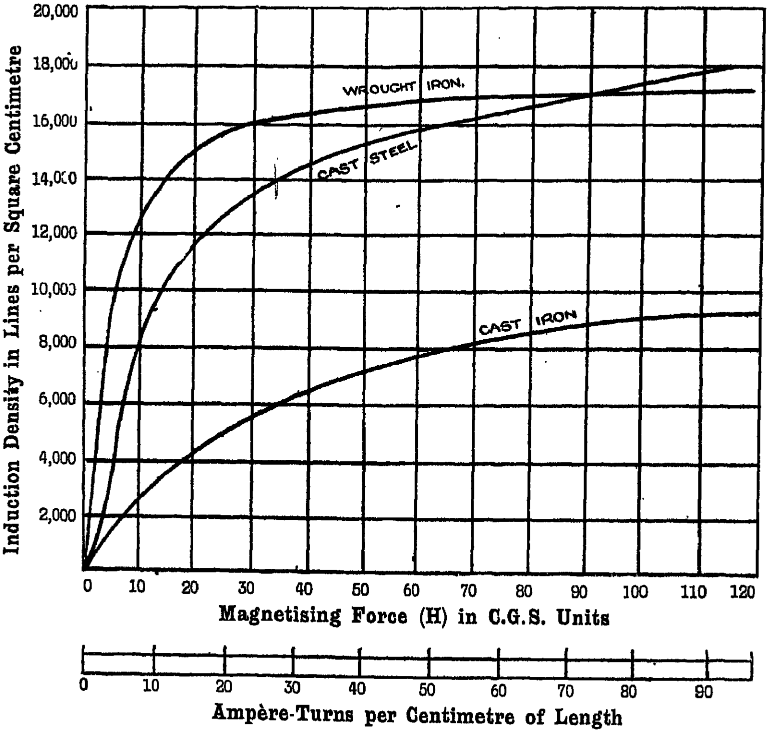

FIG. 15.—Magnetisation curves for iron and steel.

Figure 15 gives the B-H curves for wrought iron, cast steel, and cast iron, these being the magnetic materials invariably employed. It should be noted that wrought iron and cast steel give a much higher induction for a given value of H than cast iron. Reference to these curves shows that it is not economical to work wrought iron or cast steel at greater inductions than 16,000 lines per square centimetre, whereas the limiting induction for cast iron is about 9000 lines per square centimetre.

Since the magnetising force $H = 1.25 \frac{CT}{l}$ the ampère-turns per centimetre of length is given by

$$\frac{CT}{l} = \frac{H}{1.25} = 0.8\ H = 0.8\ \frac{B}{\mu}.$$

It is more convenient to express the induction density B of a circuit in terms of the ampère-turns per centimetre of length, and this relation is given for the curves in Figure 15. The ampère-turns per centimetre corresponding to particular values of H have been marked off along the abscissæ.

The makers of different brands of iron usually supply a curve showing this relation, and by making use of it laborious calculations are avoided; in fact these curves are invariably used in calculations relating to magnetic circuits. In order to make the use of these curves perfectly clear an example will be worked.

Example.—The yoke of a two-pole dynamo is of cast steel and carries a flux of 1.12 million lines; the area of cross-section is 80 square centimetres and the length 40 centimetres. What is the value of the ampère-turns necessary to drive this flux through the yoke?

$$B = \frac{N}{a} = \frac{1,120,000}{80} = 14,000.$$

From the curve for cast steel in Figure 15 a value of $B = 14,000$ requires 28 ampère-turns per centimetre of length. Therefore the total ampère-turns to drive the flux through the yoke $= 28 \times 40 = 1120$.

At any point on these curves the ratio of B to H gives the value of the permeability μ, and the relation between μ and B for wrought iron is shown in Figure 16. The permeability rapidly increases to a maximum value of 2400, corresponding to $B = 6000$; with higher values of B the permeability diminishes, and with very high inductions, such as those occurring in the teeth of armatures, the permeability may be as low as only 10 or 20 times that of air.

The value of the permeability, and, consequently, the shape of the B-H curve, also depends upon the chemical and physical state of the iron. In general all substances mixed with or alloyed with iron lower the permeability. In steel and cast iron it would appear that the decrease in the permeability is proportional to the amount of carbon present; carbon in the uncombined forms lowers the permeability less than when combined. Figure 17 shows curves for three different grades of cast iron, carbon being present in the following quantities:

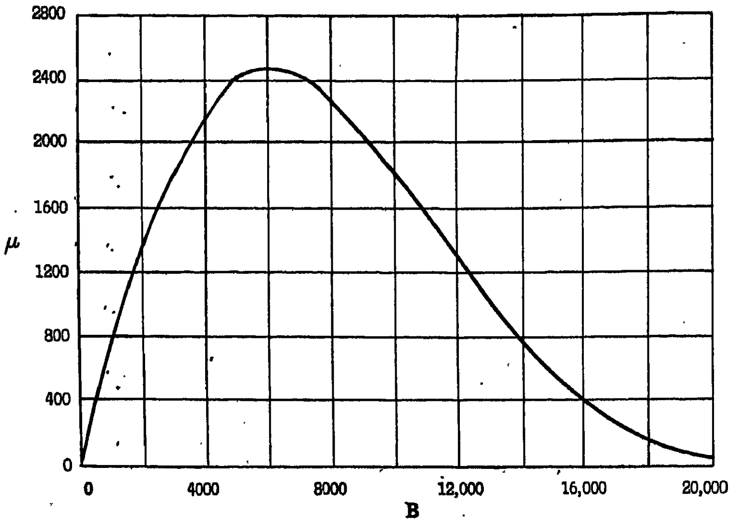

FIG. 16.—Permeability curves for wrought iron.

Sample.	Per cent. of combined carbon.	Per cent. of uncombined carbon.
A	0.78	...
B	0.565	2.98
C	0.198	3.29

These curves clearly show the effect of combined carbon upon the permeability. The cast iron in general use for magnetic purposes would be between the curves B and C.

In cast iron and steel such substances as aluminium and silicon, which tend to give softness and homogeneity to the metal, increase the permeability when present in limited quantities, say from 2 to 2.5 per cent. Figure 18 represents some of the curves from Professor Barrett's paper on "Magnetic Properties of Iron Alloys."* A, B, and C are the curves for (i) pure Swedish charcoal iron, (ii) iron containing 2.5 per cent. silicon, and (iii) iron containing 2.25 per cent. aluminium. Curve C shows the remarkable increase in permeability due to the addition of 2.25 per cent. aluminium to the pure iron.

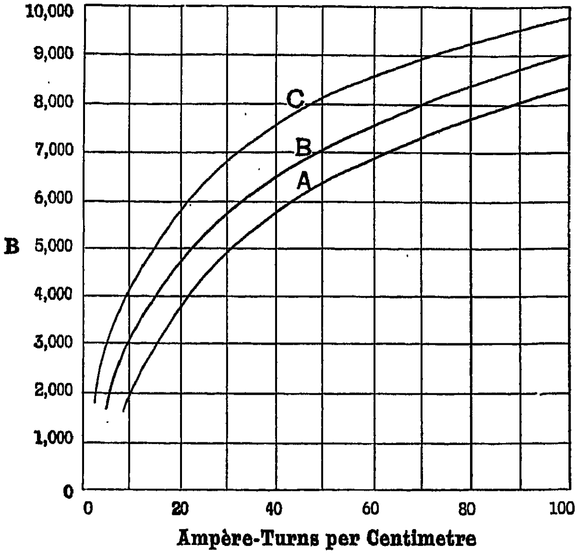

FIG. 17.—Curves for iron containing different percentages of combined and uncombined carbon.

Rate of cooling also affects the permeability of the iron, the permeability being considerably lowered if cooling takes place too rapidly. On the other hand annealing increases the permeability. Temperature also affects the permeability, but as this effect is hardly noticeable within the range of temperatures that occur in practice, it need not be further considered.

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* *Journal of Institution of Electrical Engineers*, vol. xxxi. pp. 672–721.

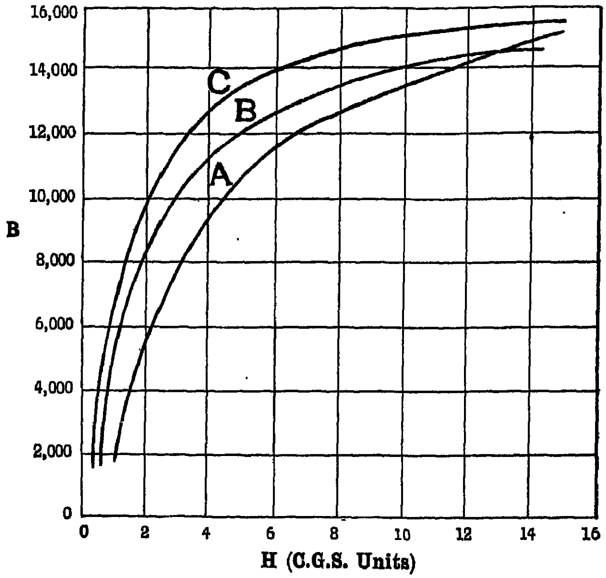

FIG. 18.—Magnetisation curves for—

A. Pure Swedish charcoal iron.
B. Iron containing 2.5 per cent. silicon.
C. Iron containing 2.25 per cent. aluminium.

METHODS OF MEASURING PERMEABILITY

In order to determine the permeability of a substance, in general, the induction density B corresponding to a given value of H must first be measured and the permeability obtained from the equation $\mu = \frac{B}{H}$. Various instruments have been devised with a view to making the permeability tests as simple as possible, but the most accurate, and also the most laborious, is that known as the Absolute or Ballistic method, which will now be described.

Ballistic Method.—In this method, advantage is taken of the fact that if the magnetic flux through any circuit (*e.g.* a coil of wire) is suddenly altered, a current is induced in the circuit during the change, the total quantity of electricity induced being proportional to the change in the magnetic flux. By connecting a ballistic galvanometer in series with the conducting circuit the quantity of electricity induced can be measured, and knowing the galvanometer constant the change in the lines of force can be computed. The value of the magnetising force H required to produce the change in the induction can be calculated by knowing the number of the turns per centimetre of length and the current flowing through the magnetising coil.

Referring to Figure 19, the sample of iron to be tested is made in the form of a ring and wound uniformly with a magnetising coil P, which is connected through a reversing switch Sw, regulating resistance R_1, and an ammeter A to a battery or other source of E.M.F. B. A search coil S is wound on the ring and is connected in series with a variable resistance R_2 to the ballistic galvanometer G.

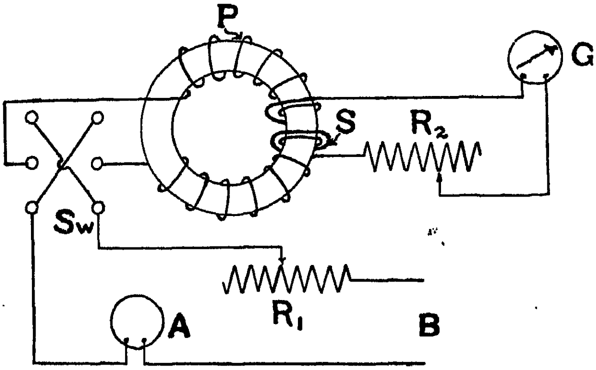

FIG. 19.—Diagram of connections for ballistic method of testing iron.

The value of the magnetising force H produced by the coil P is obtained directly from the formula $H = \frac{1.25\ CT}{l}$, where C = current in ampères flowing through the coil, and $\frac{T}{l}$ = turns per centimetre length of the coil.

The induction density B corresponding to a given value of H is obtained by suddenly reversing the current in the magnetising coil by means of the switch Sw, and noting the deflection of the galvanometer. Let N = total number of lines of force passing through the ring at the instant of reversal, and A = area of cross-section of the iron, then the induction density $B = \frac{N}{A}$. Now the E.M.F. induced in the search coil is proportional to the rate of cutting of lines of force, and is given by the equation

$$E = \frac{N_1 m}{t} \times 10^{-8} \text{ volts.}$$

Where N_1 = total change in lines of force
t = time in seconds during which the change takes place
m = turns on the search coil.

The current flowing in the search coil circuit is given by

$$C \text{ (ampères)} = \frac{E \text{ (volts)}}{R \text{ (ohms)}}.$$

Since E = induced E.M.F. $= \frac{N_1 m}{t} . 10^{-8}$

and R = resistance of galvanometer circuit, the current in ampères $= C = \frac{N_1 m . 10^{-8}}{t R}$.

The quantity of electricity flowing in the galvanometer $= Q = Ct = \frac{N_1 m . 10^{-8}}{tR} t = \frac{N_1 m . 10^{-8}}{R}$.

When the magnetising current is reversed the total change in the lines of force $= N_1 = 2N = 2BA$, therefore $Q = \frac{2BAm . 10^{-8}}{R}$.

If θ be the deflection of the galvanometer and K_1 the ballistic constant, *i.e.* the quantity of electricity required to give a deflection of one division, then $Q = K_1\theta$.

$$\text{That is, } K_1\theta = \frac{2BAm . 10^{-8}}{R}$$

$$\text{or } B = \frac{RK_1 10^8}{2Am} . \theta.$$

But R, K_1, and m are known quantities, and for a particular test can remain constant, in which case the equation may be written $B = K\theta$,

$$\text{where } K = \frac{R \cdot K_1 10^8}{2Am}.$$

So that by determining H and B as above the permeability μ of a specimen of iron is given by B/H.

The magnetising current C can be set to any desired value by means of the variable resistance R_1. The resistance R_2 in series with the galvanometer should be sufficient to keep the deflections within the scale limits, when the maximum change in the lines of force takes place.

For this test the ring should have a sectional area of about 3 square centimetres and a mean length of 30 centimetres. An inherent disadvantage of this method is that a special ring has to be forged, so that the ballistic test is very seldom used in practical work. It has been described here because it illustrates the application of a number of fundamental principles.

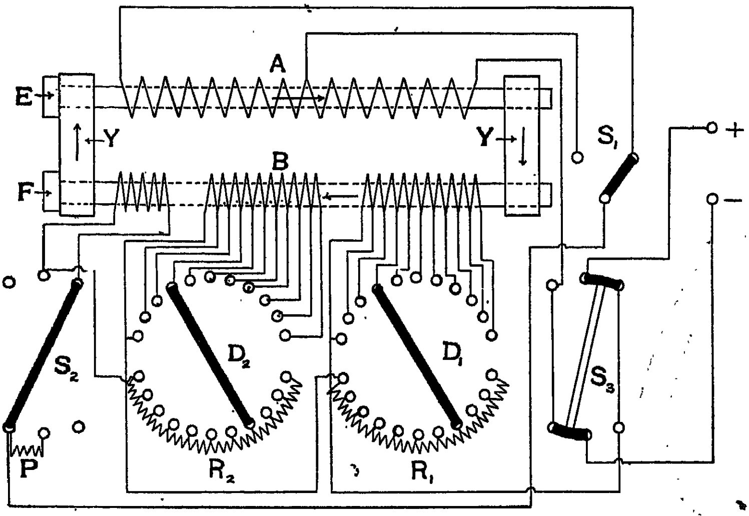

FIG. 20.—Diagram of connections for Ewing's permeability bridge.

Ewing's Permeability Bridge.—This instrument has been devised by Professor Ewing for measuring the permeability of a sample of iron by comparison with a standard test-piece, the B-H curve of which has been determined beforehand. Figure 20 gives a diagram of connections for the instrument. The bar F to be tested is turned to the same diameter as the standard bar E, and they are then placed inside two parallel magnetising coils, A and B respectively. The ends of the bars fit accurately into two soft-iron yokes Y, which make good magnetic contact between the bars by means of pinching screws. From the soft-iron yokes Y two long soft-iron horns H (Fig. 21) project vertically upwards and are so shaped that they almost meet at the top. In the gap between the ends D is placed a magnetic needle N, the sensibility of which depends on the position of the controlling magnet M. The two magnetising coils A and B are wound on brass formers and connected in series. The coil A, inside which is placed the standard bar E, consists of either 50 or 100 turns, depending upon the position of the switch S_1. The second coil B, surrounding the sample of iron to be tested, has 210 turns; of these 10 are controlled by the dial D_1, 100 by the dial D_2, and 100 by the switch S_2. By this arrangement the number of turns constituting the coil B may be varied. The coils A and B are so connected that the resultant magnetism forms a closed magnetic circuit through the two test bars, by way of the soft-iron yokes as shown by the arrows in Figure 20. + and – are the instrument terminals across which 6 or 7 cells are connected, the current taken from the cells being measured

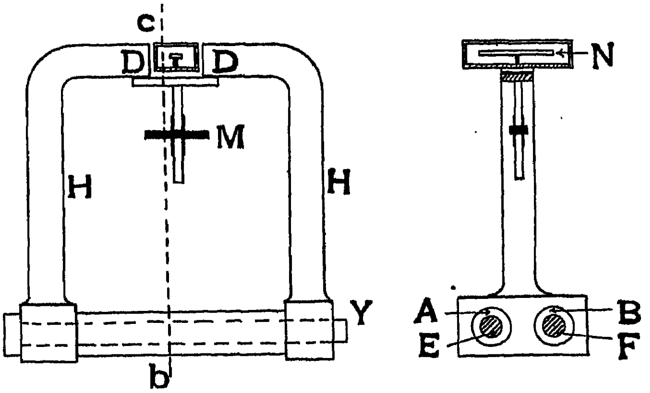

FIG. 21.—Construction of Ewing's permeability bridge.

y an ammeter. S_3 is the reversing switch for changing ıe direction of the magnetising current. To compensate ɔr the variation in resistance when the number of turns ɔrming the coil B are varied, an exactly equal resistance put into circuit by means of the dial switches. The ɛsistances are shown at R_1 and R_2 on the dials of D_1 and $)_2$ respectively. The resistance P is likewise introduced ·hen the 100 turns controlled by the switch S_2 is cut out f circuit.

The principle of the instrument is as follows:

When a current is passed through the two magnetising ɔils A and B a flux is set up in each bar, the fluxes ɛspectively depending upon the permeability of the bars. f the permeability of E and F be equal the flux set up ı each bar would be the same, and practically all the nes of force would flow in the path indicated by the rrows. If the permeability of E be greater than that f F, then the flux set up in E will be greater than that ı F, and there will be a magnetic difference of potential etween the two yokes, and leakage lines of force will ass through the horns and air-gap between DD: this auses the needle N to be deflected, and the amount of eflection is directly proportional to the number of ɛakage lines. In using the instrument the number of urns forming the coil B are varied until the needle N hows no deflection, in which case the leakage lines are liminated and the flux through the bar E equals the lux through F. Since the areas of E and F are the ame, then the induction density in E equals the induction density in F, *i.e.* $B_E = B_F$. Now $H_E = \frac{4\pi AT_E}{10l}$, but he clear length of each bar is 12.56 centimetres (4π), and he number of turns in the magnetising coil of the standard ɔar is 100. Hence the magnetising force due to coil A is 10 C.G.S. units for each ampère of current. This ıllows any required magnetising force to be easily applied, with the aid of the ammeter and adjustable resistance n series with the cells. Further, since the relation of B to H is known for the standard bar, a knowledge of he current is enough to show at what value of the nduction B the comparison is being made, and B is, of course, the same for both bars, when the condition of balance is produced. The number of turns on the bar F divided by 100 gives the ratio of the magnetising force required for that bar, to the known force applied to the standard. Because $H_F = \frac{4\pi AT_F}{10l} = \frac{4\pi AT_F}{10 \times 12.56}$ $= \frac{AT_F}{10}$, and from above $H_E = \frac{AT_E}{10} = \frac{A}{10} \cdot 100$

Therefore $\frac{H_F}{H_E} = \frac{T_F}{100}$, or $H_F = \frac{T_F H_E}{100}$.

Knowing H_E, B_E can be computed from the B-H curve of the standard, and therefore B_F, since $B_F = B_E$. Hence a point on the B-H curve of the iron under test is determined, and by changing the current, as many points as are desired may be found.

In order to get rid of residual magnetism effects in the test-pieces, yokes, and horns, the direction of the magnetising current is frequently reversed by means of the switch S_3. In testing iron of poor magnetic quality it is necessary, in order to obtain a balance, to use only 50 turns around the standard.

MAGNETIC HYSTERESIS

When a magnetic substance has been magnetised in a strong magnetic field it will be found to retain a considerable proportion of magnetism after the magnetising force has been removed. This, as will be shown, is a very important phenomenon. Take a sample of annealed wrought iron which has been previously demagnetised, then, when the magnetising force is first applied the induction B varies with H in a manner represented by the curve OA in Figure 22, which is similar in form to the curve for wrought iron in Figure 15. After having reached the point A on the ascending curve let the magnetising force be gradually reduced from A to O again. It will be found that the descending curve is not, by any means, identical with the ascending curve, but is considerably higher, as shown by the part AR. It will further be noticed that when the magnetising force is zero the induction B is not zero, but has a value represented by OR. The height OR represents the residual magnetic induction of the iron, *i.e.* the lines of force per square centimetre passing through the iron when the magnetising force has been entirely withdrawn. When the magnetising force is reversed, by reversing the direction of the current through the magnetising coil, the iron rapidly loses its magnetism, and a negative force, represented by OC, is sufficient to deprive the iron of all its magnetism. When the negative magnetising force is increased beyond OC the iron becomes negatively magnetised and reaches a maximum at the point D. At this point the induction B is of the same numerical value as at the point A, but of the opposite sign. After the magnetising force is again reversed it requires a positive force equal to OF to deprive the iron of its negative magnetism. Lastly, by increasing the magnetising force from O to the same positive maximum as at

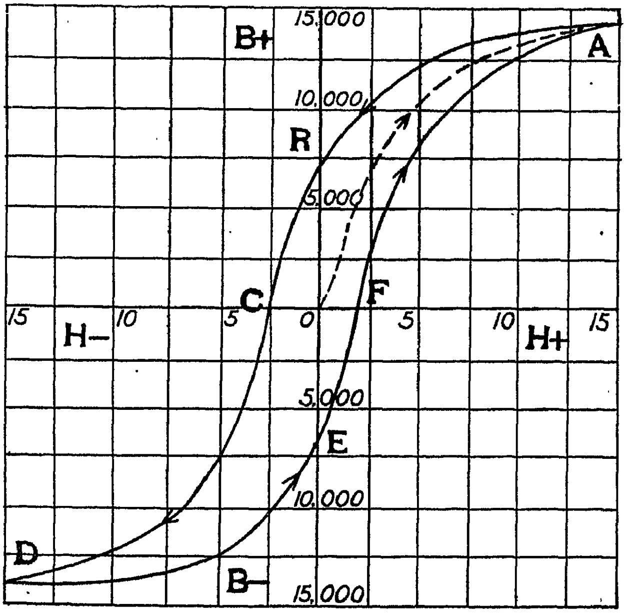

FIG. 22.—Hysteresis curve for wrought iron.

first, the curve EFA is obtained. A loop ARCDEFA has then been described, and is known as a complete magnetic cycle. If the cycle of operations be repeated a curve identical with ARCDEFA will be obtained.

Professor Ewing was the first to point out this lagging of the induction behind the magnetising force. The phenomenon is known as *magnetic hysteresis*, and the curves enclosing the area ARCDEFA is known as the B-H curve of hysteresis. On examining this curve it will be seen that when H is reduced to O, B still has a considerable value, OR; this is termed the *retentivity*. To reduce the induction to zero a negative or demagnetising force, represented by OC, has to be applied; this is called the *coercive force*. The return half DEFA of the curve is a repetition of the first half ARCD, and OE = OR, and OF = OC.

If a similar test be performed on a ring of sheet iron for armature cores, a loop such as is represented by ARCDEF in Figure 25 will be obtained. Every kind of magnetisable substance shows this phenomenon in varying degrees.

Dissipation of Energy through Magnetic Hysteresis.—One very important consequence of magnetic hysteresis is that changes of magnetisation involve a dissipation of energy. When, for instance, a piece of iron is magnetised, a certain amount of work is done in overcoming the molecular friction of the iron, *i.e.* work is done in the process of magnetisation, and if there were no energy retained by the iron it would return to its original state on withdrawing the magnetising force. This is known not to be the case; in fact, in order to demagnetise the iron a reverse force has to be applied, that is, work has to be done in demagnetising the iron. The law of the conservation of energy states that no energy can be lost, but is convertible from one form to another. Experiment shows that the energy retained by the iron is converted into heat, and the iron, or any other magnetisable substance which goes through magnetising and demagnetising cycles, becomes heated. Hence, whenever the phenomenon of magnetic hysteresis occurs, energy is expended and work done, and it will be shown that the work done is proportional to the area of the hysteresis loop.

The problem now is to find the work done when a sample of iron, having a length l and area of cross-section a, is subjected to a complete magnetic cycle. Let the magnetising coil consist of n turns per centimetre, so that the whole number of turns is ln. Suppose that the magnetic induction of the iron is increased by an infinitely small amount, dB, in an infinitely small interval of time, dt, by increasing the magnetising force by an infinitely small amount. Then the total lines of force through the iron are increased by adB, and the time rate of change of this increase is $a\frac{dB}{dt}$. This change in the lines of force will set up an opposing electromotive force in the coil, the value of which will be equal to the product of the total number of turns constituting the coil and the time rate of change in the lines of force, *i.e.* the total opposing E.M.F. $= ln \cdot a \cdot \frac{dB}{dt}$.

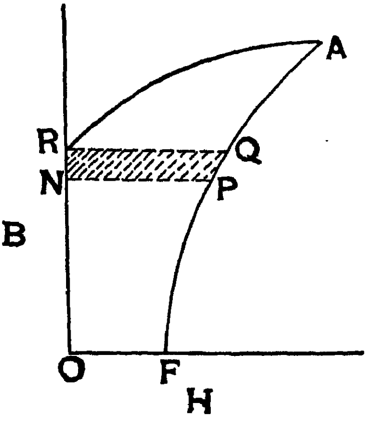

FIG. 23.

The work done in overcoming this opposing E.M.F.

$$= C \cdot dt \cdot lna \cdot \frac{dB}{dt} = nC \cdot la \cdot dB,$$

but $H = 4\pi nC$, and la is the volume of the iron, so that the work done per unit volume $= \frac{1}{4\pi} \cdot HdB$. In Figure 23 the product HdB is represented graphically as the area NPQR, and the work done per cubic centimetre of iron when B is changed by any finite amount by changing the magnetising force from a value H_1 to

H_2 is $= \frac{1}{4\pi}\int_{H_1}^{H_2} H\,dB$. Or the total work done by the cycle OFAR $= \frac{1}{4\pi}$ area OFAR. In a complete hysteresis loop, as is given in Figure 22, $\int H\,dB$ is given by the area of the loop. The work done per cycle per cubic centimetre of a substance is then given by the area of the hysteresis loop divided by 4π, *i.e.* $\frac{1}{4\pi}\int H\,dB$, and if H and B be in absolute units, the work done is expressed in ergs. The energy expended in doing this work is therefore derived from the magnetising current. The area of the hysteresis loop may be obtained accurately by means of a planimeter, but as an approximation may be taken as the value of the product of twice the retentivity into twice the coercive force. Referring to Figure 25, this would make the area ARCDEF approximately equal to $2OR \times 2OC = RE \times CF$.

The form and area of a hysteresis curve depend upon the kind of material, and the harder the physical state of the material the larger will be the area of the loop, and consequently the greater will be the hysteresis loss. In subsequent calculations of commercial apparatus it is more convenient to have the loss of energy due to hysteresis expressed in watts per kilogramme of material, and the relation between the two quantities may be obtained as follows:

Ergs per cubic centimetre per cycle

$$= \frac{\text{area of hysteresis loop}}{4\pi}$$

Watts per cubic centimetre per cycle per second

$$= \frac{\text{area}}{4\pi} \times 10^{-7}.$$

Since 1 cubic centimetre of sheet iron weighs 7.8 grammes, the watts absorbed per kilogramme per cycle per second

$$= \frac{\text{area}}{4\pi} \times 10^{-7} \times \frac{1}{7.8} \times 1000$$

$\approx 0.000001 \times$ ergs per cubic centimetre per cycle.

The ergs per cycle per cubic centimetre will depend upon the degree to which the iron is magnetised, so that for each specimen of iron a series of tests must be performed in order to determine the hysteresis loss at various inductions. This would be rather a tedious procedure, and Steinmetz has suggested the use of an empirical formula, in which the hysteresis loss is expressed in terms of the maximum induction B, and a constant η depending upon the kind of material.

The specific loss in ergs per cubic centimetre per cycle is given by $h = \eta B^{1.6}$. This formula is sufficiently correct for all practical purposes where B varies between 1000 and 14000. The values of η for different magnetic materials are given in Table IV. These values hold for inductions up to and just over the knee of the magnetisation curve. From the table it will be observed that for sheet iron employed in the construction of armature cores, η may be taken as $= 0.0025$ or 0.003, while for good transformer iron it may be as low as 0.0015.

TABLE IV

HYSTERESIS CONSTANT η FOR DIFFERENT MATERIALS

Material.	Hysteresis Constant η.
Sheet iron for transformer cores	0.0015 to 0.0025
,, ,, armature cores . . .	0.0025 to 0.003
Ordinary soft wrought iron	0.0032
Soft annealed cast iron	0.0045 to 0.0085
Tool steel (annealed)	0.009
Grey cast iron	0.015

Since 1 erg per second $= 10^{-7}$ watts, the watts lost per cubic centimetre per cycle per second $= 10^{-7} . \eta . B^{1.6}$. The watts per kilogramme per cycle per second

$$= 10^{-7} \times \eta \times B^{1.6} \times \frac{1}{7.8} \times 1000$$
$$= 0.000013\eta . B^{1.6}.$$

So that when K kilogrammes of iron are subjected to N magnetic cycles per second the total loss in watts

$$= 0.000013\eta \times B^{1.6} \times K \times N.$$

In Figure 24 is shown a curve connecting the watts expended per kilogramme per complete cycle per second with the maximum induction for iron used for armature stampings in which $\eta = 0.003$.

Hysteresis loss in iron may be produced in two ways: one when the magnetic force acting upon the iron passes through zero when changing from a positive to a negative, and the other when the magnetic force remains constant in value but varies in direction. The former is known as an alternating field, and takes place in the core of transformers; the latter is termed a revolving field, and it is the field an armature is subjected to when it revolves between the pole-pieces. The resultant hysteresis loss from these two causes is not by any means the same. Professor Baily has found that for low inductions an alternating field produces lower hysteresis loss than that of a rotating field, but at an induction of about 15,000 lines per square centimetre the loss due to a rotating field reaches a maximum and then rapidly diminishes if the induction be further increased. On the other hand, the loss due

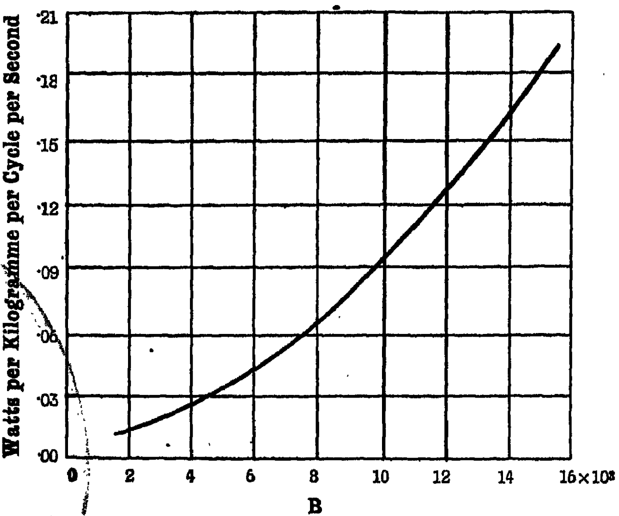

FIG. 24.—Hysteresis loss in watts per kilogramme for armature iron.

to an alternating field increases with the induction until the latter reaches about 22,000 lines per square centimetre. If the induction be increased beyond this the hysteresis loss ceases to increase, but shows no signs of diminishing, as in the case of the revolving field.

Example.—The ascending and descending values of B and H in half the hysteresis curve for a sample of armature iron are as follows:

Ascending—

B	1000	2500	5000	6000	7500
H	1.7	2.0	2.6	3.1	4.0

Descending—

B	7000	6000	5400	3000	0
H	2.3	0.5	0	− 1.2	− 1.7

From the above data find the watts absorbed by an armature having a volume of 5000 cubic centimetres of iron, revolving in a magnetic field, the induction density of which is 7500 lines per square centimetre, and the iron goes through 20 magnetic cycles per second.

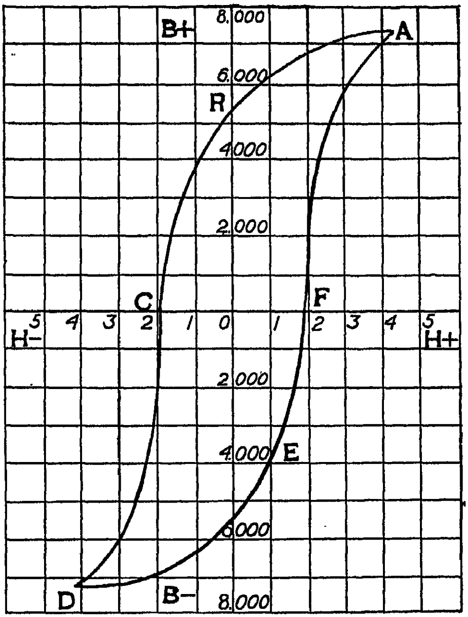

FIG. 25.—Hysteresis loop for armature iron.

Plotting the above values of B and H, the curve Figure 25 is obtained, the two halves of the curve being identical. Using a planimeter, it will be found that the curve has an area of 44,000 units.

Now, the ergs wasted per cubic centimetre per cycle

$$= \frac{1}{4\pi}\int H\,dB = \frac{\text{area of curve}}{4\pi}.$$

$$= \frac{44{,}000}{4\pi} = 3500.$$

Watts per cubic centimetre per cycle per second $= \frac{3500}{10^7}$.

Total watts absorbed $= \frac{3500}{10^7} \times$ volume $\times$ cycles per second

$$= \frac{3500}{10^7} \times 5000 \times 20 = 35.$$

Example. — Find the increase of temperature in degrees centigrade of a mass of iron having a volume of 8000 cubic centimetres. The iron revolves for 30 minutes in a magnetic field, the induction density of which is 8500 lines per square centimetre, and goes through 50 complete cycles per second. Assume 25 per cent. of the heat to be lost by radiation. When $B = 8500$, the ergs lost per cycle per cubic centimetre equals 9000.

Specific heat of iron = 0.11.
1 cubic centimetre of iron weighs 7.8 grammes.
1 calorie $= 4.2 \times 10^7$ ergs.

The total work done against hysteresis
= 9000 × volume × cycles per second × time
$= 9000 \times 8000 \times 50 \times 30 \times 60$ ergs.
The work done in heating the iron
= weight × temperature rise × specific heat $\times 4.2 \times 10^7$.
75 per cent. of work done against hysteresis
= work done in heating the iron.
That is

$$\frac{75}{100} \times 9000 \times 8000 \times 50 \times 30 \times 60$$
$$= 7.8 \times 8000 \times T \times 0.11 \times 4.2 \times 10^7.$$

Therefore the temperature rise of the iron =

$$T = \frac{75 \times 9000 \times 50 \times 30 \times 60}{100 \times 7.8 \times 0.11 \times 4.2 \times 10^7}$$
$$= 17^\circ \text{ C.}$$

HYSTERESIS TESTING

Within recent years a number of instruments have been devised for rapidly measuring the hysteresis loss in specimens of iron. Some of them indicate the loss when iron is revolved in a magnetic field, as in the case of the armature core of a dynamo or motor. Others again indicate the loss when the magnetism passes through zero in changing from positive to negative, as is the case of iron forming the magnetic circuit of a transformer. The Ewing hysteresis tester shown in Figure 26 is an example of the former type.

Ewing's Hysteresis Tester.—This instrument consists of two parts, the carrier and the permanent magnet. The iron sheet to be tested is cut into strips about 2 centimetres wide, 7 centimetres long, and formed into bundles about 1 centimetre thick. The bundle A is fixed to the carrier and rotated by means of driving wheels between the poles of the C-shaped permanent magnet M. The result is that the magnetism in the iron is periodically reversed. The permanent magnet is supported upon knife edges at B, and carries a pointer P which moves over a graduated scale S; to the lower end of the magnet is attached a vane which dips into an oil bath O

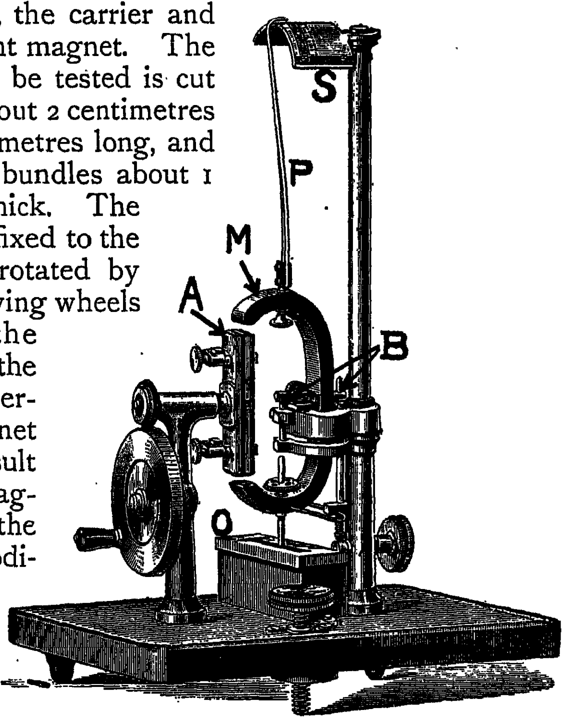

FIG. 26.—Ewing's hysteresis tester.

and so serves to damp the vibrations. The work done in reversing the magnetism due to the hysteresis loss causes a mechanical force to be exerted by the revolving iron upon the magnet; consequently the latter, along with the pointer P, is deflected through an angle which is proportional to the work expended per cycle. Since a certain amount of work is done for each reversal, whatever the frequency, the deflection is independent of the speed, provided the latter is not sufficiently high to create eddy current effects. Two pieces of iron, having different but known hysteresis losses, are supplied with the instrument; these are used as standards of reference. The deflections produced by both these samples are plotted as a function of the hysteresis loss in ergs per cycle per cubic centimetre, and a straight line drawn through the two points A and B, as in Figure 27. By observing the deflectionproduced by a given specimen the corresponding value of the hysteresis loss can be obtained from the calibration curve. To eliminate any zero error the iron should be rotated in both directions, and the mean of the two deflections may be considered the correct value. The standard hysteresis loss is based upon the assumption that $B = 4000$ lines per square centimetre in the iron. The loss at any other value of B is easily obtained from the law, hysteresis loss $= \eta B^{1.6}$.

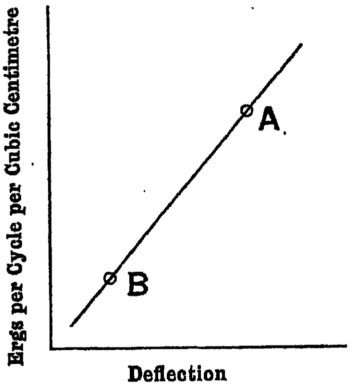

FIG. 27.

In practical work the hysteresis loss is expressed in so many watts per kilogramme weight at 50 cycles per second, and at a value of $B = 10{,}000$. A fair average hysteresis loss for armature laminations is about 2 watts per kilogramme.

CHAPTER IV

ELECTRICAL MEASURING INSTRUMENTS

INDICATING INSTRUMENTS

ERE are four distinct types of instruments intended for measurement of current, pressure, and power. They classified as follows, according to the principles upon ich they work.

A. Electro-magnetic instruments for measuring rent and pressure, which may be subdivided into

(*a*) Moving soft-iron instruments;

(*b*) Moving-coil permanent-magnet instruments.

B. Hot-wire instruments for measuring current and essure.

C. Electrostatic instruments for measuring pressure.

D. Electro-dynamic instruments for measuring rrent, pressure, and power.

Every instrument has some moving part which, when uated by the current to be measured, takes up a nporary position of equilibrium between two positions rest,—one when there is no deflecting force and the ner when the moving system can move no further.

Controlling Force.—In all instruments it is necessary provide a controlling force which opposes and controls e deflecting force so that the deflection may be pro-rtional to the deflecting force. In any position of uilibrium the controlling and deflecting forces are ual, the former force tending to bring the moving stem back to the zero position, while the latter tends to rther deflect the moving system.

The controlling forces employed in the majority of esent-day instruments are:

(*a*) Force of gravitation.

(*b*) Torsion of a spiral spring.

The force of gravitation, as a controlling force, has the great advantage that it is absolutely constant. Gravitation controls the movement of all instruments of the moving soft-iron type, hence they are often referred to as gravity instruments.

In the second form of control the force exerted by the spring is proportional to the angle of torsion. The spring is, in most cases, made of some hard and non-magnetic material, such as phosphor-bronze, and its strength varies directly as the sectional area of the material and inversely as the number of turns. It is essential that the turns be uniformly spaced, and do not touch one another at any stage of their action.

Damping.—The moving part of an instrument on receiving an impulse, due to a sudden change in the current flowing, tends to oscillate about its mean position. This is often undesirable, and a damping device is introduced so that, if the dimensions of the moving parts be suitably designed, the pointer of an instrument will move up to its reading without passing beyond it or oscillating about it. The instrument is then said to be dead-beat.

The methods of damping now in most general use depend on

1. Viscosity of liquids.
2. Electrical eddy currents.
3. Air friction.

Of these, that depending on the viscosity of liquids is the easiest to apply, and has therefore been largely employed. This arrangement consists of a light disc (attached to the moving system) immersed in a liquid (usually oil), so that the resistance offered by the liquid to the plunging of the disc damps out any vibration. This form of damping has been successfully applied to Kelvin's electrostatic voltmeters.

In the second method, damping is effected by causing a conductor, usually in the form of an aluminium disc attached to the moving system, to rotate in a magnetic field produced by a permanent magnet. Such an rrangement is depicted in Figure 28, in which the disc D otates between the poles N and S of the permanent nagnet M. As the disc moves across the magnetic field lectric currents are induced in it, and assume the form f little whirls or eddies, as shown by the dotted lines. These currents, according to Lenz's law, oppose the cause producing them, thus the motion of the disc and consequently the moving system is retarded, and any tendency to oscillate is effectively damped.

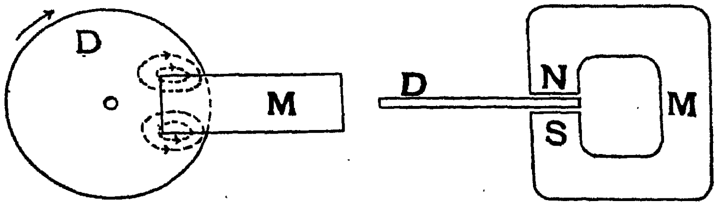

FIG. 28.—Principle of Eddy current damping.

This damping device is suitable for hot-wire and electrostatic instruments, but cannot be applied to moving soft-iron instruments. In the latter instruments the presence of a permanent magnet close to the working coil would have a considerable effect on the readings. The instruments would of course be calibrated with the magnet in position, but the small magnets with comparatively large air-gaps have a great tendency to lose their strength, so that the effect on the readings would be a variable one.

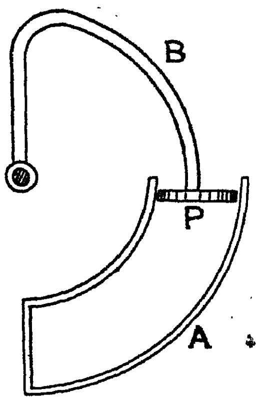

FIG. 29.—Principle of air damping.

Air damping, although used before any other system for galvanometers and other laboratory instruments, has only within recent years been applied to commercial instruments. Figure 29 illustrates a typical form of air-damping arrangement. It consists of an air chamber A closed at one end, in which works a loosely fitted aluminium piston P. The piston is connected by means of a crank B to the moving system, so that any oscillation is damped by the cushioning action of the air on the piston.

The mechanical difficulties of manufacture of this type of damping are great, since the clearance between the piston and the air chamber should not exceed 0.04 centimetre.

Ammeters and Voltmeters.—The essential difference between an ammeter and a voltmeter of the same type is in the resistance of the coil which carries the current to actuate the moving parts. The same movement does for either. An ammeter is connected in series with the circuit whose current is to be measured, and consequently must have a low resistance, otherwise the insertion of an ammeter in a circuit might alter the value of the current; moreover, the power absorbed by any part of the circuit is equal to C^2R, so that if R be large an appreciable amount of power might be absorbed by the ammeter.

A voltmeter is connected in parallel across the points where the difference of potential is to be measured. The current taken by a voltmeter should be small, and in order to fulfil this condition the resistance must be very high, in some cases being as high as 50 ohms per volt. If the resistance be low then an appreciably large current will flow through the instrument, altering the pressure across the points connected to the instrument and preventing a reading of the true difference of potential being obtained. Figure 1 shows how to connect an ammeter and voltmeter to indicate the current and pressure respectively of a dynamo supplying current to a number of lamps in parallel.

MOVING SOFT-IRON AMMETERS AND VOLTMETERS

This type of instrument includes those in which a piece of soft iron is constrained to move from one part of a magnetic field to another; the field being set up by a suitably wound solenoid energised by the current or pressure to be measured. The solenoid for these instruments, in general, is similar to that shown in Figure 10, the outside diameter being about 5.5 centimetres and the length 5 centimetres. The magnetic force H inside the solenoid is equal to $\frac{1.25\,CT}{l}$; that is, for the same size of solenoid the strength of magnetic field H is directly proportional to the ampère-turns CT.

The same movement serving for both ammeters and voltmeters, it is essential to provide the same strength of magnetic field for both when giving a full scale deflection. Also it is immaterial whether the product CT be given by a large current flowing through a few turns of thick wire or a small current flowing through many turns of fine wire. In an ammeter the working coil is wound with a few turns of thick wire capable of carrying the maximum current to be indicated, while a voltmeter has its working coil wound with many turns of fine wire.

Thus the only difference between an ammeter and a voltmeter of this type is in the turns and size of wire constituting the winding of the working coil.

It will be of advantage, before investigating the theory of this type of instrument, to describe a few representatives.

Examples of Moving Soft-Iron Instruments.—Figure 30 shows an end elevation and plan of the internal parts of one form of moving soft-iron instrument. The coil W, of cable or fine wire according as the instrument is to be used as an ammeter or a voltmeter, is wound on a

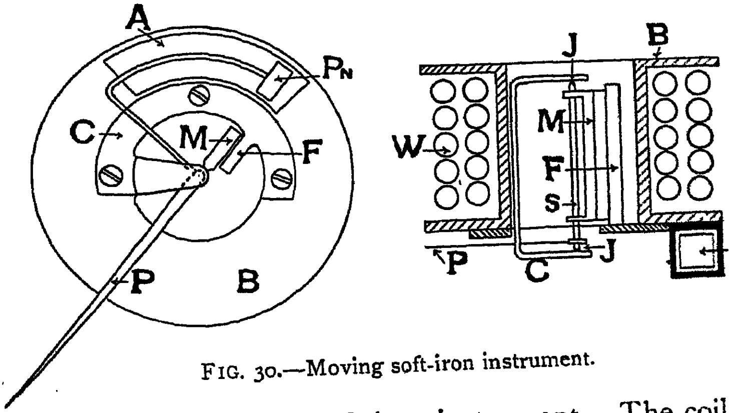

FIG. 30.—Moving soft-iron instrument.

suitably insulated brass bobbin B. Inside this bobbin is placed a brass frame C which carries the jewelled centres J between which is pivoted, concentric with the bobbin B, the steel spindle S. Inside the coil are two pieces of soft iron M and F, set axially with the coil. F is fixed to the frame C, and extends nearly the entire length of the solenoid. M is attached to the spindle, and is therefore free to move. To one end of the spindle is attached a pointer P which passes over a graduated scale.

The working of the instrument is as follows. When no current flows through the solenoid the action of gravity causes the moving iron M to lie parallel to and almost touching the fixed iron F, the pointer being set to zero on the scale. When the coil is energised by a current, lines of force are set up inside, and M and F become magnetised, developing similar polarity at the same ends, consequently repulsion ensues and the moving iron M is repelled against the controlling force of gravity from its zero position; the pointer P at the same time moves over the scale. The movement of M, and hence the deflection of the pointer P, is very nearly proportional to the current energising the coil W.

In order to render the instrument dead-beat a small aluminium piston P_N which is carried at the end of a crank attached to the spindle S, moves to and fro in the air chamber A.

In the instrument illustrated in Figure 31 advantage is taken of the fact that the magnetic field set up inside a solenoid is stronger near the inner circumference CC (Fig. 10) than along the axis AB. The moving system consists of a piece of very thin soft iron M shaped as shown, and attached to a steel spindle S. The latter also carries an aluminium pointer P and balance arm A. The spindle is set eccentric with, but parallel to, the coil in jewelled bearings carried by the bridge piece D attached to the bobbin B, and a similar bridge piece at the other end. By the control of gravity, in the zero position, the iron M lies near to the centre of the coil. When the coil is energised, the field set up being stronger at the inner circumference C, the part M is attracted towards it. As the iron is attracted, causing the spindle to rotate, the pointer P moves over a graduated scale. The position of the moving system when the pointer indicates its maximum reading is shown in the figure by the dotted lines. The zero may be adjusted by altering the position of the balance arm A.

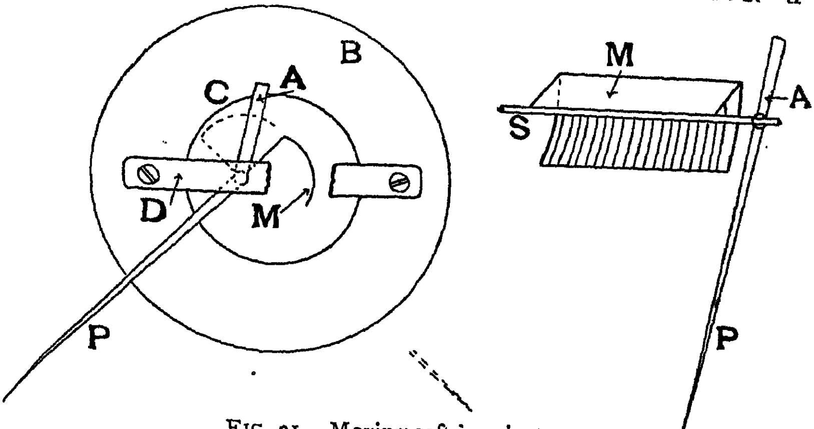

FIG. 31.—Moving soft-iron instrument.

Figure 32 shows an elevation and plan of a third form of moving soft-iron instrument. The working coil C is wound on an insulated bobbin B, and set with its magnetic axis parallel to the base of the instrument. The coil is of a flat shape, in order to give as concentrated a field as possible with the minimum expenditure of energy. The moving soft iron M, of the shape shown, is built up of three thin pieces and carried by a steel spindle S set in jewelled centres J; the latter are supported by two lugs L projecting from the frame of the instrument. To the spindle is fixed an aluminium pointer P, and balance arms carrying the adjusting weights W_1 and W_2.

The figures show the position of the moving system when no current passes through the coil C. On the coil becoming energised the moving iron is attracted against the controlling force of gravity towards the centre by an amount proportional to the strength of the current, the pointer at the same time moving over the scale.

Damping is effected by the movement of the piston P_N inside the curved air chamber A, the piston being attached by the crank D to the spindle.

Design of Moving Soft-Iron Instruments.—The force required to deflect the moving system of an instrument so that the pointer moves over the scale is proportional to the weight of the moving parts. Consequently, in order to reduce the power absorbed by the working coil, the weight of the moving system and friction at the pivots must be reduced to a minimum. This is effected by (1) making the moving iron as small as possible, (2) making the pointer of aluminium, and (3) setting the spindle in jewelled bearings.

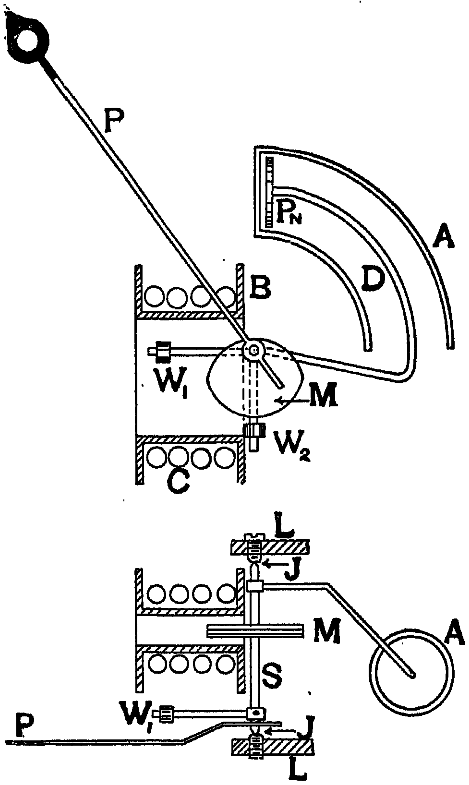

FIG. 32.—Moving soft-iron instrument.

In well-designed instruments the ampère-turns required to give a full scale deflection range from 200 to 400; so that from a knowledge of the ampère-turns required to deflect the moving system of a particular type of instrument over the scale the number of turns constituting the working coil of an ammeter can be easily determined. For a voltmeter the calculation is not so simple, for not only must the coil give the requisite magnetising force, but its resistance must be sufficiently high.

Size of Wire required for a Voltmeter Coil.—The ampère-turns CT required to deflect the needle of a particular instrument over the scale can be determined experimentally, and the voltage E impressed on the working coil is also a known factor.

Let C = current flowing through the coil.
R = resistance of the coil.
l_m = length of a mean turn on the coil.
l = total length of wire forming the coil.
a = area of cross-section of wire.
d = diameter of wire.
ρ = specific resistance of wire.
T = turns constituting the coil.

Then $$\frac{E}{C} = R = \frac{\rho \times l}{a}$$

but $l = l_m \times T$ and $a = \dfrac{\pi d^2}{4}$

Therefore $$\frac{E}{C} = \frac{\rho \times l_m \times T \times 4}{\pi d^2}$$

or $$d^2 = \frac{\rho \times l_m \times 4 \times TC}{\pi E}.$$

From the above the diameter of the wire is given by

$$d = \sqrt{\frac{4 \times \rho \times l_m \times CT}{\pi E}}.$$

Example.—The working coil of a moving soft-iron voltmeter reading up to 100 volts requires 300 ampère-turns to give a full scale deflection. Find the diameter of wire for the coil if the wire be of copper having a specific resistance of 1.6×10^{-6} ohms per centimetre cube; the mean length of one turn = 13.5 centimetres.

By the above formula

$$d = \sqrt{\frac{4 \times \rho \times l_m \times CT}{\pi E}}$$

$\rho = 1.6 \times 10^{-6}$ $CT = 300$
$l_m = 13.5$ centimetres $E = 100.$

The diameter of the wire

$$= \sqrt{\frac{4 \times 1.6 \times 10^{-6} \times 13.5 \times 300}{\pi \times 100}} = 0.009 \text{ cms.} = 0.09 \text{ mm.}$$

To obtain a uniform scale with these instruments it is necessary to have the moving soft iron saturated, the reason for this being as follows:

When the solenoid is energised the moving soft iron becomes a magnet, and the force deflecting the moving system is proportional to the product of the strength of the field and the strength of this temporary magnet. From the curves in Figure 15 it will be seen that when the iron is saturated its magnetic strength remains practically constant, in which case the deflecting force would be directly proportional to the current flowing in the coil, the field strength being proportional to the current. These instruments, when the iron is saturated, will have a perfectly uniform scale, equal increments of current producing equal increments in the deflection.

Below saturation value the variation of B with H is irregular, consequently the scale of an instrument in which the iron is not saturated cannot be evenly divided. For this reason moving soft-iron instruments are not calibrated for currents or voltages which do not produce a magnetic field strong enough to saturate the iron, and the lower part of the scale of these instruments is generally left blank.

Errors in moving Soft-Iron Instruments.—The chief errors occurring in this type of instrument are due to the following causes:

1. Hysteresis effect.
2. Effect of stray magnetic fields.
3. Change in resistance of working coil due to temperature.

The *hysteresis effect* causes the readings taken with a rising current to be lower than those obtained with a falling current, due to the property of retentivity explained in Chapter III.

In order to reduce the hysteresis error to a minimum, it is essential that the moving iron should be very soft, and further, that the mass be as small as possible. By working the iron near saturation point and observing the above conditions the hysteresis effect is practically eliminated.

Stray magnetic fields affect the accuracy of all electro-magnetic instruments. This is particularly the case with instruments used for switchboard work. The effect is somewhat difficult to predetermine owing to the complicated nature of resultant fields, but other things being equal an instrument which in itself utilises the strongest field will be least affected. In Figure 33 curves are given showing the error introduced into three types of moving soft-iron instruments when subjected to stray magnetic fields near a switchboard.

These curves are reproduced from a paper on direct-reading measuring instruments by Edgcumbe and Punga.* The abscissæ show the readings as a percentage of the maximum current, while the ordinates represent the error as a percentage of the maximum readings of the instruments. Curve A shows the effect on a 30-ampère moving soft-iron ammeter, having 450 ampère-turns and enclosed in a brass case. The movement consisted of a fixed mass of iron repelling a moving mass. It will be observed that the percentage error is practically constant. Curve B refers to a 10-ampère instrument having 400 ampère-turns and with its cast-iron case removed. It will be seen that the error falls off very rapidly as the load increases. Curve C refers

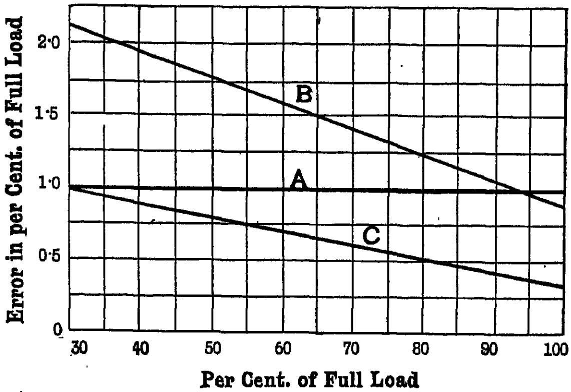

FIG. 33.

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* *Journal of Institution of Electrical Engineers*, vol. xxxiii. pp. 620–693, March 1904.

to the same instrument after replacing the cast-iron case.

From these curves it would appear that by placing the moving system in a cast-iron cover the instrument is shielded from stray magnetic fields and errors considerably reduced. The shielding action of the iron case is shown diagrammatically in Figure 34. The stray lines, say, coming from the magnet M, enter the case C, and if it be of appreciable thickness the lines of force follow the paths in the case as shown. Thus the stray field in no way influences the working of the moving parts.

The iron case may screen the working parts from stray magnetic fields, but on the other hand the lines of force emanating from the moving iron pass into the case, and consequently the hysteresis error is greatly increased. Again, the case is liable to become magnetised and cause a possible error far outweighing the reduced error due to magnetic shielding. In general, moving soft-iron instruments are enclosed in non-magnetic cases.

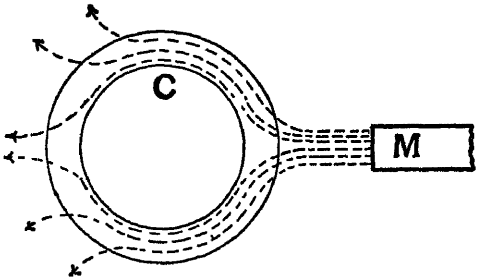


FIG. 34.

The *error due to change of temperature* occurs in all types of instruments having a high resistance coil carrying potential currents. Let C denote the current passing through the coil of a voltmeter, R the resistance of the coil, and E the potential across the voltmeter. The scale of the instrument is graduated in terms of $E = CR$; so that the reading on the instrument is proportional to C. If E remain constant but R vary, due to change of temperature, then C will alter, and consequently the reading. This is best illustrated by a concrete case.

Example.—The working coil of a moving iron voltmeter has a resistance of 4000 ohms at 20° C., at which temperature it was calibrated and read correctly when connected across a potential of 200 volts. The coil is wound with copper wire having a temperature coefficient equal to 0.0043. Find the percentage error in the reading when the temperature of the working coil increases to 60° C.

At 20° C. the current flowing in the coil

$$=\frac{E}{R_{20}}=\frac{200}{4000}=0.05 \text{ ampère.}$$

This current gives a full scale deflection corresponding to 200 volts.

At 60° C. the current flowing in the coil

$$=\frac{E}{R_{60}}; \text{ but } R_{60}=R_{20}(1+\alpha t)$$
$$=4000(1+0.0043\times 40)$$
$$=4000\times 1.17=4680 \text{ ohms.}$$

Therefore the current flowing in the coil when it is at 60° C. $=\frac{200}{4680}=0.043$ ampère.

The current energising the solenoid is thus reduced, so also will be the voltmeter reading. With 0.05 ampère through the coil the needle indicated 200 volts, therefore, if the iron be saturated, 0.043 ampère will indicate

$$\frac{200\times 0.043}{0.05}=172 \text{ volts.}$$

The error introduced due to change of temperature $=-28\times\frac{100}{200}=-14$ per cent. That is, the voltmeter indicates 14 per cent. low.

A voltmeter so constructed would be unreliable, and the method of reducing the temperature error will now be considered.

Alterations in the temperature of an instrument are caused in two ways—first, by change in atmospheric temperature; and second, by the expenditure of energy in the coil itself, the latter being proportional to C^2R. The employment of a material such as manganin, which has a negligible temperature coefficient, would practically eliminate any temperature error; but a voltmeter coil wound with manganin wire would have a much less radiating surface than a coil of copper wire of the same resistance. This would result in a considerable increase in the actual heating, the radiating surface being insufficient. A compromise has therefore to be made between the two. The working coil of a voltmeter is wound with copper wire so as to obtain as large a magnetising force as possible with the least expenditure of energy, and connected in series with it is another coil, made of a material such as manganin, having a negligible temperature coefficient.

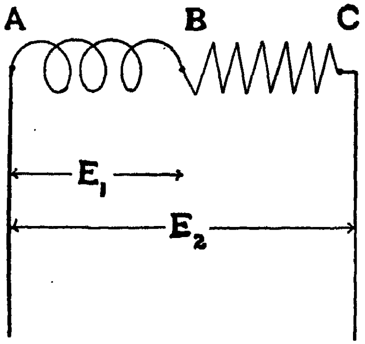


FIG. 35.

In Figure 35 AB is the working coil, and BC the resistance in series with it. Let E_1 denote the voltage required to produce a certain deflection when applied to the working coil AB, and E_2 the voltage to produce the same deflection when applied to AC. If R_1 and R_2 denote the resistance of the coils AB and BC respectively,

$$\text{then } \frac{E_1}{E_2} = \frac{R_1}{R_1 + R_2}.$$

With this arrangement if R_1 vary it is so small a fraction of $(R_1 + R_2)$ that the latter remains practically constant. The resistance BC is wound on one or more porcelain bobbins. In low-reading voltmeters the bobbins are fixed to the base of the instrument, but for high-voltage instruments several are required, so they are placed in a separate case. The working coil is generally designed so that it produces a full scale deflection with about 20 volts, the remaining voltage being absorbed by the extra resistance. In this way the same size of working coil will do for any voltmeter reading up to say 600 volts; the resistance coils being wound for each particular voltage.

Example.—If in the previous example the resistance of the working coil be reduced to 400 ohms at 20° C. by winding it with the same number of turns of copper wire having a larger sectional area, determine the error in the voltmeter reading when the temperature of the working coil increases to 60° C. In series with the working coil is a resistance of 3600 ohms, having a negligible temperature coefficient.

Current flowing through the coils at 20° C. = 0.05 ampère.

Resistance of working coil at 60° C. is equal to $R_{20}(1+at)$

$$= 400\,(1+0.0043\times 40) = 400\times 1.17$$
$$= 468 \text{ ohms.}$$

Total resistance in voltmeter circuit at 60° C.

$$= 468 + 3600 = 4068 \text{ ohms.}$$

Current flowing through the coils at 60° C.

$$= \frac{E}{R_{60}} = \frac{200}{4068} = 0.0492 \text{ ampère.}$$

Voltmeter reading at 60° C.

$$= 200\times\frac{0.0492}{0.05} = 197.$$

Error in voltmeter $= -1\frac{1}{2}$ per cent. The voltmeter now reads only $1\frac{1}{2}$ per cent. low, clearly showing the effect of introducing a resistance in series with the working coil.

MOVING-COIL PERMANENT-MAGNET AMMETERS AND VOLTMETERS

Instruments belonging to this type, though differing in the details of construction, all work on the d'Arsonval principle. Such instruments depend for their action on the rotative force experienced by a coil of wire carrying current when the latter is pivoted so as to lie across a magnetic field. The amount of movement is indicated by a pointer, attached to the moving system, passing over a graduated scale.

The principal parts of these instruments are:

(1) A permanent magnet provided with soft-iron pole pieces.

(2) A moving coil of fine wire mounted on pivots and fitted with controlling springs.

Figure 36 shows a plan and elevation of the working parts. PM is a permanent magnet which has been specially "aged" so that its strength will remain constant over a long period of time. The soft-iron pole pieces P are fixed to the ends of the permanent magnet and bored out so as to be truly cylindrical. Concentric with the two pole pieces P is placed a soft-iron core IC. This iron core has a diameter of about 2.5 centimetres, and is screwed to a brass bridge piece D. Capable of turning in the narrow air-gap formed between the poles P and the iron core IC, is a rectangular-shaped coil MC, composed of a number of turns of fine insulated wire wound on a copper or aluminium frame. The frame is carried by two pivots running in jewelled bearings J, the latter being supported from two brass brackets B attached to the pole pieces. The pivots are insulated from the frame, but

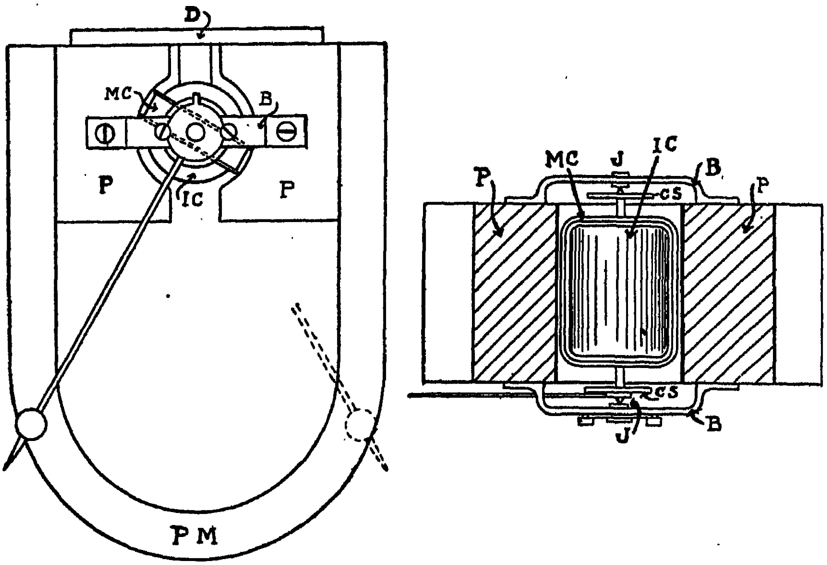


FIG. 36.—Principle of moving-coil permanent-magnet instrument.

each is in electrical connection with an end of the moving coil.

The current is led into and out of the moving coil by means of two copper phosphor-bronze hair-springs CS, one end of each being connected to the top and bottom pivots respectively, and the other ends to two insulated contacts fixed either to the brackets B or to two metal pillars fixed to the base of the instrument. Connection is made between these insulated contacts and the terminals of the instrument. The springs are set in opposite directions, so that as one coils up the other uncoils. This arrangement of the springs neutralises the effect on the position of the coil due to expansion or contraction through temperature variation.

The action of the instrument is as follows: When a current passes through the moving coil, lines of force are set up, and they tend to set themselves parallel to the lines of force from the permanent magnet, consequently the coil experiences a rotative force and moves against the spiral springs which act as a controlling force. When equilibrium is obtained the controlling force equals the deflecting force, and the pointer takes up a definite position on the scale, that position depending upon the value of the current flowing in the moving coil.

The deflecting force is proportional to the product $C . B_a$

where C = current in the moving coil,

and B_a = induction density in the air-gap, and varies between 600 and 2000 lines per square centimetre, depending on the make of instrument. Throughout the entire movement of the coil MC, the strength of the magnetic field B_a is maintained uniform by having a very narrow air-gap about 0.18 centimetre, and setting the coil MC at such an angle that the two positions of rest are within the polar arc. Under these conditions the deflecting force is directly proportional to the current, and as the controlling force due to the hair-springs is proportional to the angle turned through, an evenly divided scale is obtained throughout the entire range.

These instruments are rendered dead-beat by the damping action of eddy currents induced in the aluminium or copper frame as the coil moves from one position to another. The moving coil is wound with different sizes of wire, depending upon the use for which the instrument is intended, *i.e.* ammeter or voltmeter.

Ammeters of this type measure current indirectly by indicating the difference of potential between the ends of a low-resistance shunt made of a material having a negligible temperature coefficient. The shunt is connected in series with the main circuit, and carries the current to be measured. The voltage drop in the shunt is proportional to the current flowing through it, so that moving-coil ammeters are in reality milli-voltmeters. Figure 37 illustrates how a shunt S is connected in series with a circuit of lamps, the measuring instrument A being connected across the terminals of the shunt. The fall of potential in the shunt should be small, consequently an ammeter coil is wound with a small number of turns, and is then actuated by a small electromotive force across its terminals.

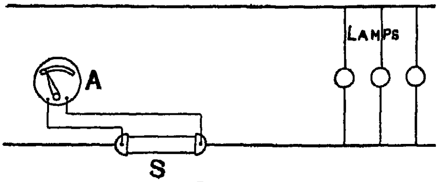


FIG. 37.

In a particular make of ammeter of this type the moving coil consists of 15 turns, having an approximate resistance of 0.15 ohm. In series with this coil is a resistance having a negligible temperature coefficient, which reduces the error caused by temperature variation, as explained on page 76.

Ammeter Shunts.—These are made of strips of manganin, and are so designed that with the maximum current flowing, the drop of E.M.F. is sufficient to give a full scale deflection on the instrument.

Suppose the current to be measured to flow through the shunt and be denoted by C_s, then if R_s be the resistance of the shunt the volts drop is given by $E_s = C_s R_s$. R_s remains constant, as the shunt is made of manganin, having a negligible temperature coefficient, so that $E_s \propto C_s$. Now the current C flowing in the moving coil and producing the deflection is proportional to E_s, so that the deflection of the instrument is directly proportional to the current flowing through the shunt.

In order that the student may thoroughly understand the function of a shunt, the following example is given:

Example.—In a particular moving-coil instrument 0.08 of a volt is sufficient to produce a full scale deflection. It is proposed to use the instrument to indicate a maximum current of 200 ampères. If the shunt be made from manganin strip 0.5 millimetre thick, calculate the dimensions of the shunt. Specific resistance of manganin = 43 microhms per centimetre cube.

Since 0.08 of a volt is required to produce a full scale deflection, the voltage drop in the shunt must equal this value when the maximum current the instrument has to indicate passes through the shunt. So that the resistance of the shunt is given by

$$R_s = \frac{E_s}{C_s} = \frac{0.08}{200} = 0.0004 \text{ of an ohm.}$$

Shunts are generally designed for a temperature rise of about 30° C. when carrying continuously their full load, and in order that the temperature increase may not exceed this limit the shunt must have a radiating surface of 12 square centimetres per watt absorbed.

In this case the watts absorbed

$$= E_s \times C_s = 0.08 \times 200 = 16 \text{ watts.}$$

Therefore the total radiating surface must be $= 12 \times 16 = 192$ square centimetres; but there are two sides of the shunt strip exposed to the atmosphere, so that the surface per side $= \frac{192}{2} = 96$ square centimetres.

Referring to Figure 38, l = length of strip, b = breath, and t = thickness; therefore $l \times b = 96$.

The resistance of the shunt is also given by the equation

$$R_s = \frac{\rho \times l}{a} = 0.0004 \text{ of an ohm}$$

where a = area of cross-section of the strip.

From this equation $\frac{l}{a} = \frac{0.0004}{\rho} = \frac{0.0004}{0.000043} = \frac{9.3}{1}$.

Now $a = b \times t = b \times \frac{1}{20} = \frac{b}{20}$ sq. centimetres

that is $\frac{l}{b} \times \frac{20}{1} = \frac{9.3}{1}$

or $l = \frac{9.3 \times b}{20}$.

Substituting for l, $\frac{9.3\, b^2}{20} = 96$

or $b = 14.4$ centimetres.

Now $l \times b = 96$

therefore $l = \frac{96}{14.4} = 6.65$ centimetres.

Length of shunt strip = 6.65 centimetres.
Breadth of shunt strip = 14.4 centimetres.

Referring to Figure 38, the shunt strip is brazed into saw-cuts made in two terminal blocks B; to the latter are attached main terminals T_1 and potential terminals T_2. If the breadth be greater than the length (as in the above example) the shunt is made up with a number of strips, the length remaining the same as that calculated, but the breadth of each part is $\frac{1}{n}$th the calculated breadth, n being the number of parts into which it is found

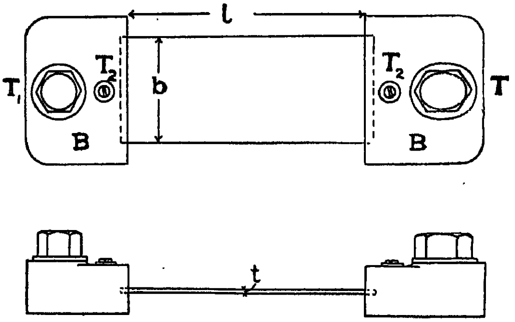


FIG. 38.—Ammeter shunt.

convenient to divide the strip. Figure 39 illustrates the shunt calculated above; it is divided into 4 strips as shown, the breadth of each strip being $\frac{14 \cdot 4}{4} = 3 \cdot 6$ centimetres.

For instruments reading up to 75 or 100 ampères the shunt usually is fixed to the instrument base; for higher reading instruments the shunt is separate and can be fixed at any convenient place on the switchboard, and leads run from the potential terminals to the instrument, as indicated in Figure 37.

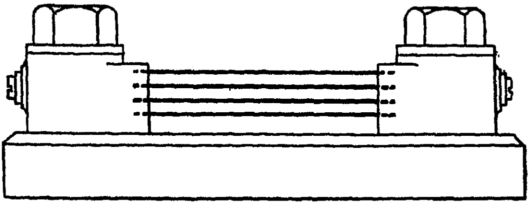


FIG. 39.—Ammeter shunt for large currents.

These instruments are calibrated in ampères, and should only be used with the shunt for which each one is calibrated.

In voltmeters the moving coil is wound with 130 to 250 turns of copper wire having a resistance of from 30 to 100 ohms. In series with the moving coil is connected a resistance of manganin having a negligible temperature coefficient and is adjusted to such a value that 0.015 of an ampère passes through the voltmeter when giving a full scale deflection. The resistance of a voltmeter may therefore amount to many thousands of ohms, so that any error due to temperature variation is practically eliminated. The actual temperature coefficient of a 100-volt instrument is often less than one one-hundredth per cent. per degree centigrade.

Example.—A moving-coil voltmeter reading up to 150 volts requires a current of 0.015 of an ampère to produce a full scale deflection. If the error due to temperature change must not exceed 0.05 per cent. of the maximum reading, with a rise in temperature of 20° C., and the moving coil is wound with copper wire having a temperature coefficient of 0.43 per cent. per degree centigrade, calculate the value of the manganin resistance in series with the moving coil. Manganin has a zero temperature coefficient.

Total resistance of voltmeter circuit at ordinary temperature

$$=\frac{E}{C}=\frac{150}{0.015}=10{,}000 \text{ ohms};$$

when the temperature increases by 20° C. the current in the moving coil is reduced by 0.05 per cent. and

$$=\frac{99.95}{100}\times 0.015=0.01499.$$

Therefore the resistance of the voltmeter when the temperature rises 20° C. above normal must not exceed

$$\frac{150}{0.01499}=10{,}007 \text{ ohms.}$$

Increase in resistance $= 10{,}007 - 10{,}000 = 7$ ohms.

The copper increases $0.43 \times 20 = 8.6$ per cent. per 20° C. So that the normal resistance of moving coil

$$=\frac{100 \times 7}{8.6}=81 \text{ ohms.}$$

The manganin resistance must therefore have a value of $10{,}000 - 81 = 9919$ ohms.

The resistance of the moving coil is quite low compared with that of the manganin resistance.

Example.—The control of a particular moving-coil instrument is such that it requires a force of 1.5 gramme-centimetres to produce a full scale deflection, and the current flowing in the moving coil when giving this deflection is 0.015 of an ampère. From the following data calculate the number of turns required for the moving coil:

Diameter of moving coil $(d) = 3$ centimetres.
Length of active conductor $(l) = 3$ centimetres.
Flux density in air-gap $(B_a) = 800$ lines per square centimetre.

The force acting on one conductor $= B_a \times l \times 0.0015$ dynes.

If t denote the turns required for the moving coil, then the force acting on one side of the coil

$$= B_a \times l \times t \times 0.0015 \text{ dynes}$$
$$= \frac{B_a \times l \times t \times 0.0015}{981} \text{grammes.}$$

The total moment producing rotation is given by

$$\frac{B_a \times l \times t \times 0.0015}{981} \times d = 1.5 \text{ gramme-centimetres,}$$

$$\text{so that } t = \frac{1.5 \times 981}{0.0015 \times B_a \times l \times d}$$
$$= \frac{1.5 \times 981}{0.0015 \times 800 \times 3 \times 3} = 136.$$

Turns required for moving coil $= 136$.

Sources of Error in Moving-Coil Instruments.—Hysteresis is of course absent, as the magnetic flux is practically constant, although theoretically the ampère-turns of the moving coil (in practice less than one ampère-turn is required) slightly affect the permanent magnetic field, weakening it over the lower half of the scale and strengthening it throughout the upper half.

The chief source of error in moving-coil instruments was formerly the variation with time of the springs. A great amount of attention has been devoted to the preparation of non-magnetic springs with as small a permanent set as possible; these can now be obtained, and if properly fixed they introduce practically no error.

Instruments of the moving-coil type are the most accurate for the measurement of direct currents. They are the least liable to derangement, have no hysteresis loss, and when enclosed in a cast-iron case are unaffected by stray magnetic fields. The cast-iron case, however, diverts some of the lines of force from the air-gap and so lowers the induction density in the latter, affecting at the same time the accuracy of the instrument

In order to avoid this error instruments of this type must be calibrated with the cast-iron case in position, otherwise an error amounting to as much as 4 or 5 per cent. may be introduced.

The current in the moving coil must always flow in the one direction, otherwise the needle would be deflected in the wrong direction. For convenience the terminals of these instruments are marked + and − respectively, so that the proper connections may be made. The zero is adjusted by altering the tension of the controlling springs. For this purpose they are generally provided with an adjusting screw, so that the operation may be performed without the removal of the case.

HOT-WIRE AMMETERS AND VOLTMETERS

In these instruments the heating effect of an electric current in a wire of suitable resistance is by special mechanism made to indicate current or potential in a circuit. The rate of generation of heat when a current of C ampères flows through a wire having a resistance of R ohms $= C^2R \times 0.24$ calories per second.

If E be the fall of potential along the wire, then

$$C^2R \times 0.24 = E \times C \times 0.24.$$

If the wire be at a temperature $t_1°$ C. before the current or potential is applied, then let $t_2°$ C. denote the temperature to which the wire is raised when a constant current of C ampères flows through it.

Also let r_2 = radius of wire in centimetres at $t_2°$
l_1 = length ,, ,, $t_1°$
l_2 = ,, ,, ,, $t_2°$
ρ = specific resistance of wire at $t_1°$
κ = coefficient of linear expansion of the wire.

Denoting the emissivity by σ,—emissivity being the rate of loss of heat by radiation per unit area of the wire per degree centigrade difference in temperature between the wire and the surrounding air,—then when the maximum temperature, due to the current flowing in the wire has been attained, the rate of loss of heat by radiation

$$= 2\pi r_2 \, l_2 \, \sigma \, (t_2 - t_1) \text{ calories per second}$$

the rate of generation of heat

$$= C^2 R \times 0.24 \text{ calories per second}$$

$$= C^2 \, . \, \frac{\rho \times l_2}{\pi r_2^2} \times 0.24 \quad \text{,,} \quad \text{,,}$$

Now the rate of generation of heat must equal the rate of loss of heat by radiation, so that

$$2\pi r_2 l_2 \, . \, \sigma \, . \, (t_2 - t_1) = C^2 \frac{\rho l_2}{\pi r_2^2} \times 0.24$$

$$\text{or } (t_2 - t_1) = \frac{\rho}{\sigma} \times \frac{C^2}{\pi^2 r_2^3} \times 0.12.$$

Let x = the elongation of the wire when its temperature rises from t_1° to t_2°, then

$$x = l_1 \times \kappa \times (t_2 - t_1)$$

$$= l_1 \times \kappa \times \frac{\rho}{\sigma} \times \frac{C^2}{\pi^2 r_2^3} \times 0.12.$$

σ and ρ are practically constant within the limits of temperature that occur in hot-wire instruments.

l_1 and r_2 are also constants for a given temperature,

$$\text{so that } x = K_1 C^2$$

$$\text{or } C = K\sqrt{x}$$

where K_1 and K are both constants.

Thus the elongation of a wire is proportional to the square of the current flowing in it. The elongation of the wire is transmitted to a pointer which moves over a graduated scale. The chief difficulty in the design of hot-wire instruments is that the expansions dealt with are so small that when the necessary multiplication has to be taken into account the available power is very small. With ammeters it is necessary to reduce the size of wire employed to a minimum, so as to avoid excessive sluggishness. Shunts are therefore employed, as with moving-coil instruments.

The first instrument of this type was the well-known but now obsolete Cardew voltmeter.

Figure 40 illustrates a more modern voltmeter of this type. The measuring or hot-wire W is of platinum silver—16 centimetres in length—and is stretched between the fixed point A and a tension adjusting arrangement B. A little to the right of the centre of the wire W is attached another wire W_1, of phosphor-bronze, at right angles, and is held taut to a fixed terminal D.

Near the centre of the wire W_1 a cocoon fibre C is attached, and passes round a grooved metal pully E fixed on a pivoted steel spindle mounted in jewelled bearings, and terminates in a small eyelet attachment to a flat steel spring S. The whole arrangement of the fibre and wires is thus subjected to tension, and any slackening or sag of the wire W is immediately taken up by the steel spring, and is transmitted by the wire W_1 and the fibre C to the spindle which carries the pointer P. By this method the smallest extension of the wire W is greatly magnified and conveyed to the pointer, thus rendering the readings easily perceptible. A magnetic damping arrangement is provided, which consists of a light disc L of aluminium, which is mounted on the spindle carrying the pulley E and the pointer P. This disc, which is about 5 centimetres in diameter, moves between the poles of the permanent magnet M, and the eddy currents induced damp the oscillations of the moving system, thus rendering the instrument

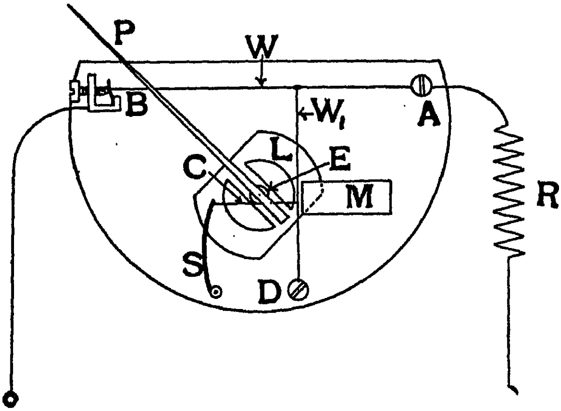


FIG. 40.—Hot-wire voltmeter.

dead-beat. The working of the instrument is as follows:

When the wire W becomes heated, due to the passage of a current, it expands, the corresponding sag being taken up by the spring S and transmitted by the wire W_1 and fibre C to the pointer P, as already explained. The whole of the hot-wire movement is mounted on a metal compensating plate, made from an alloy whose temperature coefficient is the same as the hot wire W. For high pressures the whole internal mechanism is mounted on ebonite. In all these voltmeters a fuse is introduced to protect the measuring wire in case an excessive voltage should be accidentally applied. To deflect the pointer of such a voltmeter over the scale a current of 0.2 ampère is required. For instruments up to 400 volts a series resistance R, made of constantan, is placed in the back of the instrument and connected in series with the hot wire. For higher voltages than 400, the resistance R is placed in a separate case. When the resistance is placed in the same case as the movement, means must be provided for efficient ventilation.

Hot-wire ammeters of this make are, with the exception of a slight modification in the arrangement of the measuring wire, exactly similar to the voltmeter just described. The arrangement of the measuring wire of an ammeter is shown in Figure 41. The hot wire is divided into four equal parts by thin silver foil, strips F_1, F_2, and F_3. The object of these strips is to divide the current passing through the measuring wire. Referring to the figure, the current enters the wire W at two points situated at a quarter of its length from either end, while it leaves the wire from three

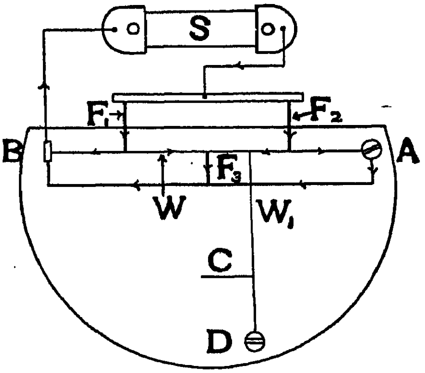


FIG. 41.—Hot-wire ammeter.

points, one being situated in the middle and the other two at the ends. The arrows in the diagram indicate the direction in which the current flows through the working wire. By suitably arranging the number of the contacts four or five ampères may be passed through the wire with a drop of potential not exceeding 0.25 of a volt.

All ammeters are provided with a shunt S, so that this type of instrument can measure currents of almost any magnitude provided the shunt is designed to give a fall of potential of 0.25 of a volt at full load.

The means provided for re-setting the pointer to zero when necessary, consists of a very fine pitched screw passing through the support B of the measuring wire. This screw, which is accessible through a small aperture in the case, moves very slightly the arm of the stud to which the wire is attached. The arm is forced back against the set-screw by means of steel springs.

Characteristics of Hot-Wire Instruments.—The main advantage of hot-wire instruments is that they are unaffected by stray magnetic fields and have no hysteresis error. Their disadvantages, however, far outweigh their advantages, and are as follows:

1. Large consumption of power, the drop in an ammeter varying from 0.1 to 0.5 of a volt, and the current taken by voltmeters being from 0.1 to 0.4 of an ampère.

2. Uncertainty of zero, and very sluggish action, as the final temperature of a wire for a particular current is only gradually reached.

3. Liability in an ammeter of the working wire to fuse. These wires are usually raised to a high temperature in order to get the maximum possible elongation, hence a momentary increase of current might destroy the wire. Further, these instruments, in common with all others giving deflections proportional to the square of the quantity measured, have very unevenly divided scales, being excessively open at the maximum readings and cramped at the lower readings. Consequently they are never calibrated below one-fifth of the maximum reading.

ELECTROSTATIC VOLTMETERS

The principle on which electrostatic voltmeters work is that of an air condenser, in which one metal part is movable about an axis so as to increase or diminish the capacity. If two sets of insulated metal plates, one of which is movable about an axis, as shown in Figure 42, be connected to two points at different potentials, they will become charged to the potentials of the respective points. The moving and fixed plates being at different potentials a force of attraction ensues, and the moving plate A tends to set itself alongside the fixed plates B, and may take up the position shown by the dotted lines.

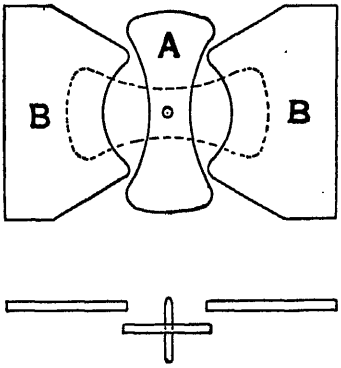


FIG. 42.

The capacity of the system will then have changed by a definite amount C, and the change in energy is equal to $\frac{1}{2}$ CV^2, V being the difference of potential between the two plates. If the moving plate be provided with a controlling force any movement will produce a couple tending to bring the moving system back to its original position. For a given form of control, this controlling force will depend solely upon the angle of rotation and may be denoted by Kθ, where θ = the angle of rotation, and K a constant. When the moving system comes into a position of equilibrium the deflecting and controlling forces must be equal, that is, the gain in energy of the system must equal the work done against the controlling force,

$$i.e.\ \tfrac{1}{2}\,CV^2 = K\theta$$

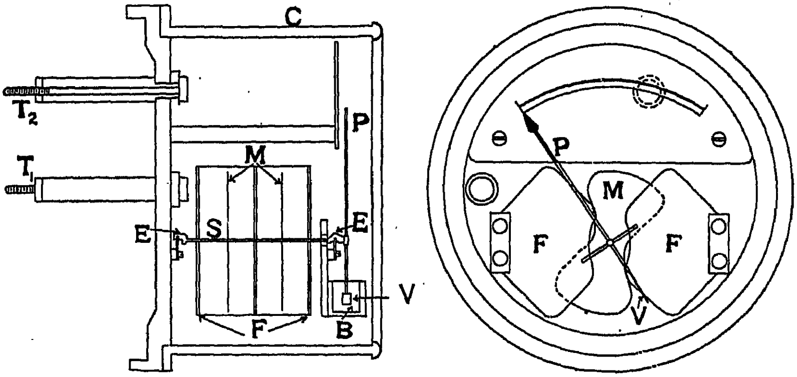
$$\text{and } \theta = \frac{CV^2}{2\,K}.$$

Hence there will be a definite relation between the pressure and the resultant angular movement; this movement may be taken as a measure of the pressure.

Instruments constructed on the above principle will therefore give deflections proportional to the square of the difference of potential between the two plates, thus causing the scale to be irregular, as in hot-wire instruments.

Kelvin's Electrostatic Voltmeters.—A type of voltmeter in which the foregoing principles are applied has been devised by the late Lord Kelvin. These instruments are made in two forms—(1) high-tension dial form, for measuring pressures above 1000 volts, and (2) multicellular form, for pressures up to 1000.

(1) *High-Tension Dial Form.*—This instrument is illustrated in Figures 43 and 44. It consists of two aluminium vanes M attached to a steel spindle S, and capable of moving between the fixed plates F. The spindle S is supported on knife-edges E and carries a light pointer P, moving over a graduated scale. The fixed plates F are insulated from the brass case C by vulcanite supports and connected to an insulated terminal T_2; the moving system is connected to terminal T_1 through the case C. The controlling force is furnished by gravity. When a difference of potential exists between the plates M and F, the moving vanes M are attracted in between the fixed plates. The moving system thus rotates against the controlling force, and when the forces balance the position of the pointer



FIGS. 43 and 44.—High-tension electrostatic voltmeter.

P on the scale indicates the pressure. Damping is effected by a small vane V, attached to the lower end of P, moving in an oil bath B. The case is surrounded by tin-foil, to screen the movement from external electrostatic effects.

(2) *Multicellular Form.*—For accurately measuring potentials below 1000 volts the dial form is not suitable, since, with only two moving vanes, difficulty is experienced in obtaining a sufficient deflecting force. This force can, however, be augmented by employing a greater number of vanes, each moving between a pair of fixed plates. Such an arrangement is referred to as a multicellular electrostatic voltmeter.

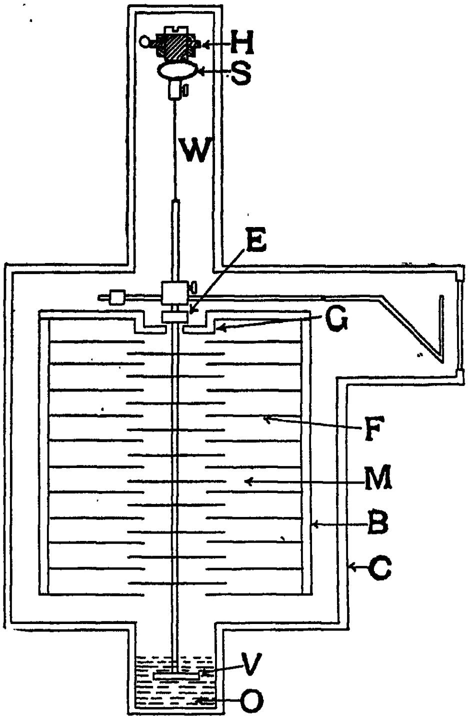


FIG. 45.—Multicellular electrostatic voltmeter

Figure 45 illustrates this instrument. The fixed plates F, of triangular shape, consist of two sets, as shown, and are fixed to brass supports B. C is the outside case to which two insulated terminals are fixed; one of these terminals is in connection with B, the other one being connected to the moving system.

The moving system consists of a number of aluminium vanes M of the form shown in Figure 42. These vanes are mounted on a spindle and suspended by a fine phosphor-bronze wire W, so that the vanes are free to move, each between a pair of fixed plates. The upper end of the suspension wire is attached through a carriage spring S to a torsion head H, provided with a tangent screw for adjusting the zero. The function of the carriage spring between the torsion head and the suspension wire is to prevent the latter from breaking when accidentally jerked. Should the vanes be jerked downwards, then the carriage spring yields sufficiently to allow the safety-sleeve E to come into contact with the guide stop G before the suspension wire is overstrained.

The working of this instrument is similar to the dial form, except that the controlling force is furnished by the torsion of the suspension wire as the moving system rotates. The controlling force thus differs from that generally employed in indicating instruments. At the lower end of the moving system is a disc V, dipping into an oil bath O, which acts as a damper on the moving system.

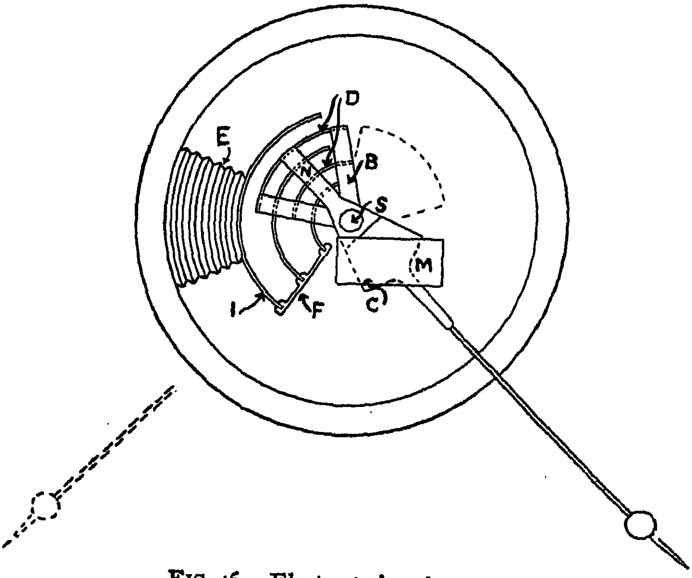


FIG. 46.—Electrostatic voltmeter.

Ayrton and Mather's Electrostatic Voltmeter.—This instrument is used for the measurement of potentials between 1000 and 3000 volts. The construction will be understood by referring to Figure 46. The plates, both fixed and moving, are curved. The fixed plates consist of three metallic inductors I fixed to a metallic endpiece F and mounted on a corrugated ebonite block E, the latter being fixed to the instrument case. The moving system N consists of two aluminium sheets D, concentric with each other and the axis about which they turn, carried by light arms B on a horizontal spindle S set in jewelled bearings. The arm B at one end of the spindle terminates in a light copper plate C, which moves between the poles of a permanent magnet M.

The fixed and moving plates are in connection with two terminals fixed outside the case; the terminal connected to the fixed plates is highly insulated, and the other is connected to the moving system through the case; when the plates are at different potentials the moving system is attracted against the force of gravity inside the fixed plates, and takes up a position of rest when the two forces balance. Damping is effected by eddy currents induced in the plate C, as it moves between the poles of the magnet M.

Instruments working on the electrostatic principle are of all instruments the least liable to change with time. Since there is no circuit through the instrument, they consume no power and are also unaffected by temperature changes. They are unaffected by stray magnetic fields, although electrostatic fields (set up, for instance, by such a simple process as rubbing the glass of the case to clean it) may cause considerable errors. This error may, however, be minimised by covering the glass with a transparent conducting varnish.

The great difficulty met with in the design of electrostatic voltmeters is that the forces dealt with are so small that frictional errors are difficult to avoid. In order to increase the deflecting force the distance between the fixed and moving plates has to be reduced to a minimum, with the result that sparking may occur across the plates should there be an abnormal rise of voltage.

In order to increase the range of electrostatic voltmeters, and at the same time to avoid sparking difficulties, a system of condensers may be made use of. Figure 47 shows the arrangement diagrammatically. An electrostatic voltmeter V, reading up to say one-third of the maximum voltage, is connected across one of the three condensers A, B, and C, the condensers being connected in series across the points M and M, whose potential difference is to be found. The voltage at the terminals of each condenser is inversely proportional to the capacity. If the condensers have each the same capacity, and that of the voltmeter V be very small, then the voltmeter will indicate one-third of the difference of potential between M and M. Should the voltmeter capacity not be small compared with the condensers, then the capacity of A and V in parallel will much exceed that of A alone, and so upset the simple voltage ratio mentioned above Voltmeters on this principle are constructed reading up to 40,000 volts.

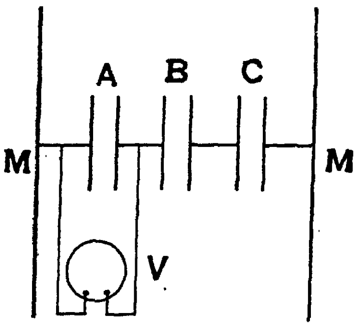


FIG. 47.

ELECTRO-DYNAMIC INSTRUMENTS

The construction of electro-dynamic instruments is based upon the mutual forces between movable and fixed portions of a circuit conveying current. The fixed portion of the circuit often consists of a rectangular coil F (Fig. 48) of one or more

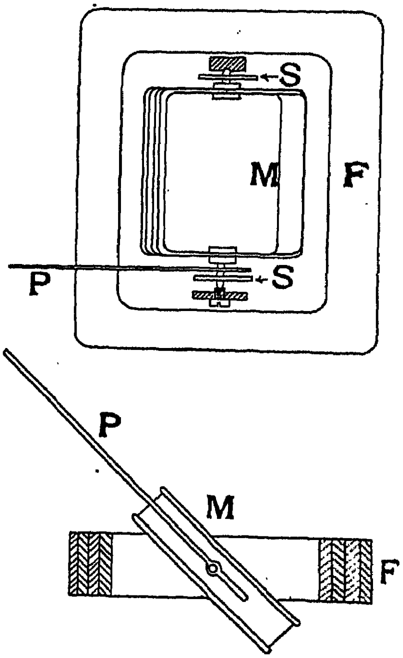


FIG. 48.—Electro-dynamic instrument.

turns of insulated copper strip, and is fixed with its plane horizontal. The moving coil M is mounted on pivots and set in jewelled bearings, so as to lie inside the fixed coil.

The current is led into and out of the moving coil by means of phosphor-bronze hair-springs S S, which also act as the controlling force. When no current flows through the coils, the moving system takes up the position shown in the figure. On the coils becoming energised a magnetic field is set up by each coil, and they tend to arrange themselves so that their fields are coincident. The moving coil therefore experiences a rotative force, and moves against the controlling force of the hair-springs into a new position, where it is in equilibrium. To the moving system is fixed a pointer P, which moves over a graduated scale.

The force of attraction or repulsion exerted between the fixed and moving coils is proportional to the product of the ampère-turns of the fixed coil and the ampère-turns of the moving coil,

i.e. deflecting force $\propto C_f N_f \times C_m N_m$.

The number of turns N_f and N_m on the fixed and moving coils respectively are constant, so that the deflecting force $\propto C_f C_m$.

In an ammeter the current in the moving coil is either equal to or proportional to the current in the fixed coil, in which case the deflecting force is proportional to C_f^2.

A voltmeter has its moving and fixed coil connected in series, so that the deflecting force is proportional to E^2.

Electro-dynamic ammeters and voltmeters give, therefore, deflections proportional to the square of the quantity being measured.

Figure 49 shows the movement of an electro-dynamic instrument. In this form the moving coil M swings outside the fixed coil F; the latter being wound in the form of a ball. The moving coil is mounted on pivots B_1 B_2 set in jewelled bearings, and the current is led into and out of the coil by means of the phosphor-bronze hair-springs H. To the top pivot B_1 is attached a pointer P, which moves over a graduated scale. Eddy-current damping is effected by causing an aluminium disc D attached to the moving system to rotate between the poles of four permanent magnets PM.

The field of these instruments is so weak that considerable errors may be introduced due to stray magnetic fields, and in order to protect them, they must be shielded with cast-iron cases. Not more than 1 ampère can conveniently be passed through the moving coil, so that a shunt becomes necessary in the case of ammeters.

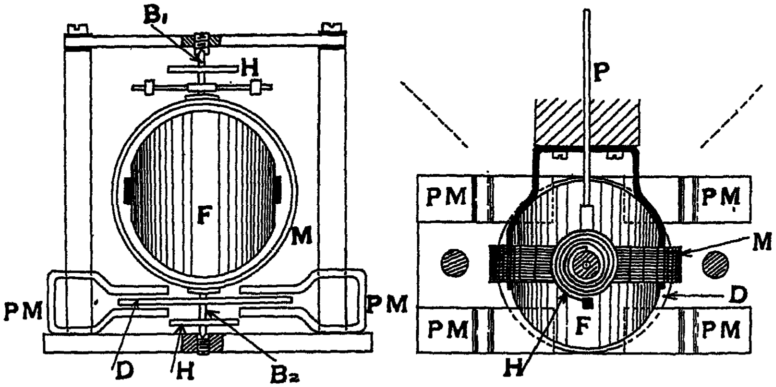
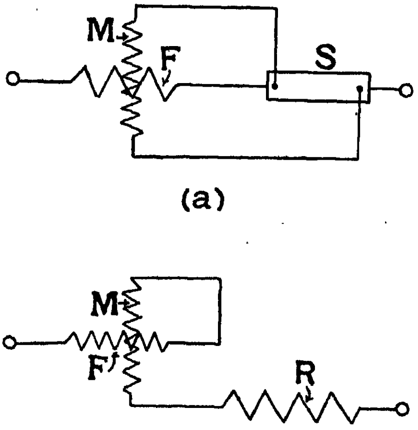


FIG. 49.—Electro-dynamic instrument.

Owing to the weak fields employed (usually not more than one-tenth of that of a permanent-magnet moving-coil instrument) the ampère-turns of the moving coil must be relatively high, so that a much greater fall of potential is required over the shunt. Again, the power necessary to overcome friction is usually greater than that required by a moving-coil instrument, owing to the greater weight of the moving parts. As a rule 20 to 30 ampère-turns are employed in the moving coils, and in order to keep down the temperature coefficient of the moving system a drop of nearly half a volt is required.

Figure 50 (*a*) shows diagrammatically the electrical connections for an ammeter indicating large currents. The fixed coil F carries the main current, and the moving coil M is connected across a shunt S; the shunt is in series with the main circuit, and is designed to give a fall of potential of 0.5 volt at full load.

In a voltmeter the moving and fixed coils M and F respectively in Figure 50 (*b*) are arranged in series, together with the necessary resistance R having a negligible temperature coefficient.



(a)

(b)

FIG. 50.

Wattmeters.—Wattmeters work on the electro-dynamic principle. The main current taken by the circuit whose power is to be measured, is passed through the fixed coil in Figure 48, and the moving coil is energised by the potential current. The deflecting force is proportional to $C_m \times C_f$, but C_m is proportional to the voltage of the circuit, and C_f is the main current; therefore the deflecting force is proportional to the product EC, where E is the pressure, and C the current in the circuit to which the wattmeter is connected.

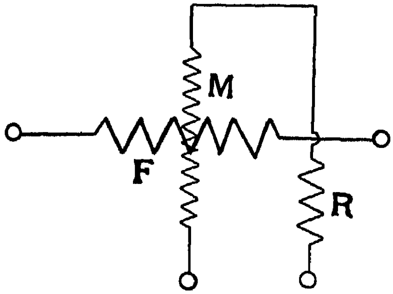


FIG. 51.

Figure 51 indicates the connections of a wattmeter: the moving coil M, carrying the potential current, together with the resistence R, is connected across the mains, while the fixed coil F is connected in series with the main circuit.

Wattmeters are seldom used on direct current circuits, being mainly employed for the measurement of alternating current power. For a fuller treatment of electro-dynamic instruments, the student is referred to text-books on alternating currents.

RECORDING INSTRUMENTS

The ammeters, voltmeters, and wattmeters which have, so far, been described, indicate the momentary value of the current, pressure, or power. It may, however, in some cases be necessary to obtain a permanent record of the variation in the value of the quantity measured during a certain period, say for twenty-four hours or more. Instruments for fulfilling this purpose are known as "recording instruments." Their electrical parts are constructed on the same principles as indicating instruments, and they are made recording by fitting them with a light cylinder or drum, which is rotated, by clock-work, at some definite speed. Long bands of paper, about 12 centimetres wide, are unwound from the drum at a rate ranging from 2.5 to 21 centimetres per hour. Holes are usually punched down one edge of the paper at a uniform distance of 1 centimetre apart; they engage in pins in a wheel driven by the clock, so that the movement of the paper is very definite and accurate.

Attached to the end of the pointer of the electrical part of the instrument, is an ink pen which presses against the paper chart as it unwinds from the drum. The chart paper is ruled longitudinally and cross-wise, the former being calibrated in time and the latter in terms of the quantity being recorded. The scale of the ruling will of course depend upon the rate of movement of the chart and the range of the instrument. The clock for such instruments is generally made to go for eight days, and is wound by means of a lever movement outside the case.

STANDARD INSTRUMENTS

Kelvin's Standard Balances.—These instruments, devised by Kelvin, work on the electro-dynamic principle. They are among the most accurate and reliable electrical instruments constructed, and are therefore used as standards. There are several forms, all alike in principle but differing in the details of construction, depending on whether the instrument has to measure current, pressure, or power.

Current Balance.—In this instrument there are two movable coils or rings, each actuated by two fixed coils, all the coils being set with their planes horizontal. Figures 52 and 53 are respectively a diagram of the connections, and a general view of the instrument. Referring to the former, the two movable rings M_1 and M_2 are attached to the ends of a horizontal balance arm, which is supported by two trunnions, each hung by elastic ligament, represented by L_1 and L_2. These consist of a number of fine copper wires laid side by side and soldered to suitably insulated contact pieces; the ligaments also serve to lead the current into and out of the movable coils. Above and below each movable coil is a fixed coil, indicated by F_1, F_2, F_3, and F_4; the fixed and movable coils are connected in series between the terminals T_1 and T_2. The current flows in opposite directions through each two adjacent fixed coils, so that the movable coil is attracted by one

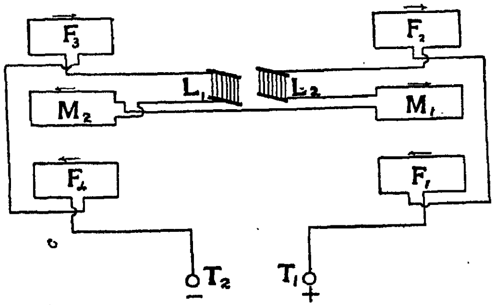


FIG. 52.—Diagram of connections for the Kelvin current balance.

oil and repelled by the other. The balance arm the zero position when the movable coils are midway between the respective pairs of fixed coils. When the current to be measured is passed through the instrument the balance arm is tilted, and the

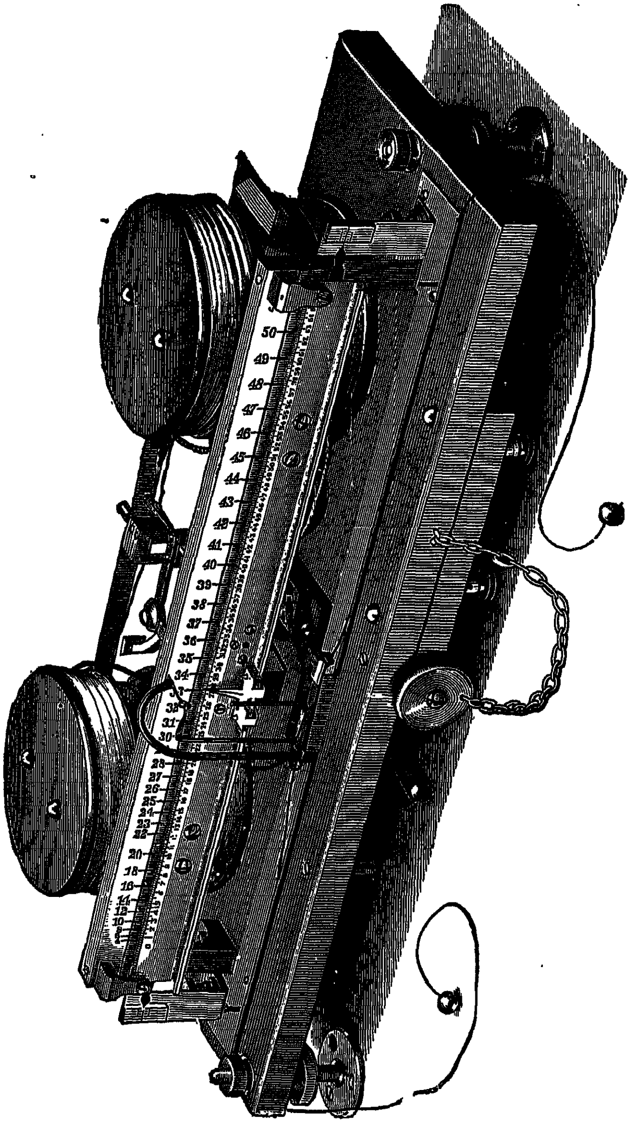


FIG. 53.—Kelvin's current balance.

deflecting couple acting on the movable system is proportional to the square of the current. The deflecting couple is balanced by a weight, which brings the movable system back to the zero position. This weight is made to move along a horizontal, finely-graduated scale attached to the balance-arm; the moving of the weight into its proper position is performed by means of a self-releasing pendant hanging from a hook carried by a sliding platform, which is pulled in either direction by two silk threads shown in Figure 53.

Since the balancing couple is proportional to the displacement of the weight, the current is proportional to the square root of the displacement; so that from the scale reading the value of the current can be obtained by taking the square root and multiplying by a certain constant.

In addition to the finely-divided scale referred to, there is a fixed inspectional scale shown above the other in the figure; the reading on which gives an approximation to the value of the current. For fine adjustment of the zero a small metal flag is provided, as in an ordinary weighing balance. This flag is actuated by a fork having a handle outside the case, as shown protruding just below the base of the instrument in Figure 53. At the right-hand end of the balance there is a trough into which a counterpoise weight is placed. The latter is for balancing the weight of the carriage and its contained weight. Several pairs of weights (moving and counterpoise) are supplied with each instrument; these weights are adjusted in the ratios of 1, 4, 16, and 64.

Composite Balance.—This instrument is in principle similar to the ampère balance, and can be used for the measurement of power, pressure, or current. It is represented diagrammatically in Figure 54. The left-hand pair of fixed coils are made of copper cable capable of carrying 500 ampères, and connected to two terminals at the back of the instrument. The other pair of fixed coils are wound with the same size of wire as the movable coils, and connected to the terminal T_2

and contact S. The movable coils are in series, one end being connected to the terminal T_1 and the other to the switch lever L. The switch allows the movable fine wire coils either to be included in the circuit by themselves when the lever is in contact with T_2, or in series with the fixed fine wire coils when in contact with S. The fixed and movable coils have a resistance of about 18 and 12 ohms respectively.

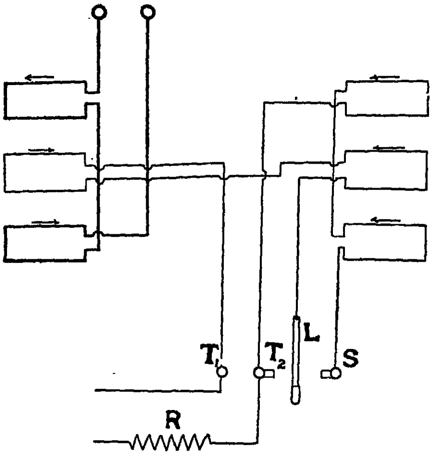


FIG. 54.—Diagram of connections for Kelvin's composite balance.

To enable the composite balance to be used as a direct reading wattmeter or voltmeter, a separate non-inductive resistance R, having four tappings, is connected as shown. The balance is provided with four pairs of weights, which, together with the extra resistance, can be adjusted to give a round number of watts or volts per division.

To use the balance as a voltmeter the switch is turned to S. When being used as a wattmeter the fixed fine wire coils are cut out of circuit by moving the switch to T_2, and the thick wire coils are connected in series with the main circuit. When using the balance as a centi-ampère meter the switch is turned to S. To use as a hecto-ampère balance the main current is sent through the thick wire coils, and a pre-determined constant current through the two movable coils.

Potentiometer.—The potentiometer is one of the most reliable instruments to use as a standard for the measurement of current, electromotive-force, and resistance. Measurements are made by opposing an unknown E.M.F. or P.D. by a known P.D., which exists between two points of a suitable resistance, frequently in the form of a wire.

The principle is illustrated in Figure 55. The potentiometer itself consists of a long wire AB of uniform cross section fixed between two terminals at A and B. In series with AB is a cell E to provide a constant E.M.F., and a variable resistance R, the positive terminal of E being joined to A. If the positive terminal of a second cell E_1 be connected to A, the former will be at the same potential as' the latter, and the difference of potential between A and the free end D will be the same as the E.M.F. of E_1. By means of the variable resistance R the P.D. across AB is adjusted, so that there will be a point on AB at the same potential as D, and if D be connected to this point C no current will flow. The P.D. between A and C is then equal to the E.M.F. of E_1, and since the wire AB is uniform the length AC is a measure of the E.M.F. of E_1. If contact be made to the right of C, current will flow from D through E_1 to A, and if contact be made to the left of C, current will flow in the reverse direction. So that by inserting a galvanometer G in series with E_1 the point C can be located.

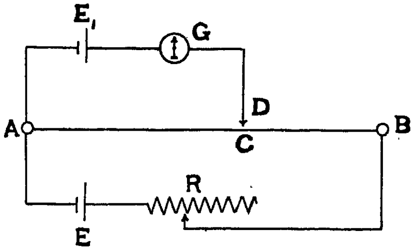


FIG. 55.—Principle of potentiometer.

There are several forms of potentiometers, but they all work on the above principle, so that a description of one form will be sufficient to illustrate the application of the principle.

The instrument illustrated in Figure 56 works entirely by adjusted resistances, so that there is no slide wire liable to injury. A secondary cell is joined to the terminals F, and sends a current through the dials C and D and the adjusting resistances K and H, the latter all being connected in series. In the dial C are 150 exactly equal resistances. The dial D is also divided into 100 equal resistances, the sum of which is equal to one of the resistances in dial C. The two together are therefore equivalent to a slide wire with 15,000 fixed contacts at equal distances. The adjusting resistances K and H are units, and this would mean an annual loss of 11 shillings. In a large supply system, where there may be hundreds of such meters, the loss might amount to hundreds of pounds per annum.

By applying the same precautions as in ammeters and voltmeters, the first part of condition 3 can be easily secured, and the second part of the same condition may be observed by so reducing the coefficient of friction of the moving parts that the meter will start when the smallest power-consuming apparatus is switched on to the supply mains.

Condition 4 is of importance to both the consumer and the supply company. If the meter read low then power is supplied for which no return is received. On the other hand, if the meter read high the consumer is paying for more than he receives.

Modern supply meters may be divided into two classes.

First, those in which the Board of Trade units are recorded by simply integrating current and time, the supply voltage remaining constant. The ampère-hours can then be multiplied by a constant to give watt-hours, though even this is not necessary, as the instruments can be calibrated to record directly in Board of Trade units. Those meters which do not take into account the difference of pressure, are known as ampère-hour or coulomb meters. To this class the majority of supply meters belong.

Second, those in which the Board of Trade units are recorded by integrating current, pressure, and time. They measure the true energy absorbed by a circuit, and are known as watt-hour meters.

Again, according to the principles upon which they work, electricity supply meters may be divided into three distinct types, each of which comprises both of the classes referred to.

1. Chemical or electrolytic type.
2. Motor type, which may be sub-divided into—
    (*a*) Meters in which the rotating part is a wound armature;
    (*b*) Meters in which the rotating part is mercury.
3. Clock type.

In order to demonstrate the application of the principles involved, a meter belonging to each of the types will now be explained.

Chemical or Electrolytic Meter.—The principle on which these meters work is that when a current is passed through an electrolyte the deposition is proportional to the quantity of electricity. Figures 57 and 58 indicate the arrangement and electrical connections of a meter of this type.

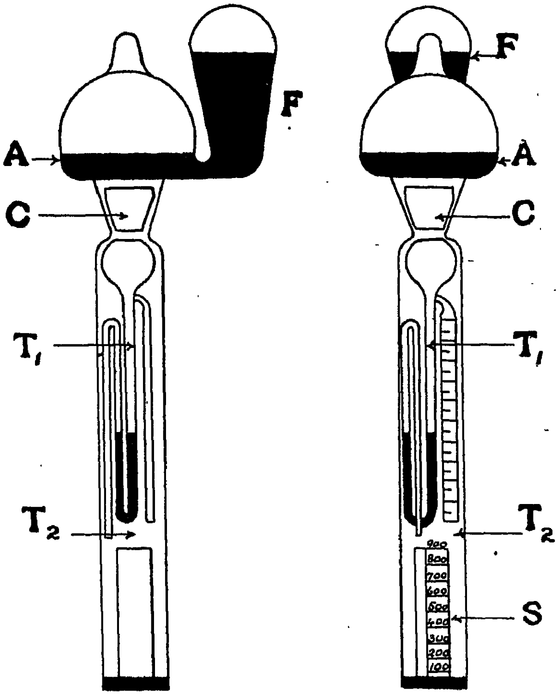


FIG. 57.—Electrolytic meter.

F is a reservoir filled with mercury and connected by a narrow neck to a flat glass vessel, the lower part of which is provided with an annular groove containing the mercury A. This circle of mercury forms the anode. The cathode C is in the form of a bowl and made of iridium; a metal which is not amalgamated by the electrolyte. The electrolyte consists of a solution of mercuric iodide in excess of potassium iodide, which forms the salt K_2HgI_4. The entire vertical tube T_2 is filled with the latter and then hermetically sealed.

In series with the electrolyte is placed a high resistance H, and both are connected in parallel with a low resistance shunt K made of manganin strip. The relation between the resistance of the shunt and the electrolyte is such that only $\frac{1}{100}$ of the current to be metered is passed through the electrolyte. Referring to Figure 58, the current enters at the terminal E, and the greater part passes round the low resistance K to the terminal D. The operating current passes from E to the anode A, through the electrolyte to the cathode C: hence it goes by way of the resistance H to the terminal D.

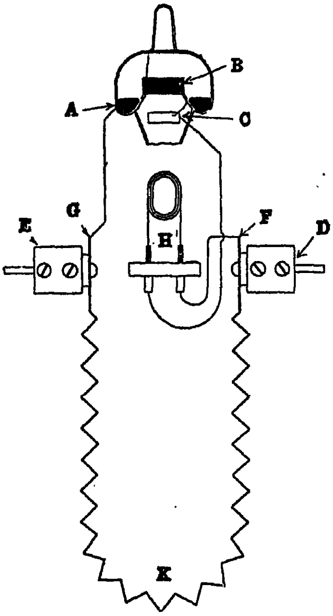


FIG. 58.—Diagram of connections for electrolytic meter.

When current is passed through the meter the electrolyte is decomposed, and the mercury along with the iodine travels to the anode, while the potassium travels to the cathode: but the potassium never appears as such, for it acts on the surrounding liquid, forming iodide of potassium and liberating mercury, which falls in minute globules into the first graduated tube T_1. At the anode, mercury dissolves to mercuric iodide, and a large excess of potassium iodide keeps this in solution by forming constantly K_2HgI_4.

The tube T_1 is graduated to read direct in B.T.U., and is made in the form of a syphon, so that when it is filled by a quantity of mercury equal to 100 units it automatically and completely empties itself into a lower tube T_2, which is provided with a scale S, of which each division is equal to 100 units. There are nine of these divisions, bringing the total capacity of the meter up to 1000 units.

The mercury as it is dissolved from the anode is simultaneously replaced by fresh mercury drawn from the anode feeder F. The mercury in the latter forces its way past the narrow neck connecting the two vessels, when the level of the mercury A falls slightly. After 1000 units have been registered the meter has to be re-set to zero. This is done by inverting the tube, so that the mercury is returned to the anode and feeder.

The point at which the mercury syphons over is first determined by experiment, and the distance between this point and zero is made equal to 100 units. The volume of the measuring tubes varies according to the voltage of supply. For example, if an instrument which reads up to 1000 B.T.U. be placed in a circuit taking 100 ampères at 500 volts, then the mercury should fill the tubes in 20 hours, recording $\frac{20 \times 100 \times 500}{1000} = 1000$ B.T.U. Now suppose the same meter is placed in a circuit taking the same current, but this time at 250 volts, since the amount of mercury liberated is proportional to the current and independent of the voltage, the measuring tubes would become filled in 20 hours as before, and 1000 B.T.U. would be indicated, when only 500 had been used. To remedy this the measuring tubes would have to be replaced by ones having double the volume, and thus taking 40 hours to fill. Each meter will only read correctly when used in a circuit having the same voltage as that for which the meter was calibrated. In the meter described the tube T_1 is so graduated that 2 millimetres represent 1 B.T.U.

Example.—In the above meter the electro-chemical equivalent of the electrolyte is 0.001037. The first graduated tube T_1 has a length of 20 centimetres, and syphons over when 100 B.T.U. are recorded. Find the internal bore of this tube for two meters, one to be used on 230 volts and the other on 460 volts. A cubic centimetre of mercury weighs 13.6 grammes, and $\frac{1}{100}$ of the main current goes through the electrolyte.

Weight of mercury deposited

$= e \times$ coulombs passed through the electrolyte.

Ampère-hours passed through the main circuit when pressure is 230 volts

$$= \frac{100 \times 1000}{230} = \frac{10{,}000}{23}.$$

Therefore coulombs passed through the electrolyte

$$= \frac{10{,}000}{23} \times 3600 \times \frac{1}{100} = 15600.$$

Weight of mercury deposited

$= 0.001037 \times 15600 = 16.3$ grammes.

Volume of mercury deposited

$$= \frac{16.3}{13.6} = 1.2 \text{ cubic centimetre.}$$

Area of cross-section of tube

$$= \frac{\text{volume}}{\text{total length of tube containing mercury}}$$

$$= \frac{1.2}{2 \times 20} = 0.03 \text{ square centimetre.}$$

If d denote the internal bore of the tube, then

$$0.03 = \frac{\pi d^2}{4}$$

$$\text{or } d = \sqrt{\frac{0.03 \times 4}{\pi}} = 0.19 \text{ centimetre.}$$

Therefore a meter used on a 230-volt circuit will be provided with a tube having an internal bore of 1.9 millimetre. By a similar calculation it will be found that for a 460-volt circuit the internal bore of the tube should be 1.4 millimetre.

The relation between the resistance of H and K is calculated in the first instance. The consideration which determines the resistance of the shunt K is the all-important one of the fall of potential across the meter terminals, which ought to be as low as possible to prevent undue loss of energy, yet high enough to give an appreciable reading for 1 B.T.U. The potential drop in these instruments varies between 0.5 and 1 volt.

The current in the main circuit when the demand is one unit per hour $= \frac{1000}{E}$, E being the pressure of supply. From the formula $W = e \times C \times t$ the following equation is obtained:

Grammes of mercury per millimetre on scale

$$= e \times \frac{1000}{E} \cdot \frac{R}{r}.$$

Where R is the resistance of K and is calculated from a knowledge of the permissible volts drop at full load, $r =$ total shunt resistance and $e =$ the electro-chemical equivalent of the electrolyte. The resistance between the electrodes is measured, and knowing this the value of the additional resistance H to be placed in the shunt can be found, so that the total shunt resistance $= r$.

The exact final adjustment is made during calibration by sliding the wires G and F up or down in the terminals E and D, and so varying the value of K. When the final adjustment has been effected the ends are brazed to the shunt as shown.

In the early forms of shunted electrolytic meters great difficulty was experienced in overcoming the back E.M.F. set up between parts of the electrolyte of differing densities. This back E.M.F. should be reduced to a minimum, as the impressed E.M.F. sending the current through the electrolyte never exceeds 1 volt. The back E.M.F. in the meter described is almost eliminated by suitably stirring the solution; this is done by gravity and is entirely automatic. Referring to Figure 59, the heavy solution formed at the anode falls, while the less dense solution at the cathode rises, the interchange of the solution being assisted by the curved surface of the mercury anode. This process is illustrated in the figure, the stream lines in the solution being shown.

Changes in temperature may also affect the accuracy of electrolytic meters if they are badly designed.

The resistance of an electrolyte diminishes with increase of temperature, while the reverse takes place in a copper wire. If, therefore, the shunt resistance H, in Figure 58, be wound with a sufficient proportion of copper, within given temperature limits, the variation in its resistance will counterbalance that of the electrolyte. The resistance between the electrodes having been measured for this purpose, the requisite amount of copper wire is first wound, and the remainder of the total calculated resistance in the shunt circuit is made up of a material having a negligible temperature coefficient.

Motor Meters: (a) With wound armature.—The meter to be described registers watt-hours, and has been devised by Professor Elihu Thomson. It consists essentially of a small motor in which the field coils carry the main current, and the armature is energised by a potential current.

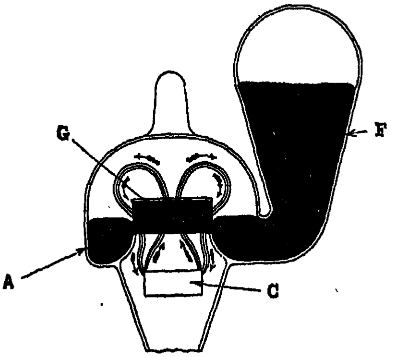


FIG. 59.—Details of top of tube, showing stream lines of circulation in electrolyte.

A. Mercury anode.
C. Iridium cathode.
F. Anode feeder.
G. Glass fence.

Referring to Figure 60. The field coils C consist of two coils of thick wire, one on either side of the armature, connected in series with each other and the circuit whose energy is to be measured. The armature A consists of a hollow ebonite frame wound with a set of coils of very fine wire, and the ends of the wire are attached to a small silver commutator E. The armature is fixed to a spindle S, which is set vertically, as shown: the lower end of S rests in a jewelled bearing B, and the upper end is geared into a chain of wheels actuating the counting mechanism. Two light springs with silver contact pieces bear upon the commutator, and constitute the brushes.

The fine wire armature is in series with a high resistance fixed to the back of the instrument and having a negligible temperature coefficient. These form the "pressure coils," the current in which varies with the voltage.

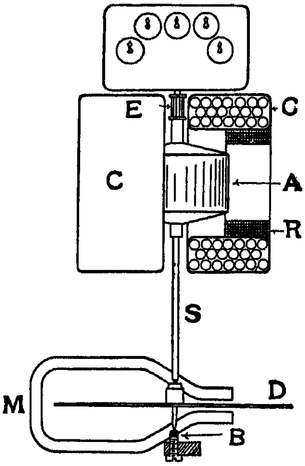


FIG. 60.—Motor meter with wound armature.

The pressure coils, having a resistance of about 3000 ohms per 100 volts, are permanently connected across the supply mains, so that, when a current flows in the field coils C, a magnetic field is set up across the armature and the latter tends to rotate. There being no iron parts, the speed of the motor will be directly proportional to the armature current × field coil current.

The torque producing rotation $= H \; l \; m \; i$ dynes, where

H = strength of field in which armature rotates, and is proportional to the main current C.

l = active length per turn of wire on armature.

m = number of turns of wire on the armature.

i = absolute units of current in the armature coils, and is proportional to the supply pressure E.

Since l and m are constant, the driving torque $\propto EC \propto$ watts.

Now, assuming there is no friction, the speed will continue to increase indefinitely, so that it becomes necessary to introduce some retarding force to make the speed vary directly as the watts. This is accomplished by fixing to the spindle a small copper disc D, which rotates between the poles of three permanent magnets (only one, M, is shown in the figure). The eddy currents induced in the disc form a drag on the motor. The ohmic resistance of the disc remaining constant, it is evident that in a constant magnetic field such as is supplied by the permanent magnets, the current generated in the disc will vary directly as the speed. Thus a retarding force has been provided, which bears the same proportion to the speed as the torque of the motor bears to the watts. The speed of the spindle actuating, the indicating mechanism will therefore be directly proportional to the watts.

Friction would very easily affect the accuracy of this instrument, and is compensated for by having a coil R in series with the armature wound along with the large coils C. This coil R being permanently connected to the mains, produces a magnetic field strong enough to just overcome the friction losses, so that the armature begins to rotate when the smallest power-consuming apparatus is switched on.

There is one inherent weakness in this method of overcoming friction, namely, should the supply voltage rise by a few per cent., then the field due to the coil R may be strong enough to start the armature.

Example.—A meter of this type has field coils which produce, at full load, a flux across the armature of intensity 200. If the meter be intended for use on a 230-volt circuit, from the following data calculate the number of turns required for the armature.

Length of armature l = 5 centimetres.
Diameter of armature d = 5 centimetres.
Resistance of armature circuit = 7000 ohms.
Torque required at full load = 3000 dyne-centimetres.
Torque producing rotation

$$= Hlmi \text{ dynes.}$$

$$\text{or } 3000 = Hlmir \text{ dyne-centimetres.}$$

$$\text{where } r = \text{radius of armature} = \frac{d}{2} = 2.5 \text{ centimetres.}$$

$$H = 200$$

$$l = 5$$

$$i = \frac{230}{7000 \times 10} \text{ C.G.S. units.}$$

Therefore the effective turns on armature

$$= m = \frac{3000 \times 7000 \times 10}{200 \times 5 \times 230 \times 2.5}$$

$$= 365.$$

The turns constituting the armature winding set up a flux which may be divided into two equal parts: (1) The effective flux at right angles to the field flux, and (2) the ineffective flux in a direction parallel to the former. Since both components are equal, and the same current flows in each coil, the total number of turns $= \sqrt{2} \times$ effective turns $= \sqrt{2} \times 365 = 515$.

(*b*) *With rotating mercury.*—A meter of this type is illustrated in Figure 61. It consists of a permanent steel magnet A, having wrought-iron pole pieces B^1 B^1 attached to it, the latter terminating in the circular poles B B. The mercury chamber is formed between the ebonite blocks E E of circular form, in which the pole pieces are embedded. The metal ring C (Fig. 62), lined with leather, surrounds the two ebonite blocks and forms the side of the mercury chamber. The copper disc D lies in the mercury and is attached to a spindle F running in jewelled centres. The lower jewelled centre is shown carried by the screw H. The upper end of the spindle is geared to and drives a counting train, the dials of this train being graduated in Board of Trade units. G is a brass weight which counterbalances the armature and prevents it from floating. Figure 62 is a horizontal section showing the position of the poles B

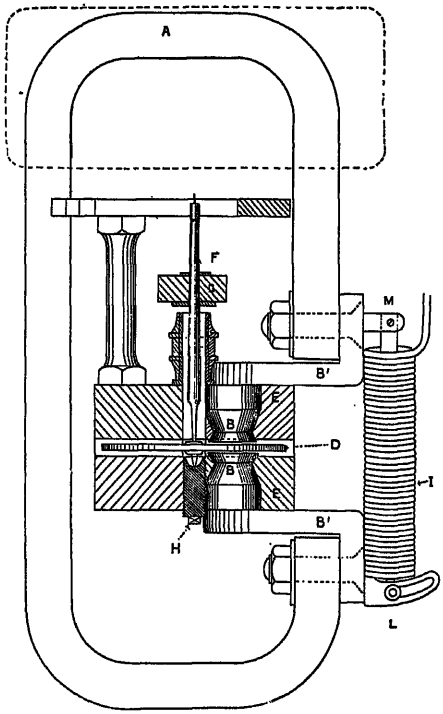


FIG. 61.—Motor meter (with rotating mercury).

and the disc D. It also indicates the wires J and K, embedded in the ebonite, which convey the current to and from the mercury.

The action of the meter is as follows:

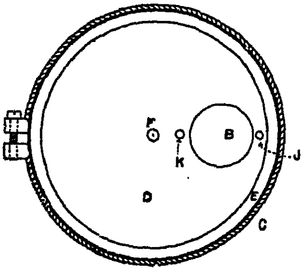


FIG. 62.

A strong magnetic field is produced between the poles B B by the magnet A. The current to be metered enters the mercury chamber at J, flows across the mercury to K, causing the former to rotate at a speed proportional to the current. The motion of the mercury is conveyed to the recording train of wheels by means of the disc D immersed in the mercury and carried round by it. The brake force of the meter is provided by the eddy currents induced in the disc D as it revolves between the poles B B. This braking force, as in the previous case, is directly proportional to the speed.

As the speed of the rotating parts increases the retardation due to the fluid friction of the mercury also increases, so that if no means is adopted to reduce the eddy current braking action, the meter would register low. This error amounts to 5 per cent., but is corrected for by winding a coil I, which carries the main current, on an iron core L M. The action of the coil is to divert part of the lines of force from the magnet poles B to the path L M. This weakens the field producing eddy currents in the disc D, and consequently reduces the retarding force. By adjusting the number of turns constituting the coil I, the reduction in the eddy current braking can be made to exactly compensate for the increased retardation due to fluid friction. The iron core L M is hinged so that the movement to or from the poles B^1 B^1 provides a sensitive adjustment of the speed of the meter. For measuring very large currents these meters are shunted across a low resistance, so that only a fraction of the main current goes through the meter.

Clock Meters.—Meters belonging to this type work by the influence of a magnetic field set up by a current upon a swinging pendulum. Figure 63 illustrates a meter of this type in which two pendulums are employed, and the difference between their periodic time is recorded on a set of dials. Great sensitiveness is obtained by accelerating one pendulum and retarding the other, the change in the periodic time in each case being produced by the magnetic attraction or repulsion between coils carrying the main current and other coils which are attached to the pendulums and energised by the supply voltage. The difference in periodic time thus produced is proportional to the watts, and is recorded through a chain of wheels on the dials of the meter.

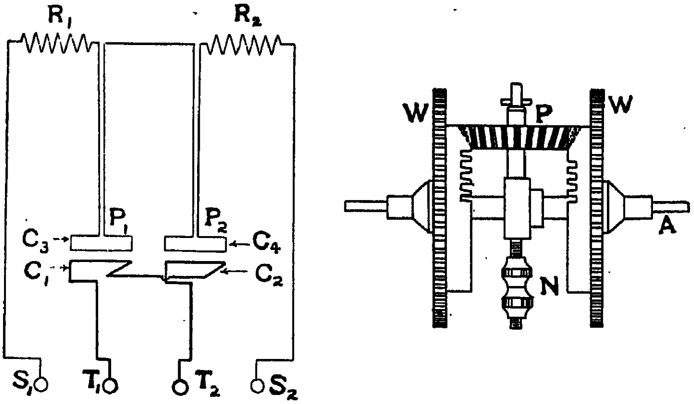


FIG. 63.—Clock meter.

Referring to the figure, T_1 and T_2 are the two main terminals. The current of the circuit to be metered enters at T_1, and passes through the two coils C_1 and C_2 to the terminal T_2. S_1 and S_2 are the terminals of the fine wire coils C_3 and C_4, which are connected in series with the resistance R_1 and R_2. The coils C_3 and C_4 are wound on the pendulums P_1 and P_2 respectively, and carry a current proportional to the voltage. C_1 and C_3 are so wound that they retard the oscillations of the pendulum P_1, while C_2 and C_4 accelerate the oscillations of P_2. The difference in the oscillations of the two pendulums is recorded on the dials through differential gear also shown in Figure 63. The two side wheels WW run loose on the main driving axle A, and are driven in opposite directions at speeds proportional to the rate of oscillation of the two pendulums respectively. P is the planet wheel which gears into WW, and so long as the latter revolve at the same speed the planet wheel will simply turn on its own axis. On the other hand, if the wheels WW are driven at different speeds the main axle, which is at right angles to them, will revolve, carrying the planet wheel and its counterpoise nuts N bodily with it. To the main horizontal axle A is geared the counting mechanism, whose indications will be proportional to the difference in speed between the two side wheels.

The winding gear consists of an electro-magnet which pulls round an armature once every 30 seconds, winding up a short stiff spiral spring which releases itself and drives a double set of clockwork. By this addition the meter is made entirely automatic.

MAXIMUM DEMAND INDICATORS

These instruments are often connected in series witl consumers' mains to determine the price to be chargec for the electrical energy consumed.

The expenses incurred in connection with a generatin station may be regarded as consisting of two separat and distinct parts, namely, standing charges and runnin charges. The standing charges are made up of suc items as interest on borrowed money, depreciation, taxes wages, and repairs; these charges are, for a station o given capacity, the same whatever the load may be The running charges include cost of fuel, oil, etc., an in a modern generating station do not exceed 0.5 of penny per unit generated.

Now consider two consumers A and B each re quiring the same amount of energy (say 20 B.T.U. over a given period. Suppose A's 20 units to be mad up of 10 kilowatts for 2 hours, while B's 20 units are mad up of 2 kilowatts for 10 hours. Plant to the extent

10 kilowatts must be provided in the station for A's requirement, while only 2 kilowatts are required to supply B, so that the standing charges in A's case will greatly exceed those of B, while the running charges will be practically the same.

It will therefore be seen that in two cases where the total energy consumed in a given time is the same the consumer requiring a large supply of power for say one hour is not so profitable as the consumer who requires a less amount of power for the greater part of 24 hours. In order to differentiate between good and bad consumers the greatest amount of power which is required at any period is measured by a maximum demand indicator, which is connected along with the supply meter in series with the consumer's mains. The consumer is then charged at the rate of say 6d. per unit for the first 100 hours at the maximum rate of demand, and 3d. to 1d. per unit afterwards. For instance, suppose an energy meter indicates that 500 B.T.U. have been consumed and the demand indicator reads 1 kilowatt, the consumer at the above rate is charged 6d. per unit for 1 kilowatt × 100 hours = 100 B.T.U., and 3d. to 1d. according to the nature of the demand for the remaining 400 units.

It is essential that the demand indicator should be sluggish in its action, so that it does not record momentary rises in current due to the switching on of a motor or an arc lamp. In fact, demand indicators should measure the average maximum value of the consumer's load.

Demand indicators are invariably constructed on either the thermal or electro-magnetic principle, and an example of the former type is shown in Figure 64. In principle the instrument is practically a differential recording thermometer, which measures the heat produced by an electric current. It consists of two bulbs A and B, of approximately the same size, connected by a U-tube C filled with a very hygroscopic liquid, and provided with a third graduated tube D closed at the bottom. A hygroscopic liquid, such as sulphuric acid, is necessary in order that the air in both bulbs may be kept free from aqueous vapour. Round the bulb A is wound a heating coil E consisting of one or two turns of platinoid strip, the ends of which are brought out to the two terminals T_1 and T_2. The coil E is connected in series with the main circuit, and the heat produced due to the passage of a current causes the air in bulb A to expand and depress the column of liquid in the left-hand limb of the U-tube, with the result that it rises in the other limb and slowly overflows into the reading tube D. The height to which the liquid rises indicates the maximum current which has passed through the coil E.

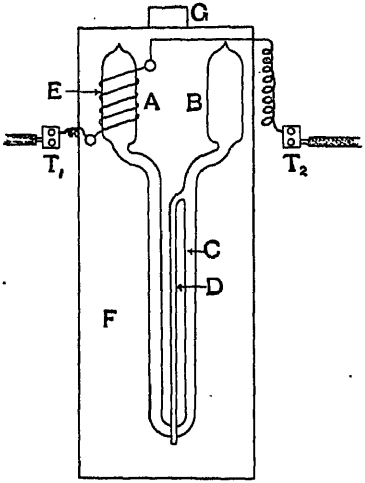


FIG. 64.—Maximum demand indicator.

The tubes are fixed to a board F, which is hinged at G inside an iron case. To re-set the instrument the board is tilted, thus allowing the liquid to flow completely out of the reading tube into the right-hand bulb.

CHAPTER V

STORAGE BATTERIES

If two plates, one of pure lead and the other of lead peroxide, be placed in dilute sulphuric acid and connected to an ammeter a current will flow through the circuit, leaving the cell at the lead peroxide plate. After a time the surface of the plates becomes changed and the current gradually falls to zero. On sending a current through the cell in the opposite direction the plates return to their original condition, and the cell will again supply current.

Such a cell was discovered by Planté in 1860, and is known as either a *storage cell*, a *secondary cell*, or an *accumulator*. The process of regeneration is known as *charging*, and *discharging* takes place when the cell supplies current. The plate by which the current leaves the cell during discharge is called the *positive plate*, and the other the *negative plate*. Positive or lead peroxide plates are of a dark brown colour, while negative plates are a neutral grey.

An electrical storage cell may be defined as a piece of apparatus which is capable of receiving, retaining, and giving up again, when required, electrical energy. The name "storage cell" or "accumulator" might convey the idea that cells store electricity *as electricity*, but this is incorrect; the electrical energy put into the cell is converted into chemical energy, and the larger portion of the latter is transformed again to electrical energy during discharge.

The lead and lead peroxide are called the active materials of the cell, and, as will be seen, must be of a spongy formation and supported by heavy grids, usually of lead.

A lead/lead peroxide-sulphuric acid cell gives an E.M.F. of about 2 volts when fully charged, and of the energy put in during charge 60 to 80 per cent. can be withdrawn.

Chemical Action.—A storage cell generates electrical energy in virtue of the chemical reactions which take place between lead, lead peroxide, and dilute sulphuric acid. There has been considerable discussion among electro-chemists as to the precise chemical changes which take place in the active materials during the periods of charge and discharge, and many theories, more or less hypothetical in character, have been evolved. There is, however, no doubt that the primary active constituents of the plates are finely divided metallic lead in the case of the negative plate and lead peroxide in the case of the positive: further, that when the plates are in a discharged state a considerable amount of lead sulphate ($PbSO_4$) is present, and the electrolyte is reduced in specific gravity by the absorption of sulphuric acid.

Discharging.—When a cell is discharging it is probable that the lead on the negative plate liberates the hydrogen from the sulphuric acid and forms lead sulphate—

$$Pb + H_2SO_4 = PbSO_4 + H_2.$$

The hydrogen thus set free travels to the positive plate and there unites with some of the oxygen, forming lead monoxide and water—

$$PbO_2 + H_2 = PbO + H_2O.$$

The lead monoxide thus formed unites with sulphuric acid, and forms lead sulphate and water—

$$PbO + H_2SO_4 = PbSO_4 + H_2O.$$

The reactions may not take place exactly in these stages, but the final result of discharge is that the lead and lead peroxide have partly become lead sulphate, and the density of the electrolyte has been lowered, due to the replacement of sulphuric acid by water. The following equation shows the change which takes place on the surface of the plates during discharge—

$$PbO_2 + 2\ H_2SO_4 + Pb$$
$$= PbSO_4 + 2\ H_2O + PbSO_4.$$

The actual composition of the active materials and electrolyte is not that indicated by the above equations. No cell contains pure sulphuric acid when fully charged, but the latter is diluted with a certain percentage of water; likewise, the water formed during discharge is mixed with sulphuric acid. The electrolyte is generally made in such proportions as to be given by $H_2SO_4 + 14\ H_2O$ and $H_2SO_4 + 20\ H_2O$ in a charged and discharged cell respectively. The symbol $PbSO_4$ also does not represent the true composition of the active material, there still remaining at the end of discharge some of the original lead and lead peroxide.

Experiments show that as much as 40 per cent. of the active material may still remain under the lead sulphate.

Charging.—During charging the elements are brought back to their original state, due to the desulphating of the active material. The charging current decomposes some of the water in the electrolyte into oxygen and hydrogen, and at the negative plate hydrogen acts on the lead sulphate, forming lead and sulphuric acid—

$$H_2 + PbSO_4 = Pb + H_2SO_4.$$

At the positive plate the oxygen reacts on the lead sulphate and water of the electrolyte near to the active material, forming lead peroxide and sulphuric acid—

$$PbSO_4 + H_2O + O = PbO_2 + H_2SO_4.$$

Again there is considerable doubt as to the accuracy of the above stages, but the resultant reaction during charge is known to be given by

$$PbSO_4 + 2H_2O + PbSO_4 = PbO_2 + 2H_2SO_4 + Pb.$$

While the active material is being desulphated no gas is evolved, but when the charging is nearly complete hydrogen is liberated at the negative and oxygen at the positive.

The sulphate of lead which is formed on the surface of the plates during discharge is insoluble and a bad conductor, so preventing the electrolyte from coming into contact with the active material. When sulphate covers the entire surface of the plates chemical action ceases, and the cell is said to be discharged. Hence the necessity for giving the active material as large a surface as possible, for the time occupied in covering the plates with sulphate will be longer, and consequently the cell will maintain a certain current for a longer period. The capacity of a cell therefore depends on the area of active material exposed to electrolytic action, and to be economical the surface should be as great as possible for a given weight of plate.

FORMATION OF PLATES

The active materials, being porous, are mechanically weak; they have therefore to be supported by massive grids. The usual practice is to make the supports of either cast, rolled, or drawn lead, which must be as free from impurities as possible. The impurities in the lead forming the grid should not exceed 0.1 per cent., as any foreign material mixed with the lead causes corrosion and oxidation by local action, as will be explained hereafter.

Of impurities, copper is the most detrimental to the life of the plates, being transferred to the electrolyte, and there setting up a continuous counter E.M.F. which prevents the cell from retaining its charge for a reasonable period of time.

The lead forming the grid is sometimes alloyed with 1 per cent. of antimony to add to its strength; but where present in larger quantities antimony may make the plate too rigid, so that it will crack, due to the expansion of the active material, whereas if the plate be more elastic it will stretch sufficiently without suffering damage. This is particularly the case with positive plates.

The structure of the grid must be sound and homogeneous. This is, however, difficult to obtain, owing to the complicated nature of the surface.

Plates are divided into the two following types, according to the process by which the active material is obtained—

1. Planté Plates.
2. Faure Plates.

Planté Plates.—These plates have their active material formed from the grid itself by electrolytic action, and are often referred to as *formed plates.*

If two plates of pure lead be put into a solution of dilute sulphuric acid, and a current sent through the electrolyte from plate to plate, the positive almost immediately gives off oxygen and becomes covered with a thin film of lead peroxide, while the other plate remains unaltered. Electrolytic action is set up, the water decomposing into its two elements hydrogen and oxygen; the latter unites with the lead on the positive plate and forms lead peroxide, while the hydrogen escapes. The chemical reaction may be represented thus:

$$Pb + 2H_2O + Pb = PbO_2 + 2H_2 + Pb.$$

After a few minutes the thickness of the lead peroxide film shows no increase, and immediately the current is switched off the film of lead peroxide is reduced to lead sulphate by the action of the sulphuric acid on the plate. On again applying the current the sulphate is peroxidised, forming a thicker coating. Repeating these cycles, at each successive reversal the oxidising and reducing action sinks more deeply into the plates until the desired surface of spongy material is obtained.

As the formation proceeds the amount of lead peroxide increases, and the periods of rest must be lengthened to hours and ultimately to days. The formation of the active material in this manner is therefore a very tedious process, and to accelerate it, methods have been adopted to open up the lead peroxide and allow the acid to penetrate it more easily. This is done by discharging the plates through a resistance after each charge, instead of leaving them on open circuit. The plates, however, should not be discharged too rapidly, because, if the peroxide is not allowed to assume a crystalline form before its reduction the soft oxide may be forced away from its support by the evolution of gas from the metallic surface. To further accelerate the process of forming, the plates are made either corrugated or ribbed; and besides these mechanical processes some manufacturers boil the plates in dilute nitric acid, so as to give to them as large a surface as possible before they are formed. By forming plates in this manner the developed surface may be increased to nine or twelve times the smooth surface.

In practice, where both positive and negative plates are to be formed, it is more economical to form both as positives against dummy negatives, and then reverse those intended to be negatives during the final charge, in which case the lead sulphate is changed into pure spongy lead.

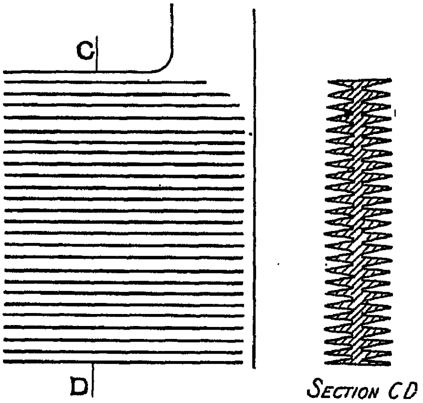


FIG. 65.—Planté plate.

Grids for Planté plates are made in several forms, of which Figures 65 and 66 illustrate two representative specimens.

The form in Figure 65 is cast in one piece, and consists of a lead core carrying a large number of horizontal ribs, upon which the active material is formed. The plate is about 8 millimetres thick and there are 8 ribs per centimetre.

The second form of plate shown in Figure 66 consists of a lead antimony frame, one centimetre thick, of great strength and stiffness. The holes, about 2 centimetres in diameter, which hold the active material, are cast countersunk on both sides. The active material is made from tapes of pure lead, which are corrugated on one side and lled into rosettes. The plate is subjected to a pressure about 200 tons, which expands the outside face of e rosettes into the counter-sinking, thereby riveting e rosettes into the plate. The corrugated ribbons

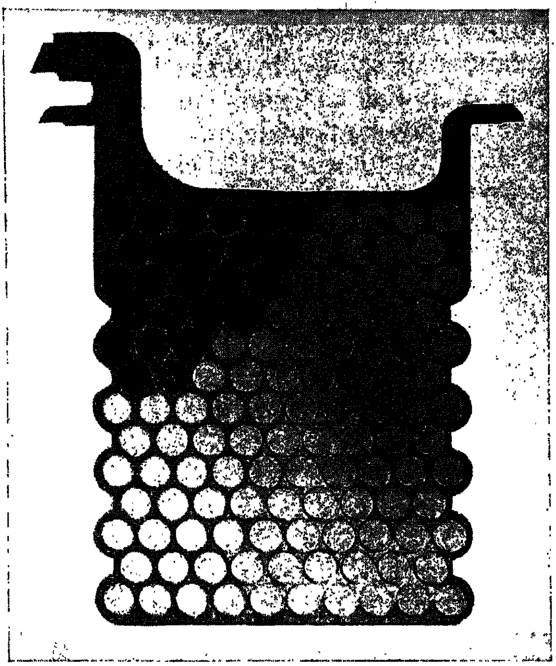


FIG. 66.—Planté plate.

ause the electrolytic action to take place right through he rosettes from one side to another.

Faure Plates.—Plates of this type are prepared by asting grids of pure lead generally containing a large umber of small pockets, into which the active material, n the form of a paste, is forced. The grids are so made hat the active material is held firmly in position.

The positive plates are pasted with minium or red ead oxide mixed in sulphuric acid; the two combine nd form sulphate of lead and lead peroxide, the hemical change being represented by the formula

$$Pb_3O_4 + 2H_2SO_4 = PbO_2 + 2PbSO_4 + 2H_2O.$$

The lead peroxide and lead sulphate eventually settle into a hard dry porous mass, which adheres to the grid

The negative plates are pasted with litharge, a mixture of lead monoxide (PbO) and sulphuric acid which also forms lead sulphate. Litharge is used for the negative plates, as it has been found that by this method a given quantity of spongy lead can be formed by a less expenditure of initial charging current. Faure plates when formed are similar to plates in a discharged condition, and when a battery built with these plates is installed, the cells require to be charged at a slow rate, for from 36 to 48 hours continuously.

As both the lead sulphate and lead peroxide with which the positive plate is coated are insoluble in sulphuric acid, the changing of the active material into lead peroxide only takes place on the surface, and requires some time to penetrate into the interior. The thickness of the active material depends on the time during which the sulphuric acid has been allowed to sink into the oxide of lead, so that the capacity of pasted plates increases with their life.

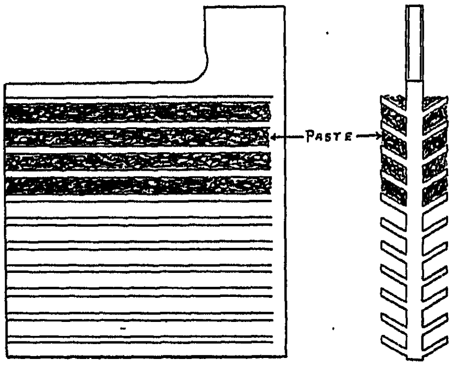


Fig. 67.—Faure plate.

The grid shown in Figure 67 consists of a lead casting having a number of thin horizontal ridges on both sides; the latter are turned upwards so as to hold the active material, which is applied in the form of a paste.

The latest form of pasted plate is illustrated in Figure 68. It consists of two grids riveted together; the outer surfaces are like sheets of lead pierced with numerous holes, and the space between them is intersected by stout horizontal and vertical ribs, which give strength and rigidity to the plate. The ribs also divide up the inside space into a number of cells, one of which is shown in Figure 69, the insides S of the frames being packed with the active material.

Plates formed by the Faure process are usually much lighter than Planté plates of the same capacity, but the active material which is formed by the Planté method adheres better to the grid than that applied in the form of a paste. In making complete cells the plates may be of either type; the general practice, however, is to use Planté positive plates and Faure negatives.

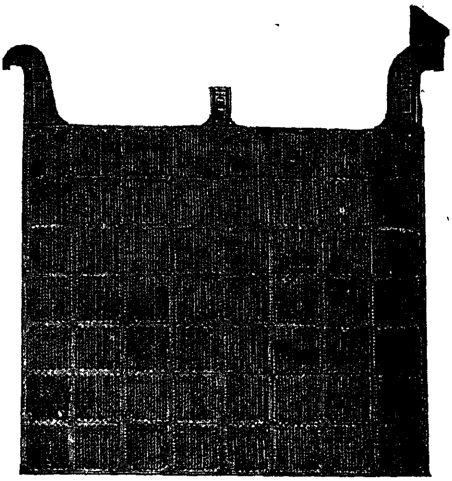


FIG. 68.—Form of pasted plate.

Pasted plates, so far as manufacturing is concerned, have a great advantage over Planté plates, in that the time required for forming is considerably less, and consequently the expenses of manufacture are lower.

The active material formed from a paste is liable to fall away from the grid, so that special attention has to be given to the method of fixing the pellets of active material into the grid.

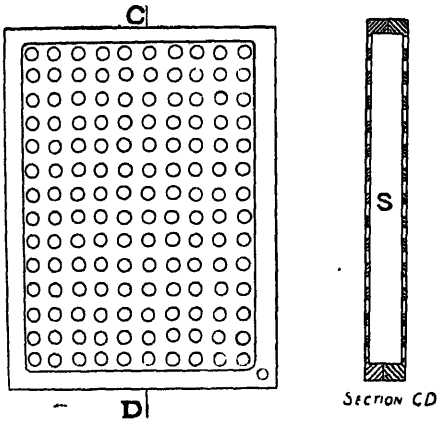


FIG. 69.—Faure plate.

Design of a Storage Cell.—To construct a cell of large capacity a number of negative plates are connected in parallel and alternated with positive plates, also in parallel, as shown in Figure 70. L. Juman has found that a high current density suits the positive much better than the negative plates, and that the best results and highest capacity are obtained by making the total area of the negative plates greater than that of the positives. It is now customary to arrange the plates of a storage cell with a negative at each end, *i.e.* there is one more negative plate than positive.

Each plate has a projection or lug L, and in paralleling a number of plates their lugs are burned into a connecting lead bar B. This connecting bar also terminates in a lug having a smooth horizontal or vertical face, by means of which two cells may be joined together.

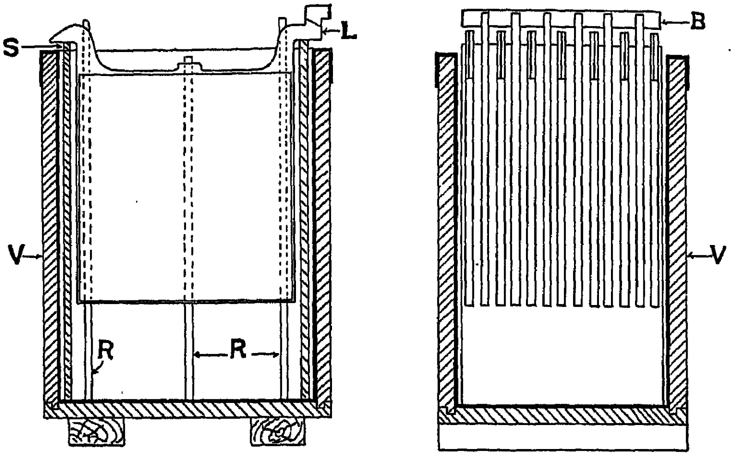


FIG. 70.—Accumulator.

The plates are contained in either a glass or teak lead-lined vessel V (the latter being the case in Figure 70), which contains the electrolyte, and are suspended by the lugs from the sides of the containing vessel when of glass, and from stout glass or wood supports S when lead-lined boxes are used. A space varying from 5 to 16 centimetres according to the depth of the plate, should be left clear between the bottom of the plates and the base of the containing vessel. This allows all waste deposit to sink to the bottom and keep clear of the plates, otherwise adjacent plates are liable to become short-circuited. The plates should be free to expand in all directions, and be separated from each other by two to four insulating rods R of glass or ebonite (for the sake of clearness these are not shown in the section) held upright by guides cast on the lugs of either plate. The distance between adjacent plates varies from 0.6 to 1.2 centimetres, depending on the type of cell. This allows any detached pieces of active material to fall to the bottom without causing an internal short circuit; and also if there be buckling, adjacent plates cannot come into contact.

The electrolyte, as will be explained, should be prepared from very pure sulphuric acid, and diluted with pure water to the required specific gravity. The specific resistance of sulphuric acid varies with the density in a peculiar manner, being a minimum at 1.220, and above and below this value the specific resistance increases, as shown in Figure 71. An electrolyte having a specific gravity of 1.22 reduces the internal resistance of the cell, but on the other hand tends to cause trouble due to local action. In general, a cell which is likely to stand for any length of time unused, should have a low density electrolyte, while a cell in constant use may have a higher density.

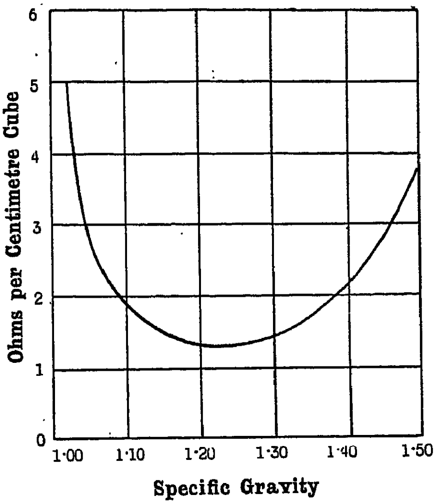


FIG. 71.—Variation in specific resistance of sulphuric acid with density.

Variation of E.M.F. during charge and discharge.—The E.M.F. of a charged cell averages about 2 volts, but varies with the density of the electrolyte, other things remaining constant. The curve in Figure 72 indicates the variation of E.M.F. with the specific gravity as obtained by Gladstone and Hibbert,* from which it will be seen that within the specific gravity limits of 1.07 and 1.300, the E.M.F. is a linear function of the specific gravity. With a lower specific gravity than 1.07 the E.M.F. falls off rapidly. The usual densities range from 1.200 to 1.220 when the cell is charged, to 1.185 to 1.195 when discharged.

During discharge the action of sulphation on the plates and the lowering of the specific gravity of the liquid goes on continually, so that theoretically the E.M.F. is always varying. However, by using a cell having a large quantity of acid, and arranging the plates so that the acid in the pores is kept at the same strength as that in the body of the liquid, the E.M.F. remains practically constant during the greater portion of discharge.

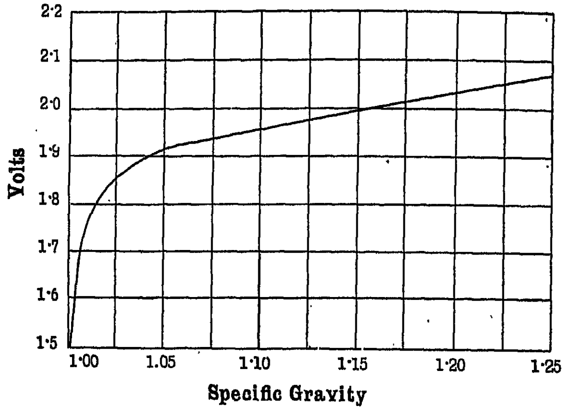


FIG. 72.—Variation of E.M.F. with density of sulphuric acid.

Curves in Figure 73 show the variation in E.M.F of a 180-ampère-hour 13-plate cell during the process of charge and discharge, the normal discharge rate being 21 ampères. These curves may be taken as typical of all lead/lead peroxide sulphuric acid cells.

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* *Journal of Institution of Electrical Engineers*, 1892, vol. xxi. p. 420.

If the discharge be commenced immediately after charging, the E.M.F. rapidly falls from 2.15 volts to about 2 volts, after which it remains approximately constant for some hours. This high but variable initial electromotive-force is probably set up by the adherence to the negative plate of some of the hydrogen evolved towards the end of charge; this, however, is readily oxidised, and the E.M.F. falls to its normal value. This high initial E.M.F. is more noticeable with cells having pasted negatives, due to a much larger quantity of hydrogen adhering to the more porous active material.

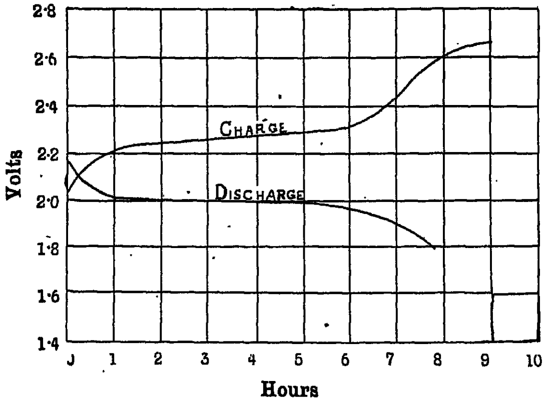

FIG. 73.—Charge and discharge curves of an accumulator.

As discharge proceeds the E.M.F. remains practically constant at 2 volts for about 4 hours, then, as the electrolyte is lowered in density due to the formation of water, falls more rapidly to 1.8 volts. If discharge be continued beyond this the lead sulphate formed on the plates completely separates the active material from the electrolyte, so that chemical action ceases. Cells discharging at their normal rate should never be further discharged when the E.M.F. reaches 1.8 volts.

On charging, the E.M.F. rapidly rises from 1.8 volts to a little over 2.20, then rises very slowly to about 2.30 volts, which is reached when the cell is about charged. After the voltage of the cell reaches 2.30 it rapidly rises to about 2.6 in a manner shown by the curve.

During the latter period hydrogen is freely evolved, the electrolyte having the appearance of boiling. The cell is then said to be "gassing."

When a cell is fully charged it has been pointed out that hydrogen and oxygen are liberated at the negative and positive plates respectively. The presence of these gases at each pole of the electrolyte sets up an E.M.F. which opposes the charging E.M.F., hence the increase in the applied E.M.F.

A cell should never be left in a discharged condition, as reactions take place which are very detrimental to the plates. In a discharged condition the active material has been converted into lead sulphate, $PbSO_4$, and it is supposed that another form of sulphate, expressed by the symbol Pb_2SO_5, is formed by the $PbSO_4$ combining with lead monoxide PbO. This Pb_2SO_5 is referred to as *white sulphate*, and when present in considerable quantities is readily recognised. It is only removed with considerable trouble, and should a cell get into this condition it requires to be subjected to a long charge with a small current, which will in time reduce the sulphate to active material. This white sulphate is very adhesive, and should it be detached from the plate during charge it generally carries with it some of the active material.

Local Action.—When a battery remains unused for a period of five or more days the above changes do not take place, but others occur which tend to lower the specific gravity of the liquid, and consequently the E.M.F. of the cell. Should the electrolyte contain any foreign materials, such as copper or iron, they will set up local currents between themselves and the plates, decomposing the sulphuric acid into water, and forming lead sulphate on the plates.

If this local action goes on for any length of time the plates become sulphated, and the acid is considerably weakened. Too great importance cannot be given to obtaining the electrolyte and plates as chemically pure as possible. With plates of the Planté type, which are given porosity in the course of manufacture by treatment in some oxidising solution containing nitrogen compounds, such as nitric acid, ammonia, and nitrates of sodium precautions should be taken to eliminate all traces of these compounds. Assuming the plates to be pure, the main source of trouble is due to impurities liable to be introduced through the medium of the electrolyte. Impurities may be present either in the sulphuric acid, or in the water with which it is diluted. According to Dr. Lunge, the concentrated acid of commerce may contain, among others, the following impurities :

Sodium Sulphate	135 parts in a million.
Ferrous ,,	291 ,,
Lead ,,	172 ,,
Arsenic	500 ,,
Copper, Zinc, and Iron	Rarely.
Sulphurous Acid	Varying quantities.

Sulphuric acid is prepared either from sulphur, by heating it in a current of air to form sulphur di-oxide, or from sulphide of iron. The former method gives what is known as "brimstone" acid, and the latter method gives an acid which may contain traces of iron, arsenic, or other metals. It is preferable to use "brimstone" acid (which is free from arsenic) for battery purposes.

Pure water should be used to dilute the sulphuric acid, as ordinary town water contains the following impurities :

Hydrochloric acid.
Sulphates.
Ammonia.
Nitric acid.

Boiler water should under no conditions whatever be used, as it contains among other impurities special alkaline softening fluids. R. W. Vicary has recently found that 10 to 0.1 per cent. of ammonia added to the electrolyte decreases the capacity immediately by some 20 to 30 per cent., thus showing the deleterious effect of nitrogen compounds. G. D. Aspinall Parr* has found that if 2 per cent. of sulphurous acid be present in the electrolyte, the capacity of a cell is reduced by 50 per cent. This local action is about three times greater at a specific gravity of 1.300 than at 1.170 for a temperature of 15° C. Hence the necessity for having a medium density of electrolyte.

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* *Journal of Institution of Electrical Engineers*, December 1905, pp 406–420.

Capacity and Output.—The capacity or rated output of a cell is the ampère-hours it will give when discharged at a particular current rate within the practical limits of charge and discharge.

The *absolute capacity* would be the quantity of electricity obtained when the cell is discharged until the E.M.F. is zero. By examining the curve in Figure 73 it will be observed that when discharging at a normal rate the E.M.F. remains approximately constant until it reaches 1.80 volts, when it falls immediately to zero. In such a case, practically the whole of the available energy has been withdrawn from the cell by the time it reaches 1.8 volts, and this energy represents the working capacity of the cell. This, as will be explained, is only the case when the cell is discharged at a slow rate, and from Figure 74 it is observed that a large amount of energy can still be withdrawn after the E.M.F. falls to 1.8 volts; in fact in some cells, discharging at a very high rate, the ampère-hours not utilised at this E.M.F. may exceed those actually withdrawn. It is, however, very detrimental to the life of the plates to discharge the cell to a very low E.M.F., no matter how small a percentage of their maximum output has been obtained.

The practical capacity of a cell is usually defined as the ampère-hours the cell will give before the E.M.F. falls to some pre-determined limit, in practice somewhere between 1.9 volts and 1.7 volts, generally 1.8 volts.

The normal charge and discharge current of a cell depends on the type and mechanical strength of the plates, and is expressed in so many ampères per square decimetre of positive, reckoning both sides of the plate. In Planté plates the support surface is large, due to the mode of formation, so that the current density may be higher than with Faure plates. The discharging current of present-day cells varies between 1 and 3 ampères, and the charging current between 0.5 and 2 ampères per square decimetre of positive plate.

In specifying the capacity of a cell the normal rate at which it may be discharged and charged must also be given. The resistance of the cell being low, the discharge current depends on the external resistance. With a low external resistance the cell would discharge at a high rate, and if the current be very high the cell may be ruined in a very short time. High rates of discharge cause the plates to buckle and active material to be dislodged. This is due to the excessive heating of the grid, and probably also to the tendency of the surface layers only of the active material to take part in the voltaic action, producing a non-uniform expansion and contraction.

In Figure 74 are a series of curves obtained from a cell which gave an output of 750 ampère-hours when discharged at its normal rate of 70 ampères. The curves were obtained when discharging at 70, 80, 100, and 140 ampères; as indicated, the outputs at these respective currents in ampère-hours were 750, 700, 650, and 560, respectively, the cell being considered discharged when its E.M.F. fell to 1.8 volts.

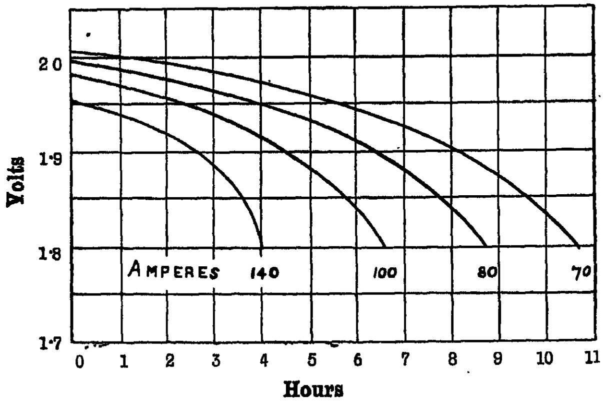

FIG. 74.—Discharge curves of an accumulator.

The rapid fall in E.M.F. when discharging at high rates is probably due to a diminution in the strength of the acid in the pores of the active material. The acid in the pores is rapidly weakened, as diffusion with the stronger electrolyte in the body of the cell does not take place fast enough, when the former is weakened due to electrolytic action. That this may be the explanation is obtained from the fact that if a cell discharging at a high rate be put on open circuit after the E.M.F. falls to say 1.8 volts, then the E.M.F. gradually rises to say 1.95 volts, for then the weak acid in the pores becomes diffused with the stronger acid in the body of the electrolyte.

Efficiency.—The efficiency of a storage cell is the ratio of energy given out by the cell during discharge, to the energy required to charge it to precisely the same condition as it was when the discharge commenced.

$$\text{Energy efficiency} = \frac{\text{watt-hours output}}{\text{watt-hours input}}.$$

This may vary between 60 and 90 per cent., depending upon the conditions under which the battery has been charged and discharged.

The quantity efficiency is expressed in terms of the ampère-hours output and input, and is given by:

$$\text{Quantity efficiency} = \frac{\text{ampère-hours output}}{\text{ampère-hours input}}.$$

With cells working under normal conditions this varies between 90 and 98 per cent.

If a cell be discharged from 2.4 volts to 1.85 volts and then recharged again to 2.4 volts, and these cycles be repeated, the energy efficiency would be about 80 per cent. for a low rate. A cell which is charged and discharged through a very narrow range during short intervals of time may have a comparatively high efficiency. For instance a cell which is discharged for say five minutes and charged for the same time repeatedly, may have an efficiency of between 90 and 92 per cent.

The longer a cell is allowed to gas during charge the lower will be the quantity efficiency, as during this period the whole of the wasted current takes part in the decomposition of the water of the electrolyte. Should charging be stopped before gassing commences the cell might have an ampère-hour efficiency of nearly 100 per cent., but the de-sulphation of the active material would be incomplete, and if continued would reduce the output and permanently injure the cell. Local action and external leakage also affect the ampère-hour efficiency.

The ampère-hour efficiency is not affected by internal resistance and gaseous polarisation during charge, but as these cause a lowering of the E.M.F. during discharge and an increase at charge, they affect the watt-hour efficiency.

G. D. Aspinall Parr* has recently made a series of tests on several large 9-plate storage cells having 4 Planté positives and 5 pasted negatives. The results given in the accompanying table are interesting and instructive, as they show the effects of a variation in the charge and discharge rates on the capacity and efficiency of the cells.

The normal charging current = 32 ampères, discharging current = 29 ampères at the 10-hour rate, and 134 ampères at the 1-hour rate. This table shows clearly that the higher the discharge rate the lower the quantity and energy efficiency.

Discharge.			Ampère-hours.		Watt-hours.		Efficiency.	
Rate in Hours.	Limit in Volts.	Current in Ampères.	In-put.	Out-put	In-put.	Out-put.	(Ampère-hour) Quantity.	(Watt-hour) Energy.
10	1.85	29	318	290	764	574	91.2	75.1
6	1.80	42.5	288	255	730	449	88.5	67.7
3	1.75	70	256	220	630	402	85.9	63.8
1	1.70	134	168	134	414	247	80.0	59.7

Erection of Storage Cells.—In arranging a battery of cells it is of importance that it be so arranged that each individual cell is easily accessible for inspection purposes. Cells should, when possible, be arranged in rows on wooden stands or shelves, and when in tiers a space of about 30 inches should be allowed between each shelf.

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* *Journal of Institution of Electrical Engineers*, December 1905, vol xxxii. pp. 412, 413.

Large-sized cells, such as are used in central station work, arranged in single tiers, need only be placed on two timbers running the length of the row, as shown in Figure 75. Small and medium-sized cells are usually arranged on stands built entirely of wood, the dimensions which must of course depend on the size of cells to erected and the nature of the connections. Before erecting the cells the stands should be well covered with some acid-resisting paint or varnish.

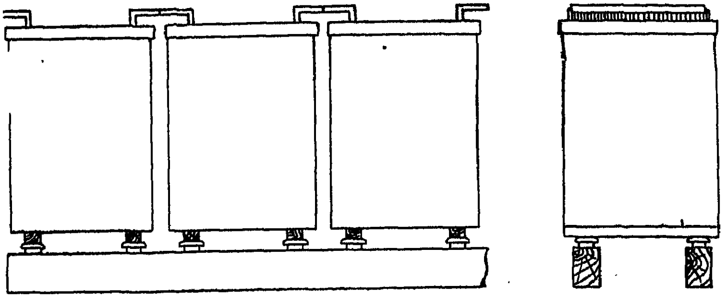

FIG. 75.—Method of erecting cells.

When a battery is almost fully charged it gives off sulphuric acid spray which covers everything, in its immediate neighbourhood, with a film of acid moisture. This would cause a leakage of current to take place from one cell to another by way of the surfaces of the containing vessels, fixings, or earth; and if no means be taken to prevent this the battery might discharge itself while on open circuit. Should the battery form part of a large distributing plant, serious consequences might result from leakages to earth. To prevent this each cell is mounted on four or six mushroom-shaped oil-insulators, shown in Figure 76. The insulators consist of a glass base B having a channel C, in which is placed a quantity of non-evaporating insulating fluid, such as resin oil. In this channel C rests the inner rim

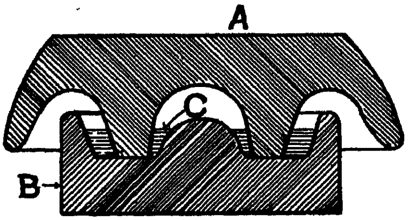

FIG. 76.—Oil insulator for accumulators.

of the upper half of the insulator. Any leakage current from the cell to earth must pass from A to B, and as a portion of the path is across the surface of the oil, leakage is practically eliminated.

Cells contained in lead-lined wood boxes are mounted directly on the insulators: glass boxes, being liable to fracture if they rest directly on the insulators, are placed in wooden trays covered with saw-dust, the tray resting on the insulators.

When connecting a number of cells in series the best connection is effected by burning the positive lug of one cell on to the negative of the next; this insures a uniform and permanent joint, unaffected by acid spray.

Where lead burning is not practicable the lugs are generally clamped together by an iron bolt and nut, protected by a lead covering. Iron is rapidly attacked by acid when exposed to the air and in contact with another metal such as lead, with which it forms a galvanic couple. Figure 77 illustrates one form of connector.

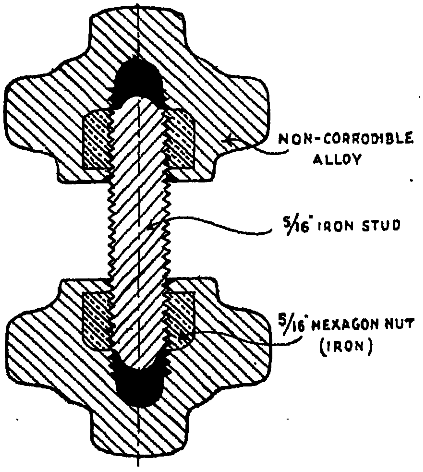

FIG. 77.—Lead-covered bolt and nut.

The condition of a cell being indicated by the density of its electrolyte, two or three hydrometers should form part of the equipment of every battery.

Application of Storage Batteries.—Storage batteries have been applied to nearly every branch of Electrical Engineering, but are particularly adapted for electric-lighting stations. During light load the dynamos may be shut down and the load supplied from the battery; the latter also acts as a stand-by when the generating plant has to be shut down for a short period of time. Batteries are also very serviceable for electric traction work. They are connected in parallel with the mains, and assist the generators during periods of heavy load, and are charged during periods of light load.

As an instance of the application of a storage battery, consider the case where it is used for storing energy obtained from a dynamo and, as required, supplies current at constant potential to a load of lamps. The arrangement of the dynamo D, battery B, and lamps L is shown diagrammatically in Figure 78. It may be inconvenient to have the dynamo working during the evening when the current is required, so that the battery has to be charged during the day. As the battery discharges the E.M.F. falls, so that to maintain the pressure across the lamps constant a number of end cells must be used for controlling the voltage. This is best illustrated by a concrete case.

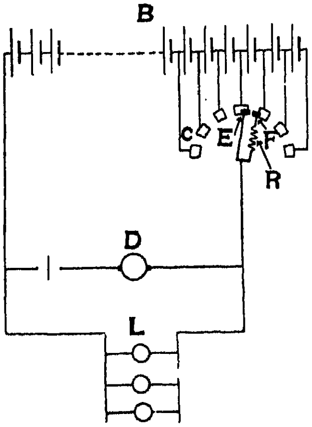

FIG. 78.—Battery, dynamo, and lamps.

Example.—A battery has a discharging E.M.F. of 2.05 volts per cell at start, and gradually falls to 1.85 volts at end of discharge. If the internal resistance of each cell is 0.0005 ohm, and the resistance of the leads from the cells to the lamps 0.011 ohm, what number of cells must be used (1) at commencement and (2) at end of discharge if they supply current to 500 60-watt 220-volt lamps?

Current taken by the lamps $= C = \frac{500 \times 60}{220} = 136$ ampères.

Terminal E.M.F. of each cell = E.M.F. on open circuit $- Cr$, where $r =$ resistance of each cell.

Volts drop in leads $= 0.011 \times 136 = 1.5$ volts.

Therefore the battery has to supply current at pressure of $220 + 1.5 = 221.5$ volts.

∴ No. of cells

$$= \frac{221.5}{\text{terminal E.M.F. of each cell}}$$

Cells at commencement of discharge $= \frac{221.5}{2.05 - Cr}$

$$= \frac{221.5}{2.05 - 136 \times 0.0005} = 115 \text{ cells.}$$

And cells at end of discharge

$$= \frac{221.5}{1.85 - 136 \times 0.0005} = 128 \text{ cells.}$$

The number of end regulating cells must equal $128 - 115 = 13$.

At commencement of discharge 115 cells give the requisite voltage. As the discharge proceeds the regulating cells are switched into circuit, one by one, by means of the regulating switch, and at the end of discharge the 13 regulating cells are in circuit.

The end regulating cells never become completely discharged, as they are only in service during a portion of the time that the battery is discharging. When the battery is being charged the end cells are placed in circuit at the commencement, and gradually cut out of circuit as each becomes charged.

A battery regulating switch requires to be of special design, so that the voltage may be controlled without interrupting the current flowing from the battery, or short-circuiting any one cell. The end cells are connected to the switch studs C, over which the movable contact arm passes. The latter consists of two pieces E and F, connected with each other through the resistance R. When passing from one stud to the next, before the contact E leaves the first stud the contact F has come on to the next stud, thus preventing an interruption of the current. For the instant that E and F are on different studs, the cell between them would be short-circuited, but by the introduction of the resistance R, the short-circuit current is limited to the normal discharge of the cell.

Example.—In the previous example what must be the E.M.F. of a dynamo to charge the above cells at

200 ampères? Each cell has an E.M.F. of 2.4 volts when fully charged. The resistance of the leads from the dynamo to the battery is 0.03 ohm.

Maximum charging E.M.F. for each cell

$$= 2.4 + Cr = 2.4 + (200 \times 0.0005)$$
$$= 2.5 \text{ volts.}$$

Suppose that near the end of charge only 5 regulating cells are in circuit, then the total E.M.F. required to charge the battery

$$= 120 \times 2.5 = 300.$$

Now the drop in leads $= 0.03 \times 200 = 6$ volts.

Therefore the dynamo must generate 306 volts.

(NOTE.—The reader who wishes to obtain further details regarding present day practice, is referred to *Storage Battery Practice* by R. RANKIN.)

CHAPTER VI

ELECTRIC LIGHTING

INCANDESCENT LAMPS

THE incandescent or glow lamp is an application of the principle that when an electric current flows through a conductor the latter becomes heated, and the quantity of heat generated in calories is given by $C^2R \times 0.24$. The energy radiated from a hot body consists of trains of waves of different lengths. The sensation of light is caused by those whose lengths lie between 40 and 80 millionths of a centimetre.

By Guillaume's law the light varies as the twelfth power of the absolute temperature of the source. This being known, attempts were made to find a conductor which could be raised to a high temperature without fusion. It is also necessary that the conductor should have a high specific resistance and be conveniently shaped, so as to provide small units of light.

From 1850 to 1890 many substances were experimented with, but carbon—specially prepared—seemed to be the only material possible. Within recent years experiments have been carried out on some of the more refractory metals, with the result that tantalum, osmium, and tungsten can now be formed into filaments for incandescent lamps; in fact, the carbon filament lamp may, within a few years, become extinct, due to the superiority of the above refractory metals.

Optical Efficiency.—Any source of illumination is simply a transformer of energy, and the vibratory motion of the ether, which is produced, requires for its support an expenditure of energy. In oil or gas lamps the energy is taken from the combustible: in an electric lamp it is furnished by the current. The transformation of this energy, in each case, is accompanied by losses which depend upon the radiant employed. Each source of light has an efficiency which indicates what fraction of the total energy expended is capable of producing a luminous impression.

Optical efficiency is the ratio of the energy reappearing in the form of luminous rays to the total energy supplied, and is given by

$$\text{optical efficiency} = \frac{\text{useful energy developed}}{\text{total energy absorbed}}.$$

The optical efficiency of an artificial source of light is low. When the temperature of the source is below 400° C., the vibrations in the ether are not rapid enough to be sensible to the eye as light, in which case the optical efficiency is zero.

The total luminous energy radiated by a glow lamp can be estimated in the following manner, as suggested by Tyndall. The lamp is placed in the water of a blackened metal calorimeter, and the increase in temperature of the water after a certain time is noted. The lamp is then transferred to a clear glass calorimeter, in which the increase of temperature will be due only to the obscure heat rays. The difference between the amount of heat given to the water by each of these methods is a measure of the luminous energy given out by the lamp.

By this method the optical efficiency of carbon filament lamps is found to be about 5 per cent.; that is, 95 per cent. of the electrical energy supplied to the lamp is merely transformed into heat, and so far as the production of light is concerned this energy is lost. In order to effect a comparison with other sources of illumination, see Table V.

Commercial Efficiency.—For practical purposes, in stating the efficiency of a glow lamp, the light energy emitted is expressed in candle-power, and the electrical energy absorbed in watts: thus expressing the efficiency

TABLE V

Illuminant.	Optical efficiency.
Gas burner	4 per cent.
Carbon filament lamp	5 "
Metal " "	12 "
Ordinary arc lamp	15 "
Flame "	25 "

by stating that the lamp develops a certain candle-power per watt absorbed. However, the more usual method of expressing efficiency is in watts per candle-power, which, correctly speaking, is the inefficiency. This is known as the commercial efficiency and is simply referred to as "the efficiency."

Commercial efficiency

$$= \frac{\text{power absorbed in watts}}{\text{illuminating power in candles}} = \frac{\text{W.}}{\text{C.P.}}$$

$$= \text{watts per candle (W.C.P.).}$$

For instance, a lamp taking 0.3 ampère at 200 volts, and giving an illumination of 16 candle-power, has an efficiency of $\frac{0.3 \times 200}{16} = 3.75$ watts per candle. This is about the average efficiency of a good carbon filament lamp, and as the optical efficiency is only 5 per cent., about 3.6 watts produce heat and the remainder light.

Polar Curves of Illumination.—No lamp gives a complete uniformity of light distribution, the intensity of light being greater in certain directions than in others. Figure 82 is a curve showing the distribution of light about the vertical plane of a carbon filament lamp: this is known as a polar curve of illumination.

To obtain the polar curve of a lamp the candle-power at a given distance is measured in different directions, and in what follows it is assumed that the radiation in any angular direction round the vertical axis of the lamp is fairly uniform. If this cannot be assumed then the illumination must be measured by rotating the lamp at a speed of three or four revolutions per second in each new angle with the vertical.

In testing a lamp it should be arranged that it can be rotated about a vertical axis in steps of 10° at a time. The intensity of light in each direction is then measured in a manner described on pages 191 to 193.

Referring to Figure 79, let L indicate the position of the lamp, which is rotated about the axis AB in equal steps of 10 degrees, shown by the respective radii. Marking off along each radius a length proportional to the candle-power in that direction, the curve LCDE drawn through these points is half the polar curve, and in general both halves of these curves are similar.

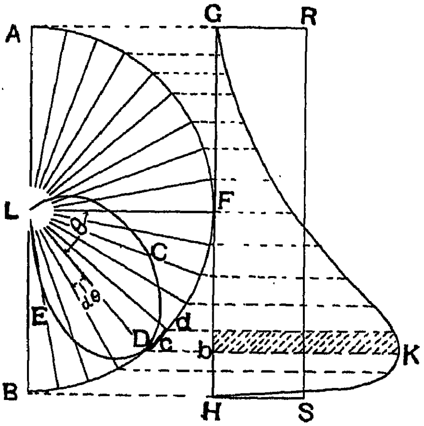

FIG. 79.—Rousseau's construction.

Mean Spherical Candle-Power of a Lamp.—By the mean spherical candle-power of a lamp is meant that candle-power, uniform in all directions, which would give the same total illumination as that from the lamp in question. Considering the light to fall on the surface of a sphere with the lamp at its centre, the total illumination will be the sum of each unit area multiplied by the intensity of light over that area, *i.e.* the total illumination is given by Σ (IA). If I_m = mean intensity of illumination, then $4\pi r^2 I_m = \Sigma$ (IA), r being the radius of the sphere. Denoting the mean spherical candle-power by P_m, $P_m/r^2 = I_m$, and $4\pi P_m$ = total illumination over the sphere.

If the distribution of light given by a polar curve is known, then the mean spherical candle-power of a lamp can be deduced from the following construction due to Rousseau.

In Figure 79, let ADB be a semi-circle, which by revolution round the diameter AB sweeps out a sphere. Through F is drawn GH, parallel and equal to AB. Set off from *b*, and perpendicular to GH, is the line *b*K, proportional to the candle-power of the lamp, in the direction LD. Carrying out the same construction for a number of different observed candle-power readings at known angles above and below the horizontal plane, the curve GKH drawn through all the points, such as K, will be described.

Suppose the candle-power to be uniform in a circle of radius r round the lamp, then in any zone of width $cd = rd\theta$, the illumination

$$\begin{aligned} &= \text{LD} \times \text{area of zone} \\ &= \text{LD} \times 2\pi r \cos\theta \, rd\theta \\ &= \quad \text{I} \times 2\pi r^2 \cos\theta \, d\theta \end{aligned}$$

and the total spherical illumination

$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \text{I} \times 2\pi r^2 \cos\theta \,.\, d\theta,$$

θ being the angle FLD.

This may be calculated for each radius at different angles, but can also be determined graphically, for $rd\theta \,.\, \cos\theta =$ projection of element cd on the vertical, and the shaded area $= \text{I}\ rd\theta \cos\theta$.

Constructing for other elements, the area of curve GJH

$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \text{I} \cos\theta \,.\, rd\theta.$$

The rectangle GRSK is drawn so that its area = area of GJH. Then $\text{HS}2r = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \text{I} \cos\theta \, rd\theta$

$$2\pi r^2 \,.\, 2\text{HS} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \text{I} \cos\theta \,.\, d\theta \,.\, 2\pi r^2,$$

that is, $4\pi r^2\text{HS} =$ total spherical illumination
therefore HS = mean spherical candle-power.

CARBON FILAMENT LAMP

Carbon is a non-metallic substance which cannot by any ordinary means be melted or volatilised. It oxidises readily when heated in an atmosphere containing oxygen, hence the air must be exhausted from the glass bulb containing the filament. The carbon forming the filament of glow lamps has a specific resistance of 0.004 of an ohm per centimetre cube, and a negative temperature coefficient = 0.00054 per degree centigrade.

Carbon used for glow lamps is specially prepared from organic substances, such as cotton wool or wood fibre. This raw material is dissolved in chloride of zinc, and the syrup so formed is purified and run into jets proportioned to the diameter of filament required. After the material has set, the threads are formed to the required shape and carbonised by placing in crucibles packed with graphite and raising the temperature to about 2000° C. This great heat is necessary in order (1) to make the carbon hard and durable, (2) to increase its conductivity, and (3) to reduce its acclusion or power to retain gases. At this stage the filaments are reduced from their previous state of high and irregular resistance to one in which their resistance is low but uniform. The process consists of electrically heating the filaments in a hydro-carbon gas or liquid to such a temperature that the latter is decomposed into its elements hydrogen and carbon, the latter being deposited in a solid form on the surfaces of the white-hot filament. This latter process is known as "flashing."

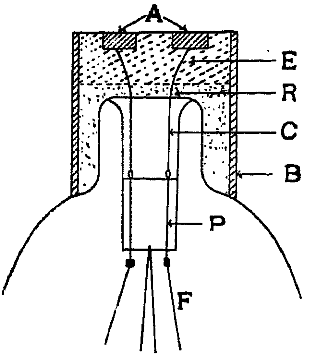

FIG. 80.—Carbon filament glow lamp.

In mounting the filaments connection is made to them by means of platinum wires P fused into the glass, as indicated in Figure 80. Platinum is used because it is the only conductor having the same coefficient of thermal expansion as glass, and hence the latter does not fracture on cooling. The cap consists of an insulated brass collar B fixed on with plaster of Paris R, and the filament F is connected by means of the platinum P and copper wire C to two contact plates A embedded in the cement E.

Relation between Voltage, Candle-Power, and Efficiency of Carbon Filament Lamps.—The efficiency of an incandescent lamp depends upon the temperature of the filament, and since the radiant light is proportional to the twelfth power of the absolute temperature, the greater the latter the higher must be the efficiency.

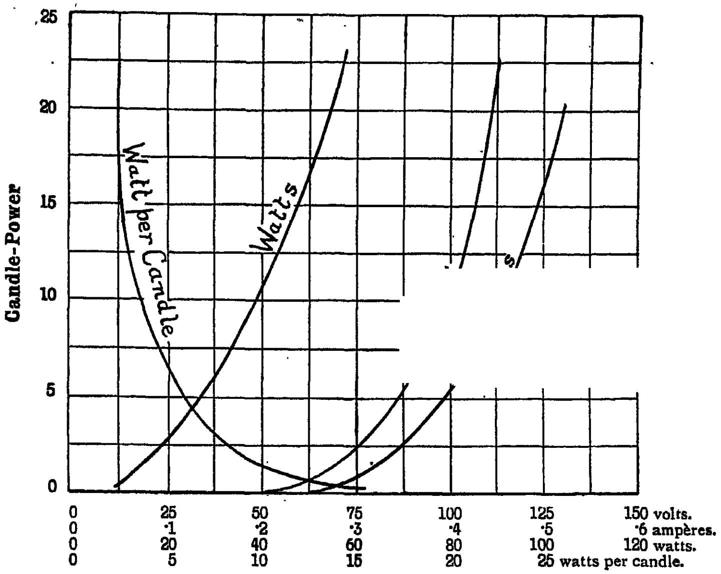

FIG. 81 —Candle-power and efficiency curves of a carbon filament lamp.

In Figure 81 are a series of curves obtained from a 105-volt 16-candle-power lamp having a normal efficiency of 3 watts per candle. The candle-power was measured in a horizontal plane, and is plotted as a function of—(1) the voltage across the lamp terminals, (2) the current flowing through the lamp, (3) the watts absorbed, and (4) the efficiency in watts per candle. The curves are characteristic of all carbon filament lamps, and show that the candle-power increases much more rapidly than either the ampères, volts, or watts.

From the voltage curve it will be seen that an increase in pressure of 5 per cent. above normal increases the candle-power by 30 per cent., or expressing this mathematically, C.P. $\propto E^6$. The candle-power varies as the third power of the watts, *i.e.* C.P. $\propto W^3$.

A large increase in the efficiency and candle-power is obtained by increasing the voltage by a few per cent. above the normal, and a thoughtful student will ask: "Why is a lamp worked at a particular efficiency and not at the maximum?" This is fully explained on page 156, but before dealing with it the theory of the incandescent glow lamp must be further examined.

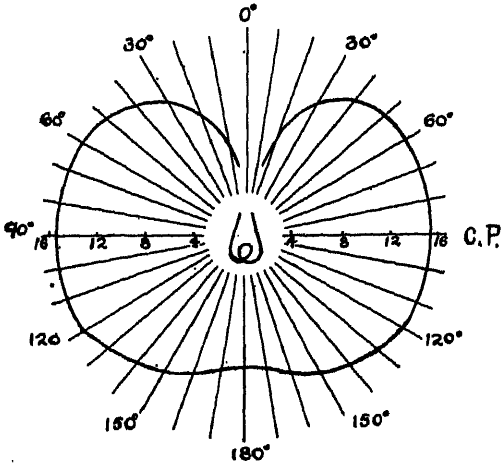

FIG. 82.—Polar curve for a carbon filament lamp.

In Figure 82 is given the polar curve for a carbon filament lamp, from which it will be observed that the intensity of light is greater in a horizontal plane than directly below the lamp. The filament was of the form indicated in the figure.

Life of Carbon Filament Lamps.—An incandescent lamp, besides having a filament which will remain unfractured for a considerable period, must also remain near to its rated candle-power and efficiency, as the most economical lamps are those which give the maximum candle-power-hours for the minimum operating cost.

In Figure 83 curves are plotted for 16-c.p. 230-volt lamps, showing the variation in candle-power and efficiency with their life. The curves are the mean of a series of tests performed on a number of lamps run at their normal pressure, the candle-power and watts absorbed being measured at intervals. These curves are typical of a good carbon lamp. It will be observed that the candle-power and efficiency increase during the first 60 hours, and then gradually diminish in a manner shown by the curves. This increase in the efficiency of a new lamp is probably due to either a better vacuum being obtained or a change in the structure of the filament, which reduces its resistance and allows a larger current to pass through it.

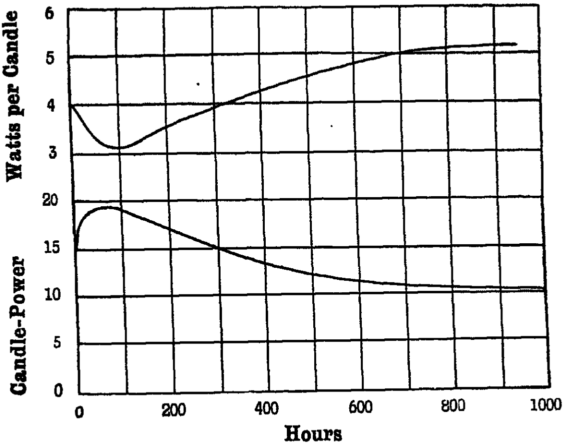

FIG. 83.—Variation in candle-power and efficiency of a carbon filament lamp.

The chief causes which lower the candle-power are (1) Alteration in the character of the surface of the filament, and (2) blackening of the bulb.

Of these, the greater effect results from the first cause, it having been shown that with a well-made lamp the deterioration in 1000 hours, owing to the blackening of the bulb, does not amount to more than

10 per cent. of the total deterioration. The deterioration by change in the character of the surface of the filament is caused by some of the carbon volatilising, and leaving the surface of the filament an aggregation of fine particles of carbon with many elevations and depressions. There will be therefore an increased radiating surface, and consequently the temperature of the filament will be lower for the same consumption of energy. The volatilised carbon which leaves the filament, besides increasing the surface, also reduces the mean area of cross-section, thereby increasing the resistance of the filament, and decreasing the current and illuminating power.

The cause of the blackening of the bulb will now be obvious. The volatilised carbon condenses on the comparatively cold glass, and as the life of the lamp increases the inner surface of the bulb becomes coated with a thin layer of carbon, which absorbs some of the radiant light rays. Experience shows that the efficiency and candle-power of lamps which have either (1) their filaments carbonised at a very low temperature, or (2) inferior vacuums, decrease very rapidly during their life.

The greater the temperature of the filament the more rapidly does this disintegration take place, and as a very high efficiency lamp means a high temperature of filament, high efficiency lamps will last a much shorter time than low efficiency ones. The useful life of a lamp is considered to be limited to the period in which the original candle-power has not diminished by more than 20 per cent. That the above is the case is confirmed by the figures in Table VI., showing the effect of over- and under-running 200-volt 16-c.p. lamps.

The average life of a carbon filament lamp, having a normal efficiency of 3.5 watts per candle at the start, is between 800 and 1200, while a very high efficiency lamp may only last 200 or 300 hours.

The life of a lamp is greatly reduced if it be supplied with current at varying pressures, the lamp then becomes blackened much sooner than would be the case if the pressure remained constant. This is obvious

TABLE VI

Volts.	Candle-power.	Watts.	Life in Hours.
184	9.0	53.2	4000
188	10.2	56.2	3100
192	12.0	59.3	2250
196	14.0	62.5	1440
200	16.0	65.6	800
204	18.0	68.7	530
208	20.2	72.2	360
212	23.0	75.7	230
216	26.0	79.3	160

from the above table, for when the voltage increases, the temperature of the filament also increases, and probably during a considerable period of its life the lamp may have been run above its normal.

METALLIC FILAMENT LAMPS

It has long been known that certain metals could withstand much higher temperatures than carbon, but the more refractory metals are very brittle at ordinary temperatures, and the difficulty experienced in forming these metals into fine wires prevented them being tried as lamp filaments. Means have now been devised for working some of the refractory metals into fine wires having a diameter of 0.05 millimetre, so that glow lamps are now made on a commercial scale having tantalum and osmium filaments.

Tantalum Lamps.—Tantalum is an exceedingly hard metal, and in a pure state is ductile. When cold it is unaffected by chemical reagents, and is attacked solely by hydrofluoric acid. The tantalum is melted in an electric furnace; the ingots are raised in temperature to a red heat, then hammered into sheets, and finally drawn into wires. From a low red heat upwards it has a great affinity for hydrogen and nitrogen, and with these it forms combinations of a metallic appearance. This latter property renders necessary the placing of a tantalum filament in a vacuum. Tantalum has a specific resistance of 16.5 microhms per centimetre cube, which is considerably less than carbon. The temperature coefficient is positive and has a value of 0.234 per cent. per degree Centigrade.

The tantalum from which the filaments are made is purified by chemical processes, and the gases contained in it are driven off by heating in a vacuum. Filaments having a diameter of 0.05 millimetre require a length of 650 millimetres for a 25-c.p. 110-volt lamp, so that special means are adopted to fix a filament of such length in the ordinary glass bulb. The low specific resistance of tantalum makes it unsuitable for high-voltage lamps, owing to the great length of filament required.

Efficiency and Candle-Power of a Tantalum Lamp.—The candle-power of a tantalum lamp is proportional to $E^{4.4}$, and since the filament can be run at a very high temperature the efficiency is much higher than that of the ordinary carbon lamp, being of the order of 1.7 to 2 watts per candle.

Tantalum having a positive temperature coefficient, these lamps will give better regulation than a carbon filament lamp, and their life is probably less affected by voltage variations. The maximum light is emitted at an angle of 85 degrees with the vertical, and is much less in a vertical plane below the lamp: the distribution of light in a horizontal plane is on an average a circle.

Life of Tantalum Lamps.—In Figure 84 are two curves, showing how the efficiency and candle-power of tantalum lamps vary with their life. The test from which these curves were obtained was on some 22-c.p. 110-volt lamps, and they indicate the general behaviour of this type of lamp.

During the first few hours of their life tantalum lamps, like most carbon ones, increase in candle-power and efficiency, the latter increasing from 15 to 20 per cent., while the power consumption drops from 1.4 watts to 1.3 watts per candle. After reaching this maximum, the candle-power gradually decreases, while a corresponding decrease in the efficiency occurs. The initial increase of illuminating power and decrease in the power consumption is probably brought about by a change in the structure of the filament, this being accompanied by a reduction in resistance and the phenomena resulting therefrom.

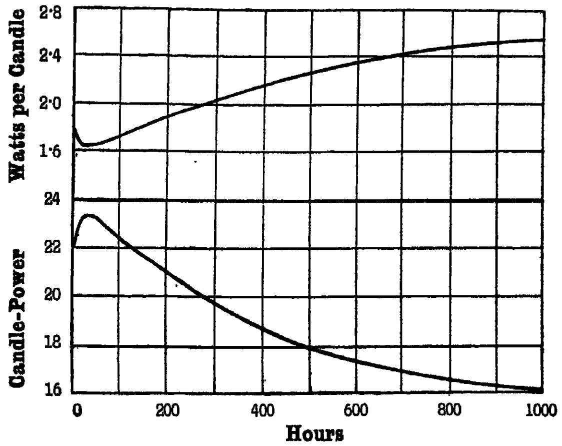

FIG. 84.—Variation in candle-power and efficiency of a tantalum lamp.

The useful life of this lamp averages between 600 and 800 hours when commencing with an initial efficiency of 1.7 watts per candle. Some specimens have had a useful life of as long as 1100 hours, but this is an exceedingly rare occurrence, in fact under normal working conditions the absolute life seldom exceeds 1000 hours.

Experience shows that the tantalum lamp blackens but little, unless it has been strongly overheated in consequence of a partial short circuit.

When a lamp has been in use for some time the structure of the filament becomes completely changed. A new filament is smooth and polished, but after some use assumes an irregular form, and at the end of its life is remarkably disjointed, as indicated in Figure 85. This change is more marked if the lamp be used on an alternating current circuit; in

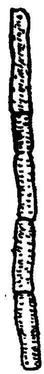

FIG. 85.—Appearance of tantalum filament after being in use for 600 hours.

fact an alternating current, due to some cause yet to be explained, is so detrimental to the life of a tantalum lamp, that the latter never exceeds 200 or 250 hours.

After being in use for 200 or 300 hours the tantalum filament loses a large amount of its mechanical strength and becomes exceedingly brittle. It is therefore advisable when lamps have been in use for some time not to remove them from their fittings, as this might easily cause the filament to fracture.

A feature which marks tantalum lamps is the ability of the filaments sometimes to repair themselves after having been broken. If the broken ends of the filament come into contact with another portion of the filament so that the circuit is completed, the lamp once more lights up. A junction of this sort may result in a very strong weld, so that it does not necessarily constitute a point of weakness. After a repair of this sort, in which a part of the filament has been cut out, the lamp glows more intensely and even too intensely, so that only a small span of life may be left for it. These occurrences of breakages and repairs account for irregularities in the life of tantalum lamps.

The behaviour of a tantalum lamp under increased pressure shows its superiority over the carbon lamp. It has been ascertained that tantalum lamps giving 25-c.p. at 110 volts and 1.7 watts per candle, only burn out when the pressure reaches about 250 volts by gradual increase. A carbon lamp designed to work under the same conditions could not withstand anything approaching this pressure.

Tantalum lamps are made for pressures of from 50 to 125 volts, so that where the current is supplied at 230 or 250 volts two lamps must be run in series. This is rather inconvenient for lighting small rooms, where one lamp might be sufficient, but for the lighting of large rooms, halls, and business premises, tantalum lamps are more economical than carbon ones.

The light is white in colour, and as the area of illumination is large the distribution is good.

Osmium Lamps.—Osmium is the most refractory metal known, and is too hard to be drawn into a wire so that in the preparation of the filaments a paste i formed by mixing the osmium with organic materials The paste is squeezed through dies so that it may assum cylindrical form, and when the foreign material has bee expelled by heating, there remains a porous brigh filament. When cold the osmium filament is brittle but becomes soft during incandescence, so that it require to be well supported.

As with tantalum, osmium has a low specific resistance ; these lamps are only made, therefore, for voltage not greater than 130, in which case the filament consist of four or five loops in series, each being anchored at it lower end.

The filament of an osmium 37-volt 22-c.p. lamp ha a diameter of 0.087 millimetre and a length of 28 millimetres.

These lamps give a white light and have an efficienc of 1.5 watts per candle. With a 10 per cent. increas in volts above the normal the current increases 6.5 pe cent. and the illumination by 40 per cent., as compare with an increase of 12 and 80 per cent. respectively i the case of a carbon filament lamp.

It is not possible to state definitely the life of a osmium lamp, but 1500 hours have been repeatedl exceeded. The surface of the filament is at first somewhat rough, but gradually becomes smoother, thus causin the candle-powe
first 250 hours or [illegible]
these lamps does not blacken under normal co[illegible].

At present they are expensive in prime cost, an this, coupled with the fact that they are only made f low voltages, makes their commercial use limited.

NERNST LAMPS

The main principles of this lamp were discover by Jablochkoff in 1877. He found that oxides of ra earths which are non-conductors when cold, beca conductors when raised to a certain temperatu Jablochkoff suggested a method of first heating t filament or rod by covering it with a thin layer carbon, through which a current was passed. The layer became hot, thus increasing the conductivity of the filament sufficiently to make it a conductor.

Professor Nernst along with others continued the research, and in 1897 produced the lamp bearing his name.

The filament of the Nernst lamp is generally a mixture of one or more of the oxides of zirconium, thorium, and cerium made in the form of little rods or spirals. Such substances as these become conductors when raised to a temperature of about 700° C., and for this purpose a heating coil is connected in parallel with the filament. This heater consists of a platinum wire having a high resistance, and is wound in a spiral over the filament. When this arrangement is connected to a supply circuit a current flows through the spiral, and the heat generated radiates out and raises the temperature of the filament. On becoming a conductor current flows through the filament, causing it to become incandescent.

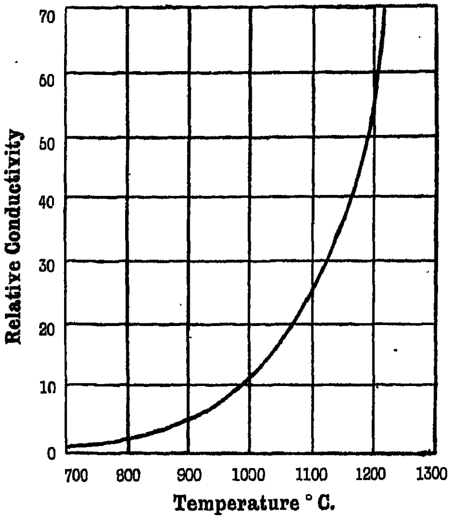

FIG. 86.—Variation in conductivity of a Nernst lamp filament.

This lamp differs from the lamps already discussed, in that the filament does not require to be placed in a vacuum. The curve in Figure 86 indicates how the conductivity of the filament varies with the temperature, the conductivity at 700° C. being taken as unity. At 900° C. the filament becomes incandescent. The rapid increase in conductivity with temperature would cause the illumination to be very unstable when the supply pressure varied within a few per cent. To reduce the effect of voltage variation a steadying resistance of fine iron wire, having a value between 9 and 15 per cent. of that of the whole lamp, is put in series with the filament. The resistance is so designed that it is at a red heat when the filament is incandescent, and any variation in the conductivity of the latter is balanced by the alteration in resistance of the iron wire, as any increase in the current through the filament increases the resistance of the iron wire.

Figure 87 is a sectional diagram of a lamp in which the filament is placed horizontal. The filament F and heater H, carried by a porcelain collar P, are connected at one end E to a socket D, the other ends being connected to C and B respectively. The heater is painted with cement to protect it from the intense heat of the filament. Connection is made to the top portion of the lamp through the thin split rods S carried by a hollow porcelain box K. The split rods fit into the sockets B, C, and D. Two of the rods carry contact guide skates, which make connection with the steadying resistance R; the latter is placed in an exhausted glass tube to prevent the iron from becoming oxidised. The porcelain box K contains a small electro-magnet M, having an armature A. The exciting coil of this magnet carries the current flowing through the filament, the coil M, resistance R, and the filament F being in series, and connected to the two contact blocks marked + and –.

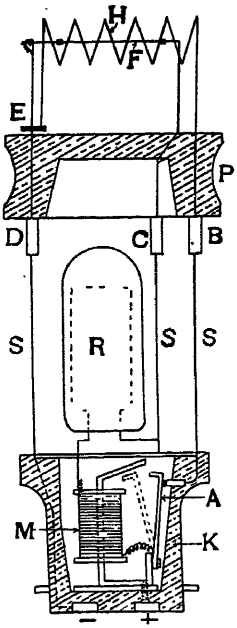

FIG. 87.—Nernst lamp.

The current enters by the + terminal, and from this point there are two paths, one through the coil M, resistance R, and filament F, but as the latter is cold no current in the first instance will flow through this circuit; the second path is completed through the armature A and heater H. On the passage of the current the heater glows dull red, and the temperature the maximum economy, *i.e.* the minimum operating expenses. When energy is cheap and lamps expensive, the efficiency should be low; but when the reverse is the case high efficiency lamps should be used. In practice the total operating expenses are generally near a minimum when the cost of lamp renewals is about 15 per cent. of the whole.

To determine the best efficiency for a lamp to work on a particular circuit, the following must be known: (1) The cost of a new lamp, (2) the average life of the type of lamp when run at various efficiencies, and (3) the cost of electrical energy. (1) and (3) are readily obtained, but a knowledge of (2) entails carrying out life tests on samples of the lamps under consideration. This information, however, in some cases can be given by the makers of the lamps.

Example.—If a particular 16-c.p. lamp costs 1s. 4d. and its life varies with the efficiency as in Table VII., find the most economical lamp to use, assuming electrical energy to cost 4d per unit.

TABLE VII

Lamp Efficiency.	Life in Hours.	Cost in Pence per 100 Hours.		
		Lamp Renewals.	Energy.	Total Operating Cost.
4.5 W.C.P.	4200	0.38 pence	29.0	29.4
4.0 "	2000	0.80 "	25.6	26.4
3.5 "	1200	1.33 "	22.4	23.7
3.0 "	600	2.65 "	19.2	21.85
2.5 "	310	5.15 "	16.0	21.15
2.0 "	160	10.0 "	12.8	22.8

The cost in pence for (1) lamp renewals and (2) energy during some period of time, say 100 hours, must first be determined.

(1) Lamp renewals per 100 hours for a 4.5 watt per candle lamp cost $\frac{16 \times 100}{4200} = 0{\cdot}38$ pence. Similarly, the

cost of renewals at other efficiencies can be calculated, and are given in the third column of the table. In Figure 89 the cost of lamp renewals per 100 hours is plotted as a function of the watts per candle and curve A is obtained.

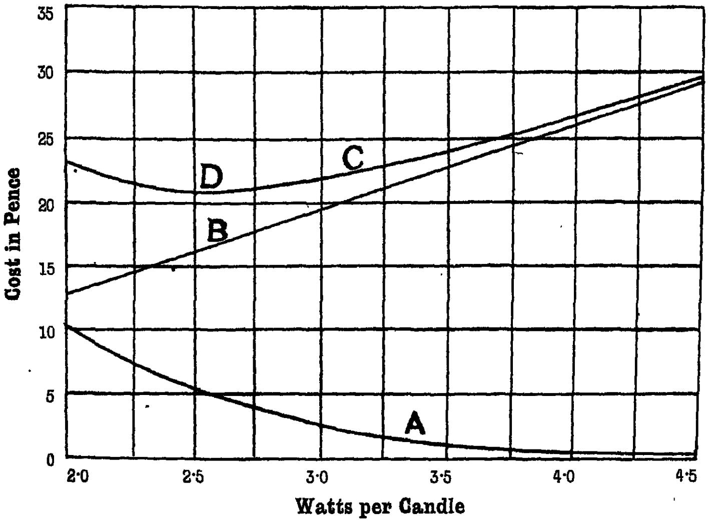

FIG. 89.

(2) Energy. In calculating the cost of energy per 100 hours for each particular lamp, let CP = candle-power of the lamp, W = watts per candle, and P = the cost in pence for 1 Board of Trade unit; then the units used by each lamp $= \frac{CP \times W \times 100}{1000}$. The cost of energy for that period per lamp is therefore

$$\frac{CP \times W \times P \times 100}{1000} = \frac{16.4 \times 4 \times 10 \times W}{1000} = 6.4W.$$

The values are given in the fourth column, and when plotted in the figure give the straight line B. Adding the ordinates of the two curves, a third curve C is obtained, which gives the total cost in pence to run the lamps for 100 hours. Their individual values are given in the last column of the table. It will be observed that in the curve C there is a minimum D, which gives the minimum operating cost per 100 hours. So that when energy costs 4d per unit, the most economic efficiency is obtained by using a lamp having an efficiency of 2.5 watts per candle.

ARC LAMPS

It is well known that when a circuit, through which a current passes, is broken the current does not immediately die down, but has a tendency to continue to flow by bridging across the gap thus formed. The existence of the current is evidenced by a flame or arc set up between the points at which the circuit was broken. The electric arc is based upon this phenomenon, and was first demonstrated by Davy before the Royal Institution, London, in 1808. He employed two carbon rods, and connected them to a 2000 volt supply provided by a battery of galvanic cells. On bringing the two carbons together a current passed from the battery through the carbons, and on being separated a luminous bridging arc appeared between them.

When two carbon rods—commonly called electrodes—are brought together heat is generated at the tips, and they begin to glow, due to the high resistance offered at the point of contact between the carbons. On the electrodes being pulled a few millimetres apart an arc is formed by the carbon vapour which is evolved from them on becoming heated; this conducts the current across the intervening air-gap. An E.M.F. of between 40 and 50 volts is required across the electrodes, for reasons which are explained later.

If the arc be allowed to burn for a few minutes the distance between the electrodes will increase, due to the volatilisation of the carbon. The length of the arc will go on increasing until the resistance becomes too great for the available E.M.F. to overcome; the arc will then be extinguished.

From the above it will be evident that in a commercial arc lamp the electrodes must be attached to some mechanism which will cause them to come together and strike the arc when the current is switched on, and also feed the carbons towards each other as they burn away, and so maintain the distance between them constant.

Form of Electrodes. — Since the temperature of volatilisation of a substance determines the intensity of light emitted, the electrodes of an arc lamp must be made of a material which has a high temperature of volatilisation. Carbon is invariably used, though in some cases it is mixed with chemicals, such as sodium, in order to increase the intrinsic brilliancy of the arc. Carbon volatilises at a very high temperature, but if foreign materials are present in the electrodes the temperature of volatilisation will be lowered, and with it the intensity of the light, for a given consumption of power, will be correspondingly reduced. Arc lamp carbons must therefore be of the purest, and formed into rods having a uniform sectional area and density.

Types of Arcs.—Of these there are two. The first type includes those in which the carbons are placed coaxially, as shown in Figure 90, and if the arc be supplied with direct current the positive carbon, *i.e.* the one by which the current enters the arc, is placed uppermost. This type will herein be referred to as *the Ordinary Arc.* The second type is known as *the Flame Arc*, and has its carbons placed at an angle of about 30 degrees with each other, as shown in Figure 98.

ORDINARY ARC LAMPS

It is essential in the first instance that the carbons touch each other, otherwise a large E.M.F. would be required to start an arc between them. When the arc is struck the intense heat produces volatilisation, and a vapour fills the space between the two carbon tips, which so reduces the resistance that the electric circuit made at the instant of contact is maintained. The current, once started, very quickly increases the amount of carbon vapour formed, and so raises the temperature and luminosity of the arc to a value depending upon the current flowing.

The carbons burn slowly away, the positive twice as fast as the negative, and after burning for some time they assume the appearance shown in Figure 90. The positive has its end hollowed into a cup-shaped form, which becomes incandescent, and is known as the crater. The negative carbon assumes a pointed shape, the tip of which also becomes incandescent, and small nodules of molten impurities solidify on the part just outside the light-giving area. The stream of carbon vapour forming the arc has a light violet colour, and is surrounded by a faint green.

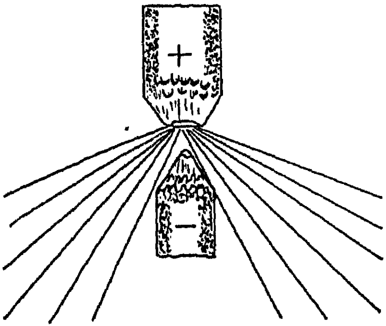

FIG. 90.

The area of the white spot on the tip of the negative carbon increases with the current, but at a much slower rate than the area of the crater; so that the ratio of crater area to the area of white spot on the negative increases rapidly with the current.

Illumination from an Arc with coaxially-placed Carbons.—The amount of light given out depends upon the temperature to which each particular part is raised. On examining an arc it will be found that all parts are not equally bright, but show at least four distinct parts. The temperature of the crater is supposed to be about 4000° C., while the temperature of the negative tip is about 2000° C. The crater is the brightest part of the arc, and gives 85 per cent. of the total illumination; the tip of the negative carbon gives about 5 per cent., and the remainder is given by the arc and the gaseous vapour round it, the arc itself supplying but a small fraction of the total light.

The crater of the positive carbon will be most usefully employed when placed above the negative carbon, as shown in Figure 90, so that as many light rays as possible may radiate downwards. An inherent defect of coaxially-placed carbons is that the negative obstructs a large proportion of the light emitted from the crater. For this reason it is usual to employ carbons for the negative electrodes of as small a sectional area as the current density permits.

Owing to the obstruction of the light from the crater by the negative carbon the illumination of the area below the lamp is not uniform, but is distributed as indicated in the figure; the maximum illumination being at an angle of 45 or 50 degrees to the axis of the carbons.

Figure 91 represents half the polar curve obtained from an arc supplied with direct current. The direction of maximum intensity will vary according to the relative sizes of the two carbons and the shape of the crater. The illumination at any point underneath the arc is proportional to the area of crater visible at that point, and varies from 140 to 170 candle-power per square millimetre of crater visible.

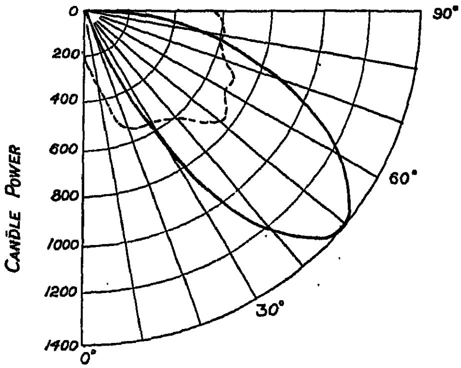

FIG. 91.—Polar curve of illumination for a direct-current arc.

What has been said up to this applies only to arcs working on direct-current circuits, but alternating currents of frequency between 40 and 80 are equally applicable. Experience shows that arc lamps connected to alternating-current circuits, having a frequency less than 40, do not work well, as the intensity of the light fluctuates greatly, due to the continual reversing of the direction of the current The latter causes the carbons to be alternately positive and negative, and the distribution of light from an alternating-current arc will consequently differ from that produced by a direct current. In the former both carbons become cratered and distribute the same amount of light, so that the distribution will be as much above the horizontal as below it. This diminishes the illumination in a downward direction as compared with a direct-current arc absorbing the same power; consequently the illumination on a horizontal plane below the lamp is considerably reduced. In an alternating-current arc both carbons are of the same diameter, and are consumed at equal rates.

Behaviour of an Arc.—In a direct-current arc the positive carbon is volatilised at twice the rate of the negative, since the incandescent portion of the former is at double the temperature of that of the latter. With the positive carbon the highest temperature is at the centre, and to this is due the cup-shaped formation called the crater.

The current flowing through an arc tends to take the path of least resistance, or, in other words, if the carbons be of uniform section the arc will maintain itself in the shortest path between them: hence as the carbon is consumed at one point the resistance increases, and causes the arc to take up a new position; consequently the arc continually alters its location, and produces a varying distribution of light. Some method must be adopted to reduce this variation to a minimum by inducing the arc to remain in a position with its axis coincident with the axis of the carbons. In order to accomplish this the best positive electrodes are formed with a core of specially purified carbon, which tends to confine the gyrations of the arc within a much-restricted area. The cored part of the carbon is about 5 millimetres in diameter. A hole of this diameter is left in the carbon when being manufactured, and afterwards filled in under pressure with the lower-resistance carbon.

The sizes of the carbons vary with the current taken by the arc, and in Table VIII. is given the approximate sizes of the positive and negative carbons for different currents. In an alternating-current arc lamp both carbons have the same diameter, the diameter being the same as the negative carbon in a direct-current arc taking the same current. The length will vary according to the time the lamp is required to burn.

Relation between E.M.F., Current, and Length of Arc.—The relation between the P.D. of the carbons, the current, and the length of arc has been investigated by many observers, with the object of giving it a mathematical expression.

TABLE VIII

Current.	Diameter of cored positive Carbon.	Diameter of solid negative Carbon.
Ampères.	Millimetres.	Millimetres.
5	13	8
6	14	9
8	16	11
10	18	12
12	20	13
15	20	13

According to Frölich the E.M.F. between the two carbons consists of two parts and is expressed by the formula $E = a + bl$, in which a is a constant equal to about 39, b a function varying inversely as the current and having a value between 1 and 2, and l the length of the arc in millimetres.

Mrs. Ayrton has found that in a direct current arc between solid carbon rods the E.M.F. across the carbons is given by $E = a + bl + \frac{c + dl}{C}$.

If the E.M.F. be in volts, the current C in ampères, and the length l in millimetres, these constants have the following values: $a = 38.9$, $b = 2.07$, $c = 11.7$ and $d = 10.5$

$$\text{Hence } E = 38.9 + 2.07l + \frac{11.7 + 10.5l}{C}.$$

The constant a in the above equations is introduced probably by a counter E.M.F. set up between the carbon vapour forming the arc and the positive carbon, as experiment shows that there is a constant drop of this value between the positive carbon and a point in the arc very near to the former. This counter E.M.F. is quite independent of the length of arc and total E.M.F. required to maintain it.

Prof. S. P. Thompson suggested that this phenomenon might be due to the fact that electrical energy is expended in volatilising a certain portion of the positive carbon without changing the temperature, so that the vapour may be considered as having latent heat. This vapour is subsequently condensed on the negative carbon with the same quantity of latent heat, and in doing so the latter may be again transformed to electrical energy and set up a back E.M.F.

Hissing Arc.—As the current increases the area of the crater also increases, while the voltage diminishes; if this be continued until the crater extends to the sides of the carbon, then the voltage falls suddenly by about 10 volts and the current rises by 2 or 3 ampères. At the same time the arc begins to hiss, and in this hissing condition, if the current be further increased, the P.D. of the carbons remains constant over wide limits. Mrs. Ayrton * has shown that the hissing is mainly due to the oxygen which gains access from the air to the crater, when the latter becomes so large as to overspread the ends of the positive carbon. According to Blondel, hissing takes place whenever the current density exceeds about 0.3 or 0.5 ampère per square millimetre of crater area.

Enclosed Arc.—The rate of combustion of matter varies with the quantity of oxygen present, the greater the latter the more rapidly does combustion take place. The carbons in an open arc burn at the rate of about 6 millimetres per hour, but this rate can be reduced to one-fifth by enclosing the arc in an almost air-tight globe.

At first completely air-tight globes were experimented with, but they quickly became obscure, due to the carbon vapour diffusing through the enclosed space and depositing on the cool sides of the glass. If, however, the arc be not completely enclosed it is then possible to admit air in such small quantities that the air oxidises the carbon vapour escaping from the arc and so prevents obscuration of the containing globe: provision must also be made for the escape of the heated and rarefied gases produced by combustion.

Since the arc is enclosed in a nearly air-tight globe, its length can be so increased as to allow it to be operated with double the pressure and half the current required for an open arc giving the same illumination.

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* *Journal of Institution of Electrical Engineers*, vol. xxvii. p. 200.

It would be impossible to operate a long, low-current arc in an open globe, because such an arc would be blown out by the least draught of air. Enclosed arc lamps are operated with pressures varying between 90 and 110 volts, the current absorbed being about 5 ampères.

The carbons when operated in this manner burn flat, thus modifying the distribution of light as compared with an open arc.

The enclosing globe will necessarily absorb some of the light and so reduce the efficiency of the lamp, which is usually 60 per cent. that of an open type of arc. The reduced consumption of carbon is therefore obtained at the expense of the efficiency, but the former more than balances the latter.

Types of Arc Lamps with coaxially placed Carbons.—The mechanism of an arc lamp is required to strike the arc when the current is switched on, and thereafter to maintain the distance between the two carbons constant. The latter operation may be performed either by causing one carbon only to move, the focus of the arc being thus constantly changed, or by causing the two carbons to move towards each other as they are consumed, thus keeping the focus of the arc in the same position.

Arc lamps are now generally controlled by electro-magnets. The electro-magnets for this purpose consist of one or two solenoids, which when carrying a current, attract an iron core running through them. The strength of the pull thus effected depends upon the ampère-turns of the coil: this property of the solenoid, in conjunction with the electrical resistance of the arc, is made use of in regulating arc lamps.

Two coils are employed, one being wound with many turns of fine wire and connected in parallel with the lamp terminals, the strength of this coil as a magnet depending upon the voltage across the arc. The second coil is connected in series with the arc, and is therefore wound with wire capable of carrying the main current. The two coils are connected in such a manner that their magnetic effects are opposite: this arrangement is referred to as a differential winding.

There are two methods of applying the above principle.

Figure 92 represents diagrammatically one form of differential arc lamp. The negative carbon A is fixed and the positive carbon B is free to move in a vertical direction, the latter being attached to an iron plunger P, which forms the core of the series and shunt coils. The series coil S_1 is wound so as to attract the plunger against the force of gravity, while the shunt coil S_2 acts in the opposite direction.

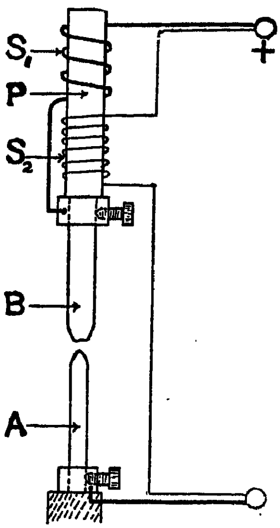

FIG. 92.—Principle of arc lamp.

Suppose the carbons are apart when the current is switched on, the shunt coil alone becomes energised and the plunger is pushed down, and with it the positive carbon, thus bringing the two carbons into contact. The current now flows through the series coil, and the pull overcomes that of the shunt winding so that the plunger and positive carbon are pulled up, thus striking the arc.

The normal resistance between the two carbons is such that the strength of the two coils equalises each other, and so the iron core is kept floating between the two balanced attractions. As the carbons are consumed the length of the arc increases, thus increasing the resistance and so reducing the current in the series coil. When the current falls to a certain value the shunt coil prevails, resulting in a lowering of the plunger and carbon B until equilibrium is re-established.

A lamp working on this principle is illustrated in Figure 93. To the lower part of the plunger P is fixed a lever L, one end of which is slotted and supported by the pin C; the other end is attached to a piston moving in the dash-pot D. This serves to steady the controlling mechanism. In the space S are wound the shunt and series coils, the former wound underneath the latter. The positive carbon is attached to the carbon holder H. which is kept in position by the guide tube T. The plunger P is tapered at its upper extremity so as to correspond to the shape of the pole-piece N, while the lower end is grooved in three places (only one shown in diagram) and each groove contains a clutch ring R_1. These rings are so fitted that when the plunger P is attracted up, they grip the carbon holder H and so raise the positive carbon. As the plunger descends the rings release their grip and allow the carbon holder to descend also.

The carbons of the lamp illustrated are enclosed in an inner globe G, which is air-tight at all parts except at the bottom, where there is an air-hole large enough to admit just sufficient air to maintain combustion. The positive carbon is guided into the inner globe G by means of three centring rings R, and the negative carbon is fixed to a frame F, which also supports the globe G. The mechanism is enclosed in a sheet-iron case E.

The current passes from the terminal + through the series winding, and is led into the positive carbon by the rings R, which provide a smooth and frictionless contact: connection is made between the negative carbon holder and the terminal –. The shunt coil is connected directly across the lamp terminals.

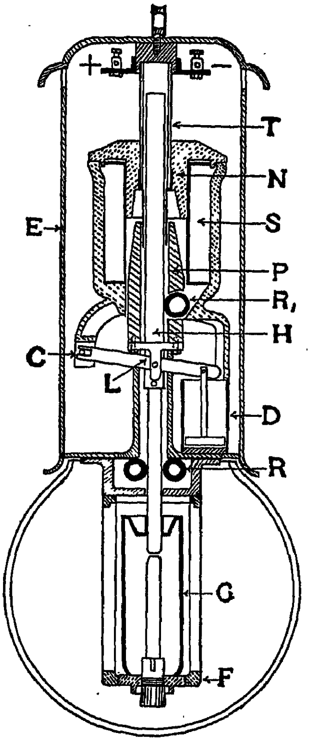

FIG. 93.—Type of arc lamp.

The second method of applying the principle of the differential coils is shown in Figure 94. There are two solenoids, one S_1 in series and the other S_2 in shunt with the arc. Each solenoid is provided with a soft-iron plunger indicated by P_1 and P_2 respectively; the upper ends of them are freely suspended from the opposite ends of a rocking lever L. W is a brake wheel which carries a brass drum D turning on the same axis. Round the former passes a brake band B, which is fixed by one end to the lever L, the other end being attached to a spring S. The drum D is made in two parts, bolted together, but insulated from each other. The carbon holders, sliding upon guide tubes T attached to the lamp, are suspended by means of flexible copper cords C, one from each half of the drum D. The suspension cords each have a turn round the drum in opposite directions, so that when the drum rotates the carbons either approach or recede according to the direction of rotation. The current is conveyed to and from each part of the drum by either spiral connections or brushes bearing on the revolving rings.

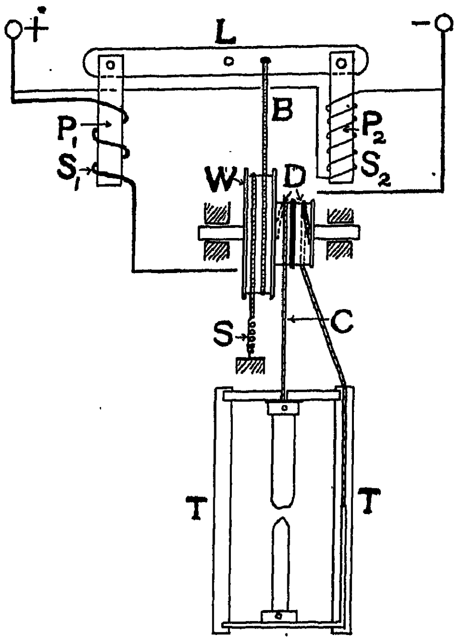

FIG. 94.—Principle of arc lamp.

The spring S is so adjusted that when no current is passing the carbons are always brought together. When current is switched on, the plunger P_1 is pulled down; this tightens the brake band and turns the brake wheel and drum. The drum D so revolves that the upper carbon is raised and the other one lowered, thus striking the arc. As the arc lengthens the series current diminishes and the shunt coil prevails, so that the plunger P_2 is pulled down and the brake band is released sufficiently to allow the carbons to move towards each other. When the series current increases the brake band holds the wheel, and so equilibrium is maintained.

The lamp illustrated in Figure 95 works on a modification of the above principle. D and E are the series and shunt coils respectively, and P_1 P_2 their plungers, which are connected to the rocking lever L pivoted at K. Attached to L are two clutch racks B, between which works the positive carbon holder C. The negative carbon is fixed and both carbons burn inside the inner globe M. Under the action of gravity the carbon G comes into contact with H, so that, when the current is switched on, the plunger P_1 is attracted up. The canting of the lever L causes the clutch racks B to come together and grip the holder C, and at the same time to move in a vertical direction, lifting C and the carbon G. When the length of arc increases the shunt plunger prevails, slackens the tension of B on the carrier C, and allows the carbons to feed. To one end of the lever L is attached a piston working in a dash pot F which serves to steady the feeding arrangement. A is a resistance coil for steadying the arc, the function of which will be explained later.

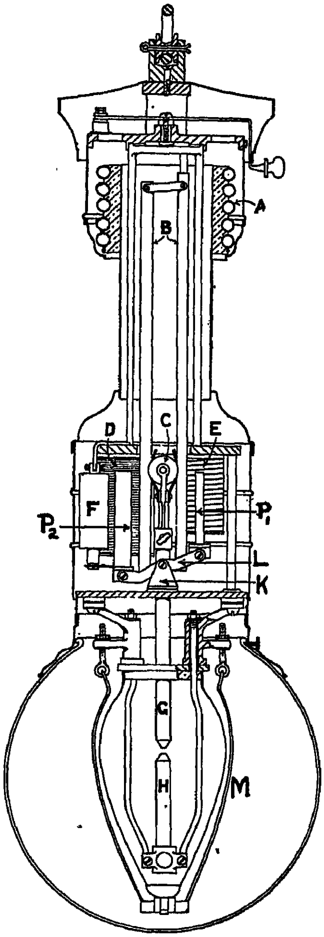

FIG. 95.—Type of arc lamp.

FLAME ARC LAMPS

In arc lamps with their carbons arranged coaxially a large percentage of the light emitted from the crater is obstructed by the negative carbon, and the light emitted by the arc itself is only a small percentage of the total. In flame arc lamps the above defects have been remedied as follows:

1. By placing the electrodes at an angle of about 30 degrees with each other, as shown in Figure 98. The arc is maintained between the tips of the electrodes, thereby forming the crater in such a position that none of the light emitted by it is obstructed by the negative carbon.

2. By impregnating the carbons with metallic salts the arc formed by the vapour is rendered more luminous. Carbons so treated are generally termed *chemical carbons.*

Composition of Chemical Carbons.—Flame arc lamps are subject to flickering, due to the composition of the carbons, and up to the present this has not been entirely eliminated, though it has been greatly reduced in more recent lamps. In the early forms of chemical carbons a large proportion of calcium salts, mixed with the carbon, produced irregularities in burning. They now usually consist of three zones: the outer zone or envelope is composed of pure carbon, which gives the necessary mechanical strength; the next zone contains carbon mixed with various salts, such as those of calcium and sodium; while the inner centring core is made of the same materials less strongly compressed.

Poisonous fumes are given off by the burning chemicals, and thus render a lamp having chemical carbons unsuitable for use in a room not sufficiently ventilated. These fumes must escape from the combustion chamber to the atmosphere, so that the design of globe does not permit of making the arc totally enclosed. In consequence of this the life of the carbons is very short—60-centimetre carbons lasting only 17 hours. This, coupled with the fact that the carbons are more expensive than ordinary carbons, renders the upkeep of chemical carbon lamps rather costly.

Distribution of Illumination.—The light emitted by the crater of a chemical carbon lamp is only about 30 per cent. of the total, the greater portion of the light being emitted from the arc itself, as is clearly shown in Figure 96,* the photograph being taken from a 38-volts 10-ampère arc. This is apparently due to the minute burning particles in the flame being raised to a very high state of incandescence. It has also been determined that the intrinsic brilliancy of the flame of a chemical carbon lamp is about one-third that of the positive crater. The author of the paper referred to in the footnote, specially points out that these chemical carbon lamps are efficient not on account of the crater light at all, but simply because they produce a flame which has a very high illuminating power, and because the light which emanates from the flame is, to a large extent, in the direction in which it is required.

FIG. 96.—Arc of chemical carbon lamp.

The distribution of light is very different from that of an ordinary arc, as will be observed from Figure 97, which indicates the distribution of light from a flame arc. The maximum luminosity is directly below the arc; the spherical candle-power, and consequently the efficiency, is much greater than other forms of arc lamps consuming the same power.

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* Taken from *Journal of Institution of Electrical Engineers*, vol. xxxvii. p. 11, December 1906. Paper on "Flame Arc Lamps" by L. Andrews.

A yellow light is given out by the majority of chemical carbon lamps, and its spectrum is said to be similar to daylight. For the illumination of public buildings and street lighting the yellow-coloured light is very effective, and experience has shown that it is the best illuminant for penetrating fogs.

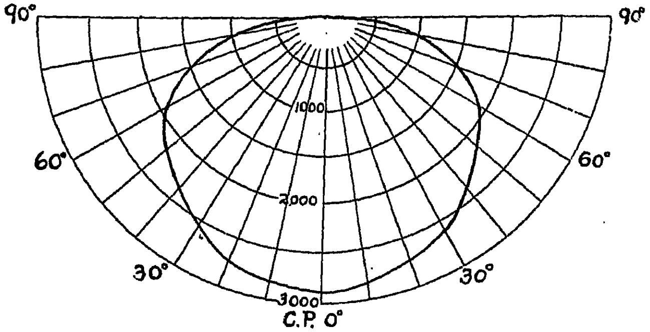

FIG. 97.—Polar curve of illumination for flame arc lamp.

Construction of Flame Arc Lamps.—The flame arc, produced by carbons arranged side by side, is, as a result of its great length and fan-like expansion, so sensitive to air currents that a very slight draught is sufficient to cause the light to be unsteady. Consequently it is necessary to construct an enclosing cover so that the arc may be protected against draughts: but it must take such a form that no impedance is offered to the free ventilation necessary to conduct away the products of combustion. Such an arrangement is called an *economiser*, and is indicated by E in Figure 98. It is made of porcelain, which can stand the high temperature of the arc, and is in two parts, kept together by a metal holder; if solid it would crack when subjected to the intense heat.

The fumes evolved during combustion are injurious to the mechanism and insulation of the coils of the lamp, so that to preserve these the combustion chamber must be separated from the compartment containing the mechanism.

Figure 98 illustrates the construction of one form of flame arc lamp. There are three compartments: the top one containing the regulating mechanism; an intermediate one containing an electro-magnet, carbons, and guide rods; and the third one is the combustion chamber enclosed by the globe.

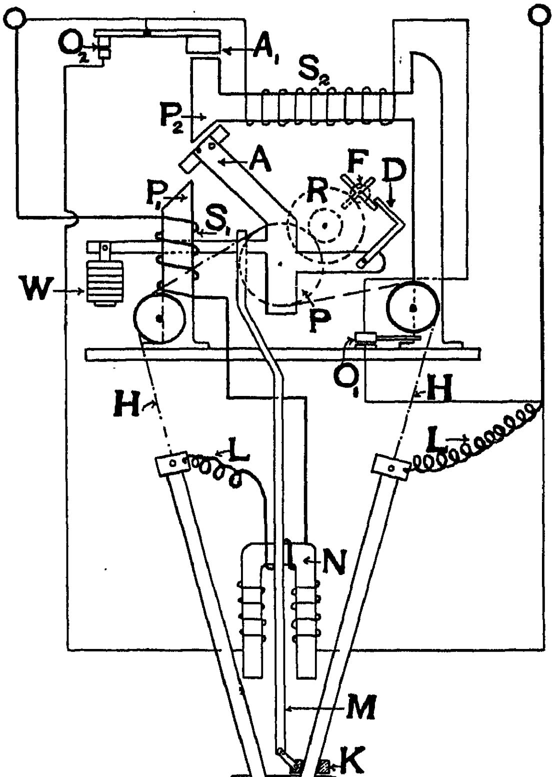

FIG. 98.—Flame arc lamp.

Referring to the figure, there are two electro-magnets at right angles to one another; the vertical one S_1 has its coil in series with the arc, while the coil of S_2 is connected in shunt. Between the respective poles P_1 and P_2, shaped as indicated, swings an armature A, forming part of a three-limbed casting. The second arm of the casting carries the detent D controlling the feeding gear while the third arm carries the balance weight W. The carbon holders running in guide rods are suspended by chains H wound round the pulley P, which is operated by the spur-wheel reduction gear R. The lower ends of the carbons are led into the economiser E, which is supported by the lower part of the frame-work. Above the economiser is the mechanism for striking the arc, which consists of a slider K mechanically connected to the rod M in such a manner that any movement of the armature A moves the extremity of the right-hand carbon. Connection is made to the carbons by means of flexible leads L insulated with glass beads.

The action of the mechanism is as follows:

When the lamp is connected to the supply circuit the shunt magnet becomes energised and attracts the armature A. The detent D then disengages the four-armed fly-wheel F, and the reducing gear R, being then free to rotate, allows the carbons to come together, due to the attraction of gravity. As soon as the carbons come into contact a current flows through the series winding, and the armature A is attracted towards P_1. The rotation of A lifts the connecting rod M and slider K, thus separating the carbons and striking the arc. As the length of the arc increases the current through the series coil diminishes so that A moves towards P_2, thus causing the detent D to release the fly-wheel F, which makes a quarter of a turn and thus allows the carbons to feed by a very small amount. The armature then returns to its position of equilibrium between the poles P_1 and P_2. The length of the arc is, in the first instance, determined by the strength of the two solenoids S_1 and S_2, but the final adjustment is made by the balance weight W.

The electro-magnet N is excited by a coil in series with the arc, the result being that its field repels the arc to the tip of the carbons.

As the carbons approach each other at a sharp angle there is a risk that, when the carbons have burned their full time, and consequently do not feed further forward, the arc itself may travel up, with the result that the economiser and some of the frame-work might be destroyed. To prevent this, when the carbons are in their lowest position and cannot feed further, the shunt circuit is automatically broken by a carbon-faced switch O_1 mechanically operated by a small detent on the chain supporting the right-hand carbon. When the shunt circuit is thus broken the armature A_1 is released and the shunt circuit of N is completed at O_2. This increases the magnetic field of the latter to such an extent that the arc is "blown out."

ARC LAMP ACCESSORIES

Globes.—The light from an arc lamp generally requires to be diffused by means of glass globes, the design and density of which are according to the illumination required. Enclosing the arc by globes considerably reduces the amount of light actually obtained from the lamp, and the following gives an approximate value of the percentage of light absorbed by various forms of globes:

Form of Globe.	Light absorbed.
Cut glass	5–20 per cent.
Clear ,,	10–20 ,, ,,
Ground ,,	25–40 ,, ,,
Opal ,,	40–60 ,, ,,

A globe alters considerably the distribution of light from an arc, making it much more uniform, as is shown in Figure 91, in which the dotted lines represent the light distribution when the arc was enclosed in an opal glass globe.

The figures in Table IX. give a comparison between the three types of arc lamps, the same form of globe being used with each lamp.

TABLE IX

Lamp.	Current in Ampères.	Volts on Lamp Terminals.	Mean power in Watts.	Mean hemispherical Candle-power.	Watts per c.p.
Open arc .	10	44	476	633	0.753
Enclosed arc .	6	77	486	338	1.435
Flame arc .	8	44	382	1352	0.282

From the above it appears that the watts per mean hemispherical candle-power are, in the case of the flame arc, only 37 per cent. of those required for the open arc, and 20 per cent. of those required for the enclosed arc; but, as already stated, the high rate of carbon consumption to a great extent nullifies this increase of efficiency.

Steadying or Line Resistance.—Any variation in the current taken by an arc alters the area of the crater on the positive carbon, and this causes the illumination to be very unstable. Hence it is necessary to introduce some compensating device which will maintain a steady distribution of light. The steadying arrangement consists of a resistance R (Fig. 99) connected in series with the lamp L; the former varies the potential across the lamp terminals inversely as the current. In practice an additional 10 to 30 per cent. of the E.M.F. required on the terminals of the lamps is necessary for steadying the arc.

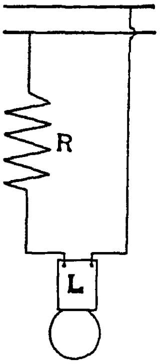

FIG. 99.—Arc lamp steadying resistance.

The action of this steadying resistance is best illustrated by a concrete case. Suppose the steadying resistance of an arc lamp, taking 10 ampères as normal current, is 2 ohms and the pressure available 65 volts. The voltage absorbed by R when the lamp takes the normal current $= 10 \times 2 = 20$ volts, thus leaving 45 volts across the lamp terminals. Next suppose the current increases by some cause or another to 12 ampères, the voltage absorbed by R is then 24 volts; this leaves only 41 volts across the lamp terminals, and so reduces the current taken by the arc to its normal value. The resistance R thus tends to maintain an approximately constant current through the arc.

The steadying resistance often referred to as the *line resistance*, generally consists of a length of iron wire wound on a porcelain cylinder, the two ends of the wire being connected to suitable terminals. In some cases this resistance is contained in the lamp itself, as shown at A in Figure 95. A certain amount of energy is absorbed by the line resistance, which in the example taken is 200 watts, so that the commercial efficiency of the lamp is considerably reduced. For lamps connected to alternating current circuits the line resistance is replaced by a choking coil, the function of which is explained in books on alternating currents.

Arc Lamp Circuits.—For street lighting and the illumination of large buildings the system invariably used is the series parallel arrangement from 230 or 460 volts. The lamps are arranged in groups of from three to ten (according to the type), connected in series, each group being connected in parallel with the supply mains. The method of grouping different types of arc lamps on supply circuits having voltages between 50 and 460, is indicated in Table X.

TABLE X

Voltage of Circuit.	Open Arcs in Series.	Enclosed Arcs in Series.
50	1	...
100	2	1
200	4	2
230	5	3
460	10	5

When four or more arcs are connected in series no steadying resistance is necessary, as any slight alteration in the length of one arc produces no appreciable alteration in the current.

Cut-out Resistance.—With a number of lamps in series, should the carbons of one lamp cease to feed and the arc increase in length so much that it cannot be maintained, then it, along with all the lamps in series, would be extinguished. To prevent the other lamps from being extinguished, some automatic arrangement must be made to complete a circuit through the lamp that has failed. If the lamps be connected to a constant potential circuit, then not only must the circuit be completed but a resistance must be inserted which absorbs the same voltage as the arc, and so maintain the normal pressure across the other lamps. Such an arrangement is called a cut-out resistance, and is indicated diagrammatically in Figure 100.

It consists of an electro-magnet M, excited by a coil S_1 in series with the shunt coil S, which controls the mechanism. When the main circuit through the lamp is broken, the voltage across the lamp terminals + and − increases, and in the same proportion the current through the coil S_1. This increase in current is sufficient to cause the magnet M to pull down the armature A against the spring K, thereby completing the main circuit through the resistance R, contact C, and armature A. The wire forming the resistance R should be capable of carrying the main current for a considerable time, and also absorb the full voltage required by the lamp.

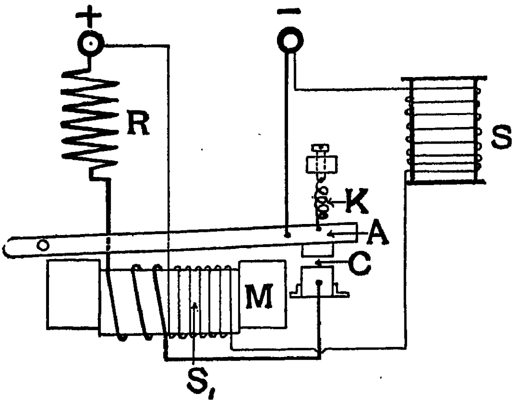

FIG. 100.—Cut-out resistance.

When the circuit is completed through R the voltage across the shunt coil falls, thus reducing the current and allowing the spring K to pull up the armature and break the circuit. The armature would then go on vibrating, but to prevent this a few turns of wire in series with R is wound round the electro-magnet M. These turns compensate for any drop in volts across the shunt coils, and so maintain the electro-magnet at such a strength that the circuit through R is maintained until the lamps are switched off.

ILLUMINATION AND PHOTOMETRY

When light falls on a surface it is said to be illuminated. The illumination of a surface depends upon two factors, namely, the candle-power emitted by the illuminant and the distance between the illuminant and the surface illuminated.

Unit of Illumination.—The standard of illumination is that produced by a standard candle at a distance of 1 foot, and is referred to as a *candle-foot.* The standard candle used as the unit of light in Britain is made of spermaceti and burns at the rate of 120 grains per hour, with a flame 1.8 inches high. It is a very imperfect standard, and when a number of these candles are tested one against another they are found to vary considerably. Other standards of light are the Carcel and the Hefner-Altenck lamps. The Carcel lamp burns colza oil at a given rate and the Hefner lamp burns amyl-acetate, the height of the flame in both cases being adjusted to a definite value. Students who wish for further information regarding these standards are referred to advanced text-books on "Light."

Distribution of Illumination.—The candle-foot is a convenient unit of illumination, being that required for reading purposes. It has been estimated that the illumination due to the sun during a bright day on the streets of London varies between 30 and 40 candle-feet, while the illumination in full moon varies between $\frac{1}{60}$ and $\frac{1}{100}$ of a candle-foot.

If c.p. denote the candle-power of a certain lamp, then the illumination on a surface d feet from the lamp $=$ c.p.$/d^2$ candle-feet, this being on the assumption that the surface illuminated directly faces the source of light. When the surface is inclined to the direction of the rays of light, the illumination will diminish in proportion as the projected area of any part of the surface diminishes when viewed from the source of light. The decrease in illumination varies with the cosine of the angle of incidence, the latter being the inclination between the rays of light and the perpendicular to the surface. A ray proceeding horizontally will therefore be at 90 degree incidence, and a ray falling on a point immediately below the lamp will be at 0 degree incidence on the horizontal plane beneath it.

The simplest case is the illumination of a horizontal plane by a light radiating uniformly in all directions, it being assumed that the intensity of the light remains constant. Referring to Figure 101, let L A represent the height of a lamp L above the horizontal plane A F, and the angle A L B ($= \theta$) the angle of incidence, then

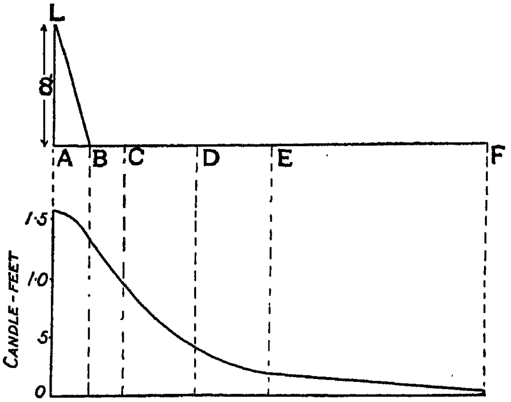

FIG. 101.

denoting the candle-power of the lamp by c.p. the illumination at B is given by

$$i = \frac{\text{c.p.}}{\text{LB}^2} \cos\theta.$$

Street Illumination.—*Example.*—A 100 candle-power lamp giving a uniform distribution of light is placed 8 feet above the street level; draw a curve showing the illumination along the street for a distance of 30 feet.

Referring to Figure 101, let L represent the position of the lamp; then the illumination is calculated for say six positions, A, B, C, D, E, and F along the street, each at a distance of 0, 2.5, 5, 10, 15, and 30 feet respectively from A, a point directly below the lamp.

$$\text{At A the illumination} = \frac{100}{8^2} = 1.56 \text{ candle-feet.}$$

$$\text{At B the illumination} = \frac{100}{\text{LB}^2} \cos \text{ALB}$$

$$= \frac{100}{2.5^2+8^2} \times \frac{8}{\sqrt{2.5^2+8^2}} = 1.31 \text{ candle-feet.}$$

Similarly the illumination at C, D, E, and F is equal to 0.96, 0.38, and 0.027 candle-foot respectively. The illumination curve is as shown, the ordinates giving the illumination at different positions. The curve indicates how the illumination along a horizontal plane decreases rapidly with the distance.

H. T. Harrison * has suggested the following as the minimum direct illumination for street lighting:

Main thoroughfares . .	0.05	candle-foot.
Side streets . . .	0.025	,, ,,
Suburban streets . .	0.005	,, ,,

In main thoroughfares arc lamps giving about 500 c.p. each should be erected at a height of from 7 to 9 metres, the distance between them being about 10 times their height. In streets which are narrow or shaded by trees, smaller units of light placed closer together will give better illumination.

Photometry. — The candle-power of a lamp is measured by comparing it with a standard light; the apparatus used is called a photometer, and its principle is as follows:

The eye is only capable of very roughly estimating the relative intensity of illumination from two sources, but it is capable of perceiving with considerable accuracy when the illuminations are of equal intensity. Hence to compare the illuminating power of two sources of light, a screen, which may consist of a piece of white paper having a uniform "grease spot" at its centre, is placed between them and adjusted with respect to the distances, so that both sides of it are equally illuminated.

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* *Journal of Institution of Electrical Engineers*, vol. xxxvi. p. 198, December 1905.

Referring to Figure 102, let P and Q represen the two sources of illumination and A the screen having the grease spot. The screen A is known as the photometer head. When both sides of the screen are equally illuminated let the distance from I to the screen be d_1, and from Q to the screen d_2. I

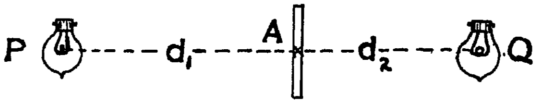

FIG. 102.

the illuminating power of the sources P and Q be in candle-power and denoted by c.p.$_1$ and c.p.$_2$ respectively, the illumination on A due to the source P is c.p.$_1/d_1^2$ candle-feet, and that due to the source Q is c.p.$_2/d_2^2$ candle-feet.

The two illuminations are equal; therefore

$$\frac{\text{c.p.}_1}{d_1^2}=\frac{\text{c.p.}_2}{d_2^2}.$$

If c.p.$_2$ be the illuminating power of the standard Q the candle-power of P is given by

$$\text{c.p.}_1=\text{c.p.}_2\times\frac{d_1^2}{d_2^2}.$$

For instance, if the standard Q be 16 c.p. and the distances d_1 and d_2 be 80 centimetres and 120 centimetres respectively, then the candle-power of the lamp being tested

$$=16\times\frac{80^2}{120^2}=7.$$

For electrical engineering work a secondary standard of light is used and is generally a carefully made glow lamp which has been previously calibrated against an absolute standard, so that the candle-power can be obtained from a knowledge of the watts consumed.

A great variety of photometers have been devised, but they all work on the above principle. The Bunsen photometer is similar to the one described above, with the addition of two mirrors inclined at an angle of 45 degrees to the screen, so that both sides of the screen can be viewed simultaneously, thus enabling a more accurate reading to be obtained.

Lummer-Brodhun Photometer.—The photometer in most general use is that illustrated in Figure 103 and is known as the Lummer-Brodhun photometer. It consists of an opaque screen S placed inside a metal case M, so that it can be illuminated from two sources of light, P and Q. The two sides of the screen are viewed through the telescope T by an arrangement of two right-angled prisms P_1 and P_2 and a double glass prism P_3. The latter prism consists of two right-angled prisms placed together so as to form a cube, the largest face of one being partly bevelled.

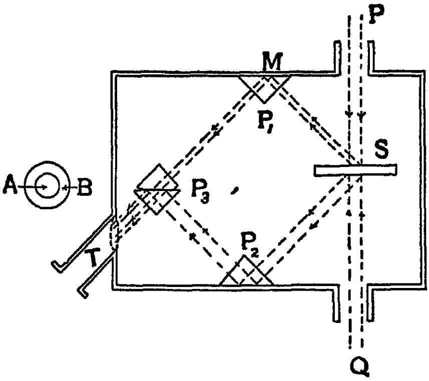

FIG. 103.—Lummer-Brodhun photometer.

The path of the light is as shown, and an observer looking into the telescope T will see an illuminated ring surrounded by another zone of different intensity. The central rays reaching the eye at T come from the top face of the screen, and the surrounding rays come from the bottom face. The observer therefore moves the photometer until the central part A, illuminated from P, and the outer part B, illuminated from Q, are of the same brightness. The intensity of illumination on both sides of the screen is then the same, and the relative candle-powers of the two sources can be determined as above.

The relative illuminating powers of two sources of light are only strictly comparable when the colour of the light emitted by both is the same, but this very seldom happens when testing electric lamps. When there is a difference in colour a rough comparison can be made by comparing the candle-power of the two sources, when a red, a yellow, and a blue coloured glass are placed in turn between the eye of the observer and the telescope T. The mean of the three values thus obtained may be taken as the true photometer reading.

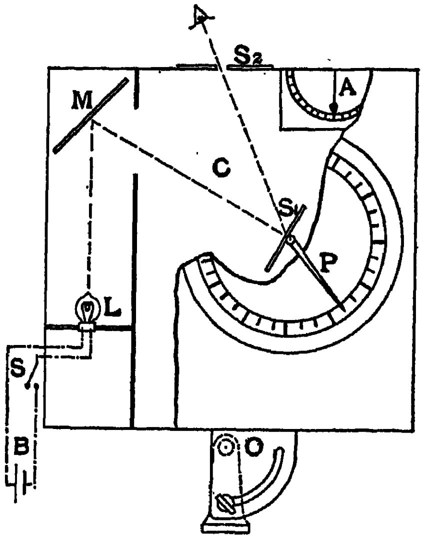

FIG. 103A.—Trotter's portable photometer.

Trotter's Photometer.—Figure 103A illustrates the design of a portable photometer for measuring either illuminating power or the illumination in candle-feet at any required angle. A small standard lamp L throws off a beam of light, which is reflected by the mirror M on to the screen S_1, the parts mentioned being enclosed inside a case C. The screen S_1 can be rotated about an axis, and is viewed through three small slits in the screen S_2, which receives the illumination to be measured. The lamp L is carefully aged and standardised, so that it can be relied upon to retain its candle-power almost indefinitely, since it is only in use during the actual time the reading is being taken, and this, as a rule, does not exceed 30 to 40 seconds. A portable 4-volt battery B supplies current to L, the circuit through which is made by the switch S. In operating the instrument the screen S_1 is rotated about its axis, and the illumination on it thereby varied, until a balance of brightness between S_1 and S_2 is obtained. The illumination of S_2 in candle-feet is then read off on a direct reading scale over which the pointer P passes, the latter being fixed to the axle of the screen S_1.

The case C is pivoted at O, and can be moved through an angle of 90 degrees, thus allowing the illumination to be measured on any plane between the horizontal and the vertical, the setting of the photometer being adjusted by the aid of a plumb-bob A, which moves over a graduated scale.

If, instead of the illumination, the candle-power of the source of light is required. the screen S_2 is turned towards it so that the angle of incidence shall be 0 degrees. The measurement of illumination is then made, and this value, multiplied by the square of the distance in feet of the photometer from the light, gives the candle-power required.

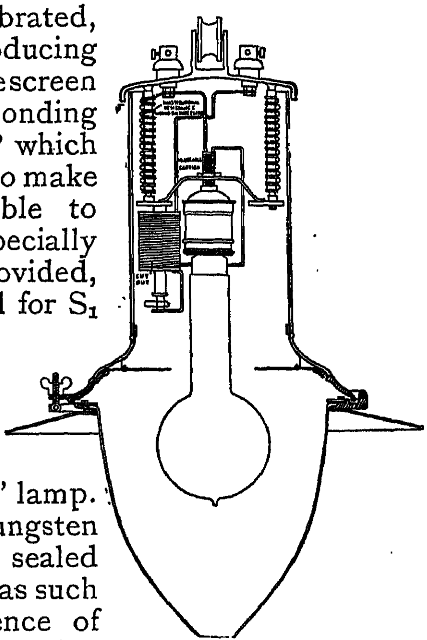

Fig. 103B.—Gasfilled Lamp and Fitting.

The instrument is calibrated, in the first instance, by producing known illuminations on the screen S_2, and finding the corresponding positions of the pointer P which give a balance. In order to make the instrument applicable to lights of every colour, specially coloured screens are provided, which can be substituted for S_1 or S_2 as required.

The Gasfilled Lamp.—The Nernst and arc lamps are now being replaced by the "half-watt" or "gasfilled" lamp. This has a closely coiled tungsten filament mounted in a sealed bulb containing an inert gas such as nitrogen. The presence of this gas prevents the volatilizing of the filament, and thus allows it to be run at a much higher temperature and efficiency than was possible in the original metallic filament lamp. Fig. 103B shows a modern gasfilled lamp and fitting.

CHAPTER VII

UNDERGROUND AND AËRIAL CONDUCTORS

ALL conductors have a definite resistance, so that when current is transmitted along them a certain expenditure of energy takes place. The energy expended generates heat, and is proportional to C^2R, where C is the current and R the resistance of the conductor. With a given current the temperature gradually increases until the rate of generation of heat is equal to the rate of loss of heat by radiation, at which stage the temperature remains constant for a particular current.

For a given current the increase in temperature of a conductor will depend upon:

1. Its electrical resistance.
2. The nature of its radiating surface.
3. The material and condition of the surrounding medium.

As mentioned in Chapter II., copper, on account of its low specific resistance, is generally used as the conductor in the transmission of electrical energy, although, under certain conditions, aluminium and iron are employed for aërial lines. The copper used for such purposes should be electrolytically refined, and of not less than 98 per cent. conductivity, according to the standard suggested by the Engineering Standards Committee. (The E.S.C. standard for annealed high conductivity copper is a uniform wire 1 metre long, weighing 1 gramme, and having a resistance of 0.1508 standard ohm at 15.6° C., *i.e.* 60° F.)

Copper produced by the process of smelting is less expensive, but in the best practice is not now used for electrical purposes, as its conductivity, due to the presence of small quantities of foreign materials, is never as high as that electrically deposited.

As regards radiation, a blackened or rough surface radiates heat at about twice the rate of a polished surface. Professor S. Forbes has determined that gutta-percha is a better radiator of heat than rubber in the ratio of 48 to 41.

Copper wires used as conductors are of circular cross-section drawn into certain definite sizes or gauges, the usual practice among British cable manufacturers being to adopt the standard wire gauge (S.W.G.). Appendix I. gives details of the various sizes of copper wires employed in electrical engineering, the values being expressed in metric units.

For the transmission of electrical energy the standard practice is to use stranded conductors or cables, the wires forming the cable being twisted into strands having 3, 7, 19, 37, 61, or 91 separate wires; for instance, a 37/20 cable consists of 37 wires of size No. 20, S.W.G. This arrangement ensures the necessary flexibility, and largely eliminates the inherent mechanical weakness of solid copper to fracture when subjected to repeated bending.

Dimensions of Conductor.—The size of conductor required to transmit a given current must be settled from a knowledge of—

1. The permissible percentage transmission loss.
2. The maximum permissible temperature rise.

The Standardising Committee of the Institution of Electrical Engineers recommend the following formula for determining the carrying capacity of a conductor:

$$C = 2.6\, a^{0.82},$$

where C is the current in ampères and a the area of cross-section of the conductor in square inches. The current density determined by this formula is such that the maximum temperature rise will not exceed 17° C., while conductors of small sectional area will be worked at a relatively higher current density than those of larger sectional area. This follows because the radiating surface per unit area of cross-section decreases with increase in diameter; consequently in order to maintain a uniform temperature rise the current density must be reduced as the area of cross-section (or diameter) increases.

The more general practice among British engineers is to limit the current density to 155 ampères per square centimetre (1000 ampères per square inch) of sectional area, thus obtaining a drop in pressure of 2.7 volts per 100 metres.

Table XI. gives the dimensions, capacity (on the basis of (1) I.E.E. formula, and (2) 155 ampères per square centimetre), resistance, and weight of stranded copper conductors used in practice. The values are given both in British and metric units.

In certain cases the values of current given in the accompanying table are not adhered to, but each problem is considered on its merits, as shown by the following example:

Example.—Current is required for a group of 200 230-volt lamps taking 60 watts each: the lamps are at a distance of 100 metres from the source of energy. What must be the size of cable, if the loss of pressure is not to exceed 2.5 per cent. of the required P.D. across the lamp?

Current transmitted along the cable

$$= \frac{200 \times 60}{230} = 52 \text{ ampères.}$$

$$\text{Volts drop in cable} = 2.5 \times \frac{230}{100} = 5.75.$$

$$\text{Resistance of cable} = \frac{E}{C} = \frac{5.75}{52} = 0.11 \text{ of an ohm.}$$

$$\text{Area of cross-section of cable} = \frac{\rho \times l}{R}.$$

ρ for copper $= 1.6 \times 10^{-6}$ ohms per centimetre cube.
$l = 100 \times 2 = 200$ metres $= 2 \times 10^4$ centimetres.
Area of cross-section of cable

$$= \frac{1.6 \times 10^{-6} \times 2 \times 10^4}{0.11} = 0.29 \text{ square centimetre.}$$

From Table XI. a 19/17 cable has the equivalent required sectional area.

INSULATED CABLES

The material forming the insulation of cables is generally referred to as the di-electric, and should

1. Be mechanically strong, so that it is not easily injured.
2. Have flexibility, so that it can be bent without cracking.
3. Be capable of standing moderately high temperatures without injury.
4. Be unaffected by gases or acids with which it is liable to come into contact.

The materials used for insulating cables may be broadly divided into two classes; namely, hygroscopic and non-hygroscopic. The hygroscopic materials chiefly used are jute and paper. To the non-hygroscopic class of di-electric belong materials such as pure rubber, vulcanised rubber, and bitumen. Table XII. gives the specific resistance of the materials most generally used for insulating cables. The resistance is given in megohms per centimetre cube, and the temperatures are those at which the values were obtained: the latter are necessary, as the specific resistance of all insulating materials decreases with increase of temperature.

TABLE XII

SPECIFIC RESISTANCE OF INSULATING MATERIALS USED FOR CABLE DI-ELECTRIC

Material.	Megohms per centimetre cube.	Temperature.
Gutta-percha	250×10^6 to 450×10^6	15° C.
Bitumen	400×10^6	15° C.
Paper (resin oiled) . . .	3000×10^6	15° C.
Vulcanised Rubber . .	4000×10^6	15° C.
Pure Rubber	5000×10^6 to $10{,}000 \times 10^6$	15° C.
Air	Infinity.	—

Pure rubber, as will be seen, has the highest specific resistance, and the best class of rubber for insulating purposes is that obtained from the Para rubber tree. Pure rubber, however, readily oxidises, and has a great affinity for moisture, so that its physical state changes with time, thereby lowering its specific insulation. Vulcanised rubber, prepared as described below, is now largely used, and although its specific resistance is lower than that of pure rubber, it does not deteriorate so rapidly under atmospheric conditions. Gutta-percha is now seldom used; for, although it originally has a high specific insulation, it becomes brittle when exposed to atmospheric changes and softens at a comparatively low temperature.

The necessary thickness of a di-electric depends upon the specific resistance of the insulation, the working pressure, and the diameter of the conductor. For pressures up to 600 volts the thickness varies between 2 and 3.3 millimetres, while for pressures between 1000 and 10,000 volts the minimum thickness of di-electric is about 3.7 millimetres, and increases by 1.2 millimetres per 1000 volts.

From a knowledge of the specific resistance of the insulating material, the insulation resistance of a length of cable can be determined in the following manner:

From the formula $R=\frac{\rho\times l}{a}$, the resistance of a thin circular lamina of the di-electric, having a radius r, thickness dr, and length l, is given by

$$R_1=\frac{\rho\times dr}{2\pi rl}.$$

The di-electric may be considered as a series of concentric laminæ similar to above, in which the radii vary from r_1 the internal radius of di-electric, to r_2 the external radius. The total insulation resistance of the cable will then be given by

$$R=\frac{\rho}{2\pi l}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$=\frac{\rho}{2\pi l}\log_e\frac{r_2}{r_1}.$$

Example.—Calculate the insulation resistance of a kilometre of cable, the di-electric of which is vulcanised india-rubber, having a specific resistance of 4×10^{15} ohms per centimetre cube. The radius of the core of the cable is 5 millimetres, and the di-electric is 2.5 millimetres thick.

$r_1 = 0.5$ centimetre.
$r_2 = 0.5 + 0.25 = 0.75$ centimetre.
$\rho = 4 \times 10^{15}$ ohms per centimetre cube.
$l = 10^5$ centimetres.

The insulation resistance is given by

$$R = \frac{4 \times 10^{15}}{2\pi \times 10^5} \times \log_e \frac{0.75}{0.5} = \frac{2 \times 10^{10}}{\pi} \times 0.4$$
$$= 2.5 \times 10^9 \text{ ohms} = 2500 \text{ megohms.}$$

Cables for electrical purposes may be divided into three classes:

1. Cables insulated with vulcanised rubber.
2. Cables insulated with plastic materials.
3. Cables insulated with fibrous materials, impregnated with oil or bituminous compounds, and covered with some waterproof material such as lead.

Vulcanised Rubber Cables.—In the early days of electrical engineering, rubber-insulated cables were the only ones used, but owing to the increasing cost of rubber they have been largely superseded by classes two and three, and are only used where high insulation and easy manipulation are of first importance.

Vulcanised rubber is prepared from pure rubber mixed with about 5 per cent. of sulphur, the vulcanising taking place at a temperature of about 150° C. Some of the sulphur forms crystals in the vulcanising process, and when the cable is exposed to moisture or chemical action the crystals dissolve, causing the di-electric to become perforated, and lowering the insulation resistance of the cable. Manufacturers, however, take the necessary precautions to reduce this deterioration to a minimum.

The strands of conductors insulated with vulcanised rubber should always be tinned, and the cable covered with a few layers of pure rubber tape to prevent the sulphur used in the vulcanising process from attacking the copper. The importance of perfect tinning cannot be over-rated for the following reasons:

The deterioration to which rubber is most liable is that due to oxidation, and this takes place to a greater extent with pure rubber than with vulcanised rubber. Copper has a great affinity for oxygen, but will readily part with its oxygen to a substance which has a greater affinity for it. Rubber is one of these substances. Hence in a rubber-insulated cable there are two substances—copper and rubber—in close proximity which, for chemical reasons, should be kept apart. The coating of tin on the conductors acts as the separating medium, and from the above facts the great importance of its efficient application is clear.

The first layer of di-electric should consist of rubber without admixture of any kind, and for manufacturing purposes should have certain good mechanical properties. The rubber used for such purposes is that obtained from the Para rubber tree. In order to give it the necessary mechanical properties, the rubber is "aged" by hanging in dark rooms at a certain uniform temperature for several months. In the case of cheap cables, pure rubber is substituted by a lower grade rubber in which the requisite elasticity and mechanical properties are obtained, by treating the surface with chloride of sulphur in carbon bi-sulphide. Such "rubber" thus contains an ingredient which is inconsistent with the function of the rubber; also in many cases traces of hydrochloric acid are present as an after-result of this chemical treatment. It will be obvious that there are considerable risks in using such substitutes.

The vulcanised rubber forming the body of the di-electric should have for its basis fine Para rubber, be vulcanised with the minimum proportion of sulphur, and mixed with mineral matter only to the extent required for the particular grade of cable which it is intended to produce. Rubber substitutes should never be used; in fact no organic matter other than rubber should be present. Any known substitutes increase the liability to oxidation, because they only form mechanical mixtures, and neither combine chemically nor cohere physically, but form cables which soon become electrically and mechanically defective.

The usual test for the presence of such substitutes is to place a sample of the di-electric in a steam chamber at a temperature of 150° C. for, say, four hours. The presence of any of the deleterious substances indicated becomes apparent by their saponification.

In order to protect the di-electric from slight mechanical injury, it is covered with specially treated braiding, woven so as to resist moisture. Rubber itself being of a highly inflammable nature, cables insulated with rubber compounds should preferably be served with an overall layer of some non-inflammable covering in the form of tape or braiding; tape and braiding together are commonly used, and such cables are commercially known as "Fireproof" or "Fire-resisting" cables.

Figure 104 shows a section through a single core cable, insulated in the manner described.

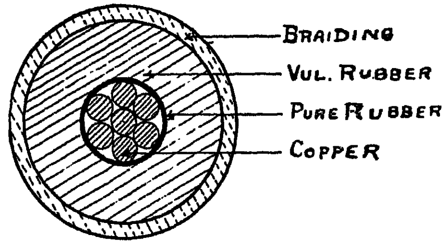

FIG. 104.—Vulcanised rubber insulated cable.

Condensed vapour, if present in appreciable quantity, is very detrimental to rubber-insulated cables. The deterioration of rubber under such conditions is not, as is commonly supposed, due to the direct soaking-in of the water, but to chemical actions which gradually decompose the rubber. In some cases moisture from wet plaster finds its way into the pipes or ducts in which cables are laid, and this being usually of an alkaline nature is fairly rapid in its deteriorating action. This action is most noticeable where low-grade di-electrics are used, as they are much less homogeneous than pure rubber compounds, and therefore the reactions between the moisture and the sulphur (which ingredient plays the chief part in these reactions) can take place with greater facility.

The increasing cost of rubber-insulated cables, due to the immense development within recent years in electrical engineering and other industries in which rubber is extensively used, has stimulated investigations into the di-electric properties of other materials. These investigations have resulted in the adoption, with more or less success, of bitumen, paper, fibre, etc., as insulating media.

Vulcanised Bitumen Cables.—Cables insulated with pure bitumen were found to be liable to decentralisation. Vulcanised bitumen is now used, and is produced by sulphuretting the products of distillation of certain bituminous oils. This alone, or with other compounds, forms the basis of several cable-insulating compounds. In cables thus insulated the conductor is first covered with a thin layer of fibrous insulation, such as jute or paper impregnated with insulating compounds. The di-electric proper is vulcanised bitumen applied warm, and then subjected to special treatment to give it the necessary mechanical strength. The di-electric is finally covered with the necessary protecting tapes and braidings to suit the conditions under which the cable is to be used.

The disadvantages of this type of cable are:

First. Its tendency to soften and decentralise under undue heat applied externally, or internally through overloading. Bitumen-insulated cables should not be worked at a greater current density than 155 ampères per square centimetre.

Second. Their lack of mechanical strength necessitates more careful handling than with other classes of cables.

A di-electric of this nature for ordinary cables, which may be subject to considerable variation in temperature, should be such that it does not get too soft at fairly high temperatures, or too brittle at low temperatures. Hardened compounds are made by loading the vulcanised bitumen with mineral matter which can only form a mechanical mixture, and therefore subdivides the vulcanised bitumen, so that instead of a homogeneous resilient mass a hard mixture is obtained which is highly susceptible to the deteriorating influences to which cables are usually exposed.

The compounds commonly used contain elements which are affected by various external agencies. Air, for instance, has a gradual hardening and oxidising effect on them, producing brittleness, particularly if the adulterants be free oils or other oxidisable materials. Acids, too, have much the same effect, but alkaline solutions produce most rapid deterioration.

Fibrous Di-electric Waterproof-Sheathed Cables.—The fibrous insulating materials generally used are paper, jute, and cotton: the former generally consists of manilla paper and is the more extensively used; the latter are frequently applied in the form of a yarn. These fibrous materials are impregnated with oil or bituminous compounds to enhance their insulating properties, and also to render the cable more pliable and reduce the liability to absorb moisture. Resin oil, or resin and resin oil are frequently used for this purpose.

Inferior forms of fibrous di-electric cables usually contain small proportions of manilla paper and large proportions of straw, esparto, and wood, all chemically treated. The introduction of such substances has the following effects:

First. The di-electric strength and insulation resistance is lowered. A cable for 10,000 volts working pressure, having chemical wood for the di-electric, requires 70 per cent. greater thickness than a cable of the same di-electric strength in which manilla paper is the di-electric.

Second. The mechanical strength and resilience are impaired. The extent and rapidity of the deterioration of woody papers under the influence of heat are shown in Figure 105. The curves are reproduced from a paper on Electrical Cables by Atkinson and Beaver.* Two samples of di-electric were experimented with, one prepared with manilla paper and the other containing sulphite (chemical wood) paper: these were placed in oil at a temperature of 155° C., and the breakdown stress in kilogrammes was obtained at various periods as indicated by the curves. Curve A is for manilla paper, and B for chemical wood-paper. Subsequent analysis of the latter indicated that traces of residual chemicals probably contributed largely to its more rapid deterioration. The curves represent the average results of about 100 separate tests on each class of di-electric.

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* Manchester Section of the Institution of Electrical Engineers, 31st January 1905.

Third. There is a danger of disintegration in course of time due to the action of residual chemicals. Manilla paper can be prepared with very little chemical treatment, but the fibrous material prepared from straw, esparto, or wood fibres, requires to be subjected to severe and drastic chemical treatment, and it becomes practically impossible to free the fibre from every trace of the chemicals used.

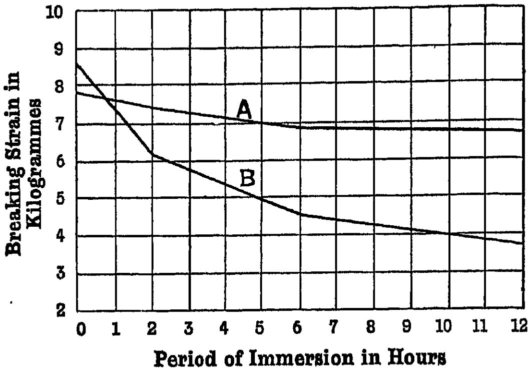

FIG. 105.

The impregnating compounds used by different manufacturers vary considerably. From an electrical point of view they should have the highest possible specific resistance, their specific inductive capacity should approach as nearly as possible that of the fibrous material, and their consistency should be plastic but not fluid.

Fibrous di-electrics have rather a high temperature coefficient, and Figure 106 is a curve illustrating the variation of insulation with temperature for a standard paper-insulated cable. At 16° C. the insulation resistance is 500 megohms, but at 26° C. it is only 100 megohms, this being equivalent to a variation of 20 per cent. per degree centigrade.

The weak point of this class of cable is that, however well the fibrous material is impregnated, it is always liable to more or less readily take up moisture from the atmosphere, whereby the insulation resistance is reduced; hence the necessity of covering with a waterproof sheath. In most cases the sheath consists of lead. The thickness of the lead is such that it forms an additional mechanical protection against damage in handling and external corrosion. The lead is applied in a plastic state under hydraulic pressure, the greatest care being taken to ensure an absolutely non-hygroscopic covering. Failure to attain this appears to have been the cause of many breakdowns with this class of cable, though many more have been due to corrosion in various forms.

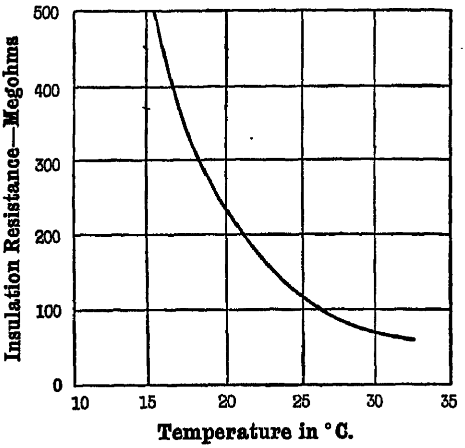

FIG. 106.—Variation of insulation resistance with temperature for paper-insulated cables.

The chief causes of corrosion in lead coverings under working conditions may be summarised as follows:

1. Impurities in the lead causing local galvanic action.
2. Corrosion by chemical action.
3. Electrolysis.

The first cause of corrosion is now very rare, as analyses show that the total proportion of impurities in practically any brand of new soft British pig lead will not average more than 0.5 per cent. The only impurity likely to get into the lead during the manufacturing process is lead oxide, but the chances are remote owing to improved methods of manufacture.

Corrosion due to the second cause is fairly common, and may occur in certain soils where organic acids are produced by the decomposition of wood or vegetable matter, or where cables are laid in "made-up" ground consisting of ashes, domestic refuse, etc.

Electrolysis is by far the most serious form of lead corrosion, and is also the most common. The word electrolysis is here taken to mean the electro-chemical action due to the passage of a current to or from the lead. The corrosive effect will depend upon the magnitude of the current, and the character of the corrosion may vary from minute holes to large patches, though faults would not be apparent until the di-electric was sufficiently exposed to absorb moisture and cause a dead earth or short circuit. The deterioration, due to electrolytic action, can be greatly reduced by careful attention to bonding at joints, which ensures electrical continuity throughout the sheath, the latter being well earthed at suitable points.

Instead of a covering of lead a waterproof envelope of non-hygroscopic material, such as vulcanised bitumen, may be provided. Such cables have all the advantages of the high specific insulation and di-electric strength of the lead-covered paper-insulated cables, and, in addition, the electrolytic action is eliminated.

Cables of this class are comparatively low in first cost, and if constructed of good materials are very efficient. They have been largely used for lighting and power mains at all voltages. Again, with the use of paper as an insulator, greater overloads can be carried with safety than would be possible with vulcanised rubber or bitumen. This in many cases is an advantage.

The great disadvantage of paper-insulated cables lies in the extreme care necessary to exclude moisture, as the insidiousness with which it creeps when once absorbed is not less than amazing. Figure 107 illustrates a special terminal for sealing the ends of paper-insulated cables. The conductor is sweated into the socket S, and the joint is covered with a hollow glazed porcelain insulator P attached as shown, the inside being filled up solid with bituminous compound B poured in through the aperture A. The joint is entirely surrounded by bitumen, which effectually excludes moisture from the di-electric. When the cable is lead-sheathed the lead terminates in a metal end-piece M which is screwed into the porcelain, the end-piece being "wiped" to the lead sheath.

Multiple Core and Concentric Cable.—Where two, three, or four conductors of a system of feeders or distributors follow a common route, it is sometimes considerably more convenient in handling and laying to have them in the form of a single cable than as separate single conductors. Cables are therefore made consisting of two or more conductors, arranged either side by side or concentrically.

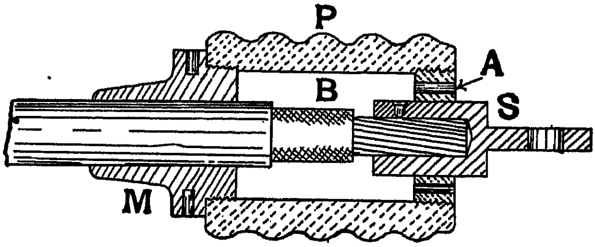

FIG. 107.—Sealing end for paper-insulated cables.

A three-core cable is shown in section, Figure 108. It consists of three conductors of circular section, each separately insulated, bound together, and insulated in common as shown.

Figure 109 shows a section through a two-conductor concentric cable consisting of the inner conductor or core; the first di-electric; the second or outer conductor (the wires of which are selected to have an equal sectional area to the inner conductor); the second di-electric; a lead sheath; and finally braiding or armouring, depending upon the conditions under which the cable is to be used. Two-conductor concentric cables are usually referred to as simply "concentric cables," and the three-conductor variety as "triple concentric cables," The latter are chiefly used on three-wire direct-current systems.

In concentric cables the inner conductor should always be the negative, in order to reduce the effect of osmosis. The phenomenon of electrical osmosis, whereby moisture tends to accumulate at the negative conductor but is repelled from the positive, can be readily observed by placing two metal plates, connected to a source of E.M.F., into some moist clay. The osmotic action commences immediately there is a difference of potential between the two conductors and varies in intensity as the E.M.F., and if the outer conductor be the negative there is a tendency to lower the insulation resistance between that conductor and earth, until finally minute currents pass from the negative conductor to earth causing electrolytic action. Hence the advisability of keeping the negative conductor as far away from earth as possible, owing to the natural tendency to become earthed.

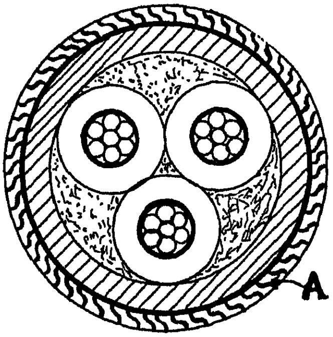

FIG. 108.—Three-core armoured cable.

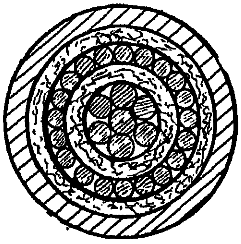

FIG. 109.—Concentric lead-sheathed cable.

Armouring.—If a cable is to be laid directly in the ground or placed where it is likely to meet with rough usage, a mechanical protection or armouring should be applied. Armouring may take the form of steel tape, usually in two layers, the lays being in opposite directions and lapped over the cable in such a manner that the space between adjacent convolutions of the first is covered by those of the second. Joints in the tape should be either brazed or welded, no other joint being permanent. After application, the armouring is coated with some waterproof material, and the cable finished off with an overall serving of yarn, or jute braiding soaked in tar.

Armouring is also sometimes in the form of galvanised iron or steel wires of a suitable gauge and having a tensile stress of not less than 4000 kilogrammes per square centimetre. The wires must closely envelop the cable, and are applied over a bed of tarred jute, lapped on in the opposite direction to the lay of the wires forming the conductor. This armouring is finally treated as in the case of steel tape. In special cases the wires are of the cross-section shown at A in Figure 108, each wire locking into the side of its neighbour. This forms the best and strongest mechanical protection devised for direct application to cables.

METHODS OF LAYING INSULATED CABLES

Insulated cables for the transmission of electrical energy are invariably laid underground in one of three systems:

1. Solid.
2. Draw-in.
3. Direct.

Solid System.—In this system the cables are laid in wood, earthenware, or iron troughs, filled up solid with an insulating compound poured in when heated to a fluid state. Cables laid in this manner do not require to be armoured, as the troughing affords a good mechanical protection.

Wood troughing is the least expensive, and can be conveniently made in greater lengths than troughing of either of the other materials. It should be of well-seasoned material impregnated with a suitable preservative, great care being taken to ensure that the preservative contains no ingredient likely to be injurious to the filling-in compound.

Glazed earthenware troughing of various forms is also largely used, and has the advantage over wood that it is not liable to rot. Its cost does not materially exceed that of wood, and is considerably less than the cost of cast iron. It cannot, however, be conveniently obtained in greater lengths than 1 metre, so that a large number of joints is inevitable.

Cast-iron troughing is the most costly, even when of very thin section. Cast-iron and earthenware troughs are usually supplied with socket ends, whereby each length of troughing is jointed with the next.

Cables laid in troughing are supported every few metres on insulating bridge-pieces, so that the filling-in compound entirely surrounds the cables. The bridge-pieces are usually of wood or asphalt. Asphalt bridge-pieces are far more efficient, because under proper conditions of filling the trough, the surfaces which come into contact with the hot compound are melted, and, in cooling, the whole forms a practically homogeneous mass.

The material used for filling the trough should be either refined bitumen or the best coal-tar pitch. Coal-tar pitch is liable to crack, but to render it plastic it is mixed with a certain quantity of shale-oil. Trinidad bitumen is the most efficient material to use, and in a semi-refined state contains natural oils which render it more adhesive and less liable to crack.

After the trough has been filled up solid it is generally covered with a single layer of bricks or tiles, except in the case of cast iron, when the cover is usually of the same material. Several cables may of course be laid in one trough by making it of sufficient width. Figure 110 shows a section through a trough made of earthenware. The cables C rest on the bridge-pieces P and so are entirely surrounded by the bitumen B. The cover A is fluted longitudinally on the under surface, and so held firmly in position laterally.

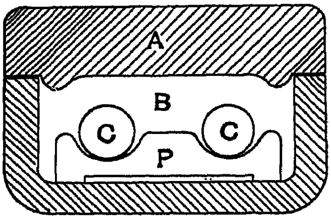

FIG. 110.—Cables laid in bitumen.

Draw-in System.—In this system the cables are drawn into ducts, of which there are several forms. In the early days of electrical engineering cast-iron pipes were invariably used, but owing to their high initial cost have been superseded by stoneware conduits. The latter may be either in the form of ordinary drain pipes of 7 to 10 centimetres internal diameter, or of casing, in which a number of ducts are made up in one block. The latter is the more general practice, and Figure 111 illustrates a 3-way conduit.

Such conduit is usually made from hard-burnt glazed stoneware. This material is strong and tough, and is thoroughly proof against the chemical action of the soil, and hence very durable. Earthenware conduits are invariably of rectangular section, and by grouping a number of ducts in one block a given number of ways can be provided within the minimum space. It is found a convenient plan to adopt a 3-way or 4-way conduit as a unit, and to build up multiples to the required number.

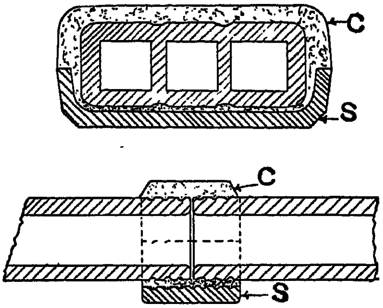

FIG. 111.—Three-way conduit.

The ducts are laid at a depth of about 40 centimetres or less below footways and 80 centimetres or more under roadways. At intervals of from 40 to 80 metres draw-in boxes are built in line with the conduit. These boxes should be of at least sufficient area to conveniently pass in the maximum size of cable without undue bending, and are usually constructed of brick, provided with a cover C, as illustrated in Figure 112.

The jointing of lengths of conduit is effected with compound or cement C, as shown in Figure 111. A mandril, capable of being mechanically expanded *in situ*, is inserted at the joint to prevent the intrusion of the jointing material. The mandril is, of course, released and withdrawn as soon as the joint is sufficiently set. The lengths of conduit are kept in alignment by means of permanently placed earthenware saddles S, which also serve as moulds for the joints. The conduits are sometimes laid on a bed of concrete, in which case saddles at the joints may be dispensed with.

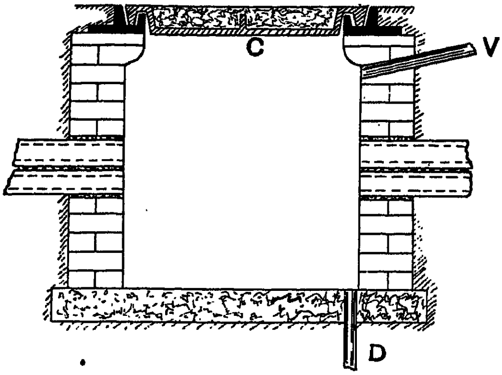

FIG. 112.—Draw-in box.

Some of the London Electrical Power Companies have their conduits built up of single ducts of circular interior and octagonal exterior cross-section, as shown in Figure 113, the conduits being built up in blocks of any desired number and the whole encased in concrete.

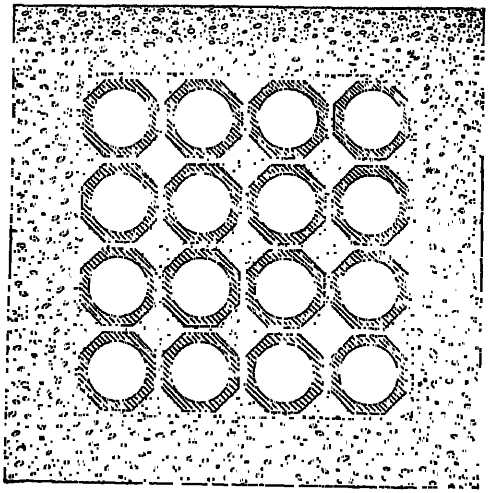

FIG. 113.—Built-up conduit.

An advantage claimed for the draw-in system is that when once the conduit has been laid, repairs and additions to the system can be made without excavation, whereas with cables laid on the solid system, unless sufficient cables for all future requirements are laid down at first, it is necessary to open the whole length of the road in order to lay an additional cable. The best practice is to use the draw-in system for feeder cables which go direct from the generating station to the feeding points, and the solid system for distribution.

Conduits are liable to become charged with explosive gases caused either by leakage from gas mains in the vicinity, or by leaks from the cables themselves, the insulation of which when heated gives off explosive gases. It is therefore necessary to provide efficient ventilation. The draw-in boxes as well as the conduits themselves should be effectually drained. These desiderata are generally accomplished by connecting a drain pipe D and ventilator V to each draw-in box, as shown in Figure 112.

Direct System.—Armoured cables are generally laid directly in the ground, and have no further protection than that afforded by their own armouring. This method of laying is the least expensive, and in certain soils, which are free from sulphur, has been attended with great success. The cables are laid at a depth of from 60 to 90 centimetres, and each individual cable should be surrounded with at least 7 centimetres of soft earth well pounded. In many cases these armoured cables are left entirely unprotected; in others a line of bricks or lengths of creosoted boards are laid directly over the cable before reinstatement of the trenching. This precaution gives an indication, to future excavators, of the presence of the cables.

Cable Joints.—All joints in cables should be made in proper joint boxes, and the jointing performed in such a manner that a thoroughly good electrical contact will be obtained. Figure 114 shows the arrangement of a "straight-through" joint box for single-core cable. The ends of the cables to be jointed are passed, one from each end, into a cast-iron box, which should be designed so as to allow ample space for making the joint.

In making the joint the insulation is cut away from the cable ends so as to leave 5 to 7 centimetres of the conductor bare. The free end of each cable is set into a tight-fitting metal sleeve M, the two ends butting together. In dealing with paper-insulated cables, it is necessary to tin the strands before inserting the ends into the sleeve, as tinning is not a precautionary necessity with this as with other forms of insulating materials, and forms no part of the process of manufacture of paper-insulated cables. The sleeve, which is usually of brass, should be well tinned. The joint is made by heating the sleeve, so that its tinned inner surface adheres to the tinned strands, and solder is poured in at a small aperture shown in the figure.

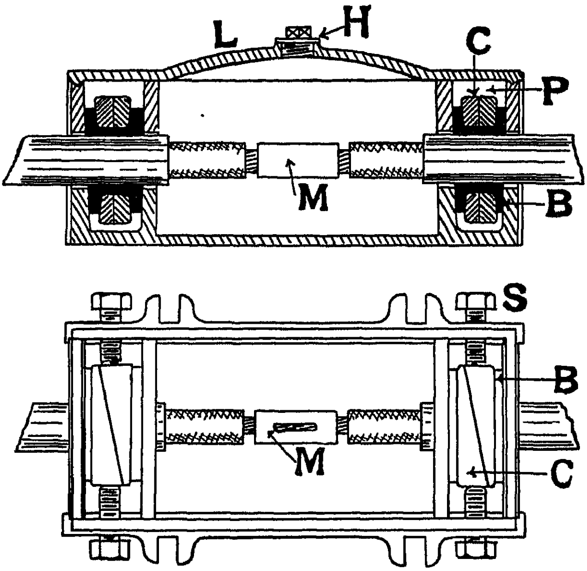

FIG. 114.—Joint box for single-core cable.

No other flux than resin should be used for soldering. Analyses of the fluids, pastes, and solid compounds made for soldering purposes show that they nearly all contain chloride of zinc in various forms, some of which distinctly promote the fluxing action; and though most of them are suitable for jointing exposed solid conductors, there is a great risk in using them on insulated stranded cables, due to their creeping quickly up the strands, setting up a corrosive action on the conductor, and attacking the di-electric.

After the conductors have been soldered, the interior of the box is filled up solid with bituminous compound. The compound used should be of a high adhesive and insulating quality. Some jointing compounds contract as much as 25 per cent. on cooling, particularly if poured in at too high a temperature. It is therefore best to pour in the compound at the lowest temperature which permits of its flowing uniformly over the box, as too high a temperature may also injure the cable insulation. When full, the box is covered by the cast-iron lid L, and after cooling, the shrinkage of the compound is made up by pouring in more, through the "filling-in" hole H, which is afterwards stopped with a screw-plug.

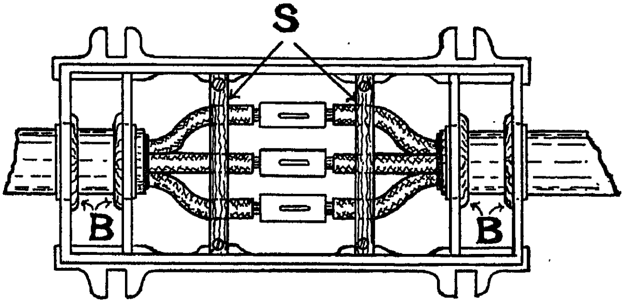

FIG. 115.—Joint box for three-core cable.

In jointing lead-covered cables the electrical continuity of the sheath should be preserved. A good method is to bond the sheath on to the box by lead bonds fitted into pockets placed outside the joint box, as shown in the figure. The bonds B are so designed that by adjusting the set-screws S on either side of the pockets P two wedge-shaped pieces of cast iron C are forced towards one another. This action forces the flanges of the lead bonds, previously put round the cable, against the sides of the pockets, and also causes the lead bond to make good contact with the sheath itself.

Figure 115 illustrates a joint box for a three-core cable. The three cores are separated by wooden spreaders S so as to keep the separate joints at least 5 centimetres apart. The glands of the box are packed with wood bushes B, treated with preservative compounds, the bushes being of such dimensions as to tightly grip the cable and prevent the movement of the internal fittings The method of making the joints is similar to tha described above.

A joint in a concentric cable is illustrated in Figure 116. The core of the cables is sweated into a brass sleeve S_1, which is then covered with three or four layers of special cotton tape T treated with resin oil. A spli sleeve S_2, tapered at each end, is next placed over the core joint, and the several strands of the outer conductor are cut to suitable lengths, twisted round the tapered portion of the fitting, bound with No. 18 or 20 S.W.G. tinned copper wire, and then efficiently soldered The second joint is taped as before, and the joint bo (the outline only of which is indicated in the figure) fille with compound.

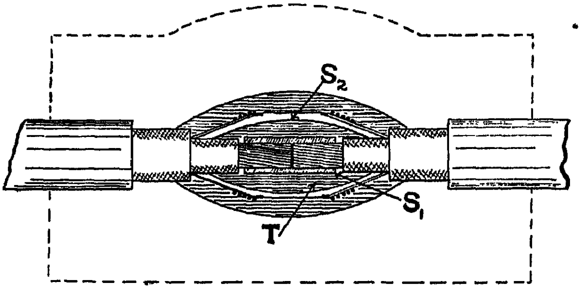

FIG. 116.—Joint in concentric cable.

AËRIAL CONDUCTORS

Aërial lines are constructed by supporting bar conductors on insulators placed on cross arms attache to poles or lattice columns. The poles, either of wood o iron (more often the former), are selected of sufficien strength to carry the weight of the conductors. Aëria conductors carried overground in this manner form th cheapest method of distribution, and have been largel used in America and on the Continent, although, eve yet, in Great Britain they have only been adopted to very limited extent.

Until quite recently hard-drawn copper was the only conductor used, but owing to the development of improved methods of manufacture aluminium is coming into more extended use. Aluminium has a low specific gravity, but its tensile strength and conductivity are also low compared with hard-drawn copper.

	Aluminium (97.5 per cent. pure).	Copper (hard drawn, 100 per cent. pure).
Specific gravity	2.7	8.9
Specific resistance (microhms per centimetre cube)	2.66	1.6
Tensile strength (in kilogrammes per square centimetre)	2000	3500
Coefficient of linear expansion per degree C.	0.0000231	0.0000168

The difficulty of jointing aluminium wires was an obstacle which delayed the general use of this material as a conductor. Joints in large aluminium wires are now made by inserting the ends to be jointed into an aluminium sleeve, and a positive metal to metal union is effected by hydraulic pressure. The joint thus formed has a shearing stress in excess of that of the conductor itself.

Poles and Cross-Arms.—In districts where there is no liability to the attacks of destructive insects, etc., wooden poles are used: white Canadian cedar or Norwegian fir make the best poles, but very often from transport and other considerations local timber, where available, is requisitioned. The cross-arms for carrying the insulators are generally made of some strong and tough wood, such as oak, and are fastened to the poles by galvanised iron bolts. For ordinary construction the cross-arms have a section of about 8×8 centimetres, and vary in length from 0.75 to 2.5 metres, depending upon the number of conductors to be supported. The height of the poles ranges from 7 to 13 metres, depending upon the nature of the district traversed and the rules enforced for the regulation of supply.

To increase their durability, the poles are usually creosoted or treated with some similar preservative. In the creosoting process they are placed in iron vats from which the air can be exhausted. The vacuum allows all the sap, etc., to leave the wood, after which steam at a pressure of about 7 kilogrammes per square centimetre is applied so as to swell the wood and open its pores for the reception of the preservative. The wood is again placed in a vacuum, and the water which gained admittance during the steaming process is withdrawn. Finally, preserving compounds, such as crude petroleum oil, are applied at high pressure; this replaces the moisture formerly in the wood. Experience has shown that poles thus treated remain sound for as long as twenty years under normal conditions.

In special cases, as in India and other tropical climates, where wood poles would be quickly destroyed by ants, tubular iron poles or lattice columns are used.

Calculation of Pole Dimensions.—Poles are subject to stresses due—

First—To the weight of the conductors. This weight acts vertically downwards, tends to break the cross-arm at the centre where it is fixed to the pole, and to crush the pole as a column. The weight is liable to be augmented in winter, due to the accumulation of snow.

The length of a cross-arm is determined by the number of conductors to be carried and also the transmission voltage. The simplest construction is illustrated in Figure 117, where a cross-arm, of length $2l$, breadth b, and depth d, carries two conductors. These dimensions must be such that they satisfy the equation

$$S=\frac{6Wl}{bd^2}$$

where W is the total weight in kilogrammes resting on one insulator, S the maximum permissible fibre stress of the cross-arm in kilogrammes per square centimetre, and l, b, and d are expressed in centimetres. The permissible values of S for different materials are given in Table XIII.

Second—To unequal tension of the conductors. These are the most important stresses to be dealt with, and occur (1) where the pole line changes in direction, and (2) at the termination of one or more conductors. Under such conditions the poles are subject to a bending stress equivalent in the first case to the resultant of the tension of all the conductors, pulling the poles sideways. If the value of the tensions on both sides of the insulators and the angle of change of direction be known, the magnitude and direction of the resultant force tending to deflect the pole can be determined from the principle of the parallelogram of forces. The cross-arm of a corner pole is usually set parallel to the direction of the resultant force, so that the latter, except at terminal poles, does not produce a bending stress on the cross-arm.

In the second case of unbalanced tension the bending moment is determined from a knowledge of the algebraic sum of the tensions in all the conductors tending to pull the pole over. If W be the resultant horizontal force in kilogrammes due to the pull of the wires, then the diameter d of the pole at the ground and the height h (Figure 117), both in centimetres, must satisfy the equation

$$S = \frac{32\,Wh}{\pi d^3}$$

where S is the maximum permissible fibre stress of the pole in kilogrammes per square centimetre as given in Table XIII.

TABLE XIII

TENSILE STRENGTH OF TIMBER

Material.	Tensile Strength in Kilogrammes per Square Centimetre.
Ash (white)	700 to 1100
Birch	500 to 1500
Cedar (Canadian)	800
Deal (Christiania)	900
Elm	400 to 700
Oak	700
Pitch pine	600
Yellow pine.	350 to 850
Norwegian fir	550

Third—To wind pressure. The action of the wind against the conductors develops lateral stress, which tends to produce a dangerous vibration of the poles. Standard practice is to select poles of sufficient mechanical strength to withstand a wind pressure of from 1 to 1.5 kilogrammes per square decimetre of surface, depending upon the climatic situation of the line. In calculating the force on a cylindrical surface the exposed area is taken to be two-thirds of the product of the mean diameter and the length of the exposed part. If P denote the wind pressure in kilogrammes per square decimetre, d the diameter of pole at base, and h the height of the pole above the ground, the bending moment

$$= \frac{\frac{2}{3} \times d \times h \times P}{2} = \frac{d \times h \times P}{3}.$$

Staying.—Whenever possible all lateral strains, whether at angles or terminals, should be counteracted by stays. The strain on a stay is always greater than that in the line, and is given by the formula $P = S \cos \theta$, where S = the strain in the stay, P the line strain, and θ (Fig. 117) the angle the stay makes with the ground. Stays therefore should be of sufficient strength to withstand the strain obtained as above. The stay wires or rods are fixed, at one end, to the pole; the other end is attached to a stay anchor buried in the ground, as illustrated in the figure.

In using the foregoing formulæ it is customary to allow a factor of safety of between 4 and 10, so that the actual tensile stress in kilogrammes per square centimetre varies from 0.1 to 0.25 of the value given in the table.

Pole Construction.—Single poles, of the construction shown in Figure 117, are the more often used, but for withstanding heavy loads the construction illustrated in Figure 118 is well adapted. Two members, having the requisite strength, are set at an angle of about 10°. The feature of this form of construction, due to C. Wade,* is the method adopted to hold the members at the top. In the form illustrated the two members are scarfed to such an extent as to reduce their diameter by one-third, the length of the scarfing depending upon the angle at which the members are set. They are held together by bolts. At the base the two members are held by a stout piece of timber bolted to each, about 0.6 of a metre from the butt. A centre tie bolt and tubular distance piece are also attached to the two members about 2.5 metres above the ground level.

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* *Journal of Institution of Electrical Engineers*, vol. xxxviii. p. 312, May 1907.

The result of Wade's tests was that an "A" pole would sustain, in a direction across its two members, *i.e.* at right angles to the line, a load four or five times as great as an unstayed single pole of the same diameter as either of the members. As regards flexibility, some single poles 12 metres long projecting 10 metres above the ground have been tested and deflected from 4 to 4.5 metres before breaking, and those that were released before breaking showed a very small permanent set. On the other hand, an "A" pole would only deflect a few centimetres, and in nearly every instance failed through buckling of the member in compression.

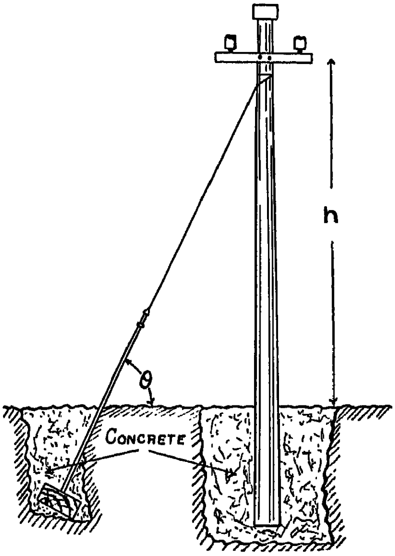


FIG. 117.—Single pole.

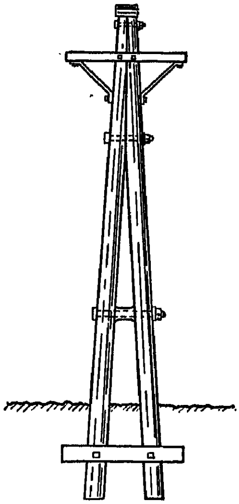


FIG. 118.—"A" pole.

The cross-arms are sometimes tied to the pole by flat iron stays; this helps to prevent twisting, and in the case of "A" poles also assists in preventing the two members from parting.

In places where the soil is soft and marshy, or where the stresses brought upon the poles by tension of the wires are excessive, the pole should be erected on a concrete foundation, with 1.5 to 2 metres of the pole butting below the ground.

Insulators for supporting Conductors.—Insulators used for supporting aërial conductors should be

(1) Made of a hard and non-hygroscopic material.

(2) Sufficiently strong to withstand (*a*) crushing due to the dead-weight of the conductors, and (*b*) fracture when subject to lateral strains due to the swaying of the conductor.

(3) So constructed that the length of the leakage paths along the surface of the insulator may be as great as possible.

Various materials, such as glass, porcelain, ebonite, and stoneware, have been used in the manufacture of insulators, but present forms are generally made only from the first two materials.

Glass is considerably more hygroscopic than porcelain, and, as is well known, has a great affinity for moisture, due to the alkali in it. These properties rapidly lower the insulating value of glass, so that glass insulators are chiefly used in dry climates. Blown glass is superior in every respect for this purpose to that which is cast, but, being more expensive, moulded insulators are generally used.

Vitrified porcelain insulators are invariably used in Britain and on the Continent, and when glazed with a non-alkaline material make very efficient insulators. Stoneware was largely in use at one time, but porcelain, as now made, has better insulating properties. The composition of porcelain used for insulators varies considerably, and consequently also its insulating properties. The material under the glaze in some forms of insulators is very spongy, so that if the glaze cracks the porcelain rapidly absorbs moisture and so destroys the insulator.

If the insulators are made of glass or thoroughly vitrified porcelain the leakage current through the material of the insulator is practically negligible so long as it is free from fractures. The main trouble in all insulators is the leakage of current over the surface. This leakage current may assume considerable proportions, but can be reduced to a minimum by suitable design. The leakage will vary directly as the length of the leakage path over the surface of the insulator, and inversely as the area of the exposed surface. The best insulators, therefore, should have the maximum surface length with the minimum area of exposed surface, so far as is consistent with obtaining the requisite mechanical strength. This is accomplished by designing insulators so that they have a series of deep grooves on their under-side.

There are two distinct types of insulators: first, those in which the entire leakage surface is of glass or porcelain; and second, those in which the leakage over the surface is intercepted by a surface of oil. Of the two, the former is the simplest in construction, and the more often used.

Insulators of the first type are of single-, double-, or triple-cup form; the double-cup insulator is the form invariably used for overhead direct-current lines. Single-cup insulators are seldom if ever used for power transmission lines; multiple-cup forms are preferred because a flaw in one cup does not destroy the whole insulator. Multiple-cup insulators are either made in one piece or a series cemented together.

Figures 119 and 120 illustrate a double- and triple-cup insulator of this type. Each insulator is carried by a galvanised-iron bolt (not shown in the figures), the interior B of the insulators being screwed with internal thread to receive the bolt. C indicates the cups forming the insulator, the outer of which is shaped as shown, so that drops of water accumulating at the rim fall away, the inside of the insulator being thus kept perfectly dry. The insulators have an exterior groove at the top, in which the conductor rests, so that the path of any leakage current from the conductor to the iron bolt is in the direction indicated by the arrows.

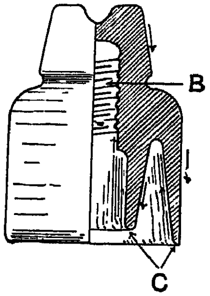


FIG. 119.—Double-cup insulator.

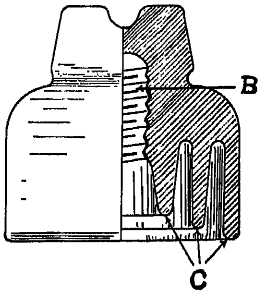


FIG. 120.—Triple-cup insulator.

An insulator of the second type is illustrated in Figure 121. It is of the single-cup form, and the lip of the cup is turned inwards so as to form an annular channel in which is placed some highly insulating non-evaporative oil. The leakage current must take the path indicated by the arrows, and has to cross the clean surface of the oil. The insulator is provided exteriorly with a top groove for carrying the conductor, and is fitted with a galvanised-iron bolt for attachment to the cross-arm. The bolts are either screwed or cemented into the insulator. The former is preferable, as the insulator can be more easily removed and replaced in case of breakage. When the bolt is cemented into the insulator it should be provided with a ragged end.

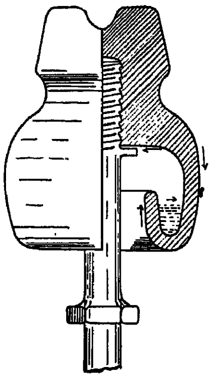


FIG. 121.—Oil insulator.

Stresses in Wires.—In erecting conductors on poles the general practice is to adjust the tension to one-fourth that of the breaking strain when the wire is shortened by the coldest weather. When the tension of a wire, suspended between two supports, is such that the sag at the centre of the span is less than one-twentieth of the length, the curve formed by the wire may be considered a parabola, and the tension of the wire in kilogrammes is given by the formula

$$T = \frac{l^2 w}{8d}$$

where l is the length of the span in metres, d the sag at the centre of the span also in metres, and w the weight of the wire in kilogrammes per metre. Solving for l

$$l = \sqrt{\frac{8aT_m}{w}}$$

where T_m represents the maximum safe tension of the wire in kilogrammes, *i.e.* the tensile strength in kilogrammes divided by the factor of safety, which is generally taken as 4. The tensile strengths of copper and aluminium are given on page 219. From the above formula it would appear that poles can be spaced farther apart the greater the tensile strength of the wire and permissible sag.

The spacing of the poles depends greatly upon the nature of the transmission line and the district through which the line is constructed. In the case of heavy power lines the poles should be placed close together, so as to distribute the weight of the conductors over a large number of insulators: this is necessary to prevent insulators being crushed. The general practice is to space the poles 24 metres apart for heavy power lines (this is the spacing on the Niagara-Buffalo line), and 30 to 36 metres for ordinary electric lighting and power lines. Wherever the line passes round a curve the poles should be spaced closer together, so that stresses due to unbalanced tension may be reduced.

When the sag is a small fraction of the length of the span the length in metres of the wire is given by

$$L = l + \frac{8d^2}{3l}$$

l and d having the same meanings as in the previous equation.

Overhead conductors should, if possible, be erected during warm weather, so that as the temperature falls the wire contracts, the tension of the wire at the same time increasing. It therefore becomes important to erect the wire with such a sag that at the minimum temperature the safe tensile strength of the wire is not exceeded.

Let t denote the temperature at which the wire is erected, L and d the corresponding length of wire and sag respectively; also let L_1 and d_1 be their values at the minimum temperature t_1. From the equation

$$T_m = \frac{l^2 w}{8d}$$

the sag d_1 in the wire at the minimum temperature can be calculated; and substituting in the equation

$$L = l + \frac{8d^2}{3l}$$

the corresponding length of wire L_1 can be obtained.

The length of wire L at the time of erection is given by

$$L = L_1\{1 + a(t - t_1)\}$$

where a is the coefficient of linear expansion of the wire. Substituting the value of L in the equation

$$L = l + \frac{8d^2}{3l}$$

the sag d at the time of erection can be calculated. Again, by substituting d in the equation

$$T_m = \frac{l^2 w}{8d}$$

the safe tensile stress may be obtained, and when erecting the wires the tension should be adjusted to this value. The effect of stretching of the wire as the tension increases is generally neglected in practical work, as the error introduced is small and always on the safe side, so that the tension at minimum temperature is even less than that calculated as above.

Example.—An aërial transmission line, in which the poles are spaced 30 metres apart, is constructed of No. 1

S.W.G. copper wire. If the minimum temperature be 5° C., and that when the wires are erected 35° C., determine the tension to which the wires can be adjusted.

No. 1 S.W.G. wire weighs 0.405 kilogrammes per metre. Tensile stress of hard-drawn copper wire = 3500 kilogrammes per square centimetre, so that, allowing for a factor of safety of 4, the safe tension of the wire at minimum temperature

$$= 3500/4 = 875 \text{ kilogrammes per square centimetre.}$$

Now No. 1 S.W.G. has a cross-sectional area of 0.456 square centimetres, so that the maximum safe tension of the wire $= T_m = 875 \times 0.456 = 400$ kilogrammes.

The sag at centre of span at 5° C.

$$= d_1 = \frac{l^2 w}{8 T_m} = \frac{30^2 \times 0.405}{8 \times 400} = 0.114 \text{ metre.}$$

Length of wire in span at 5° C.

$$= L_1 = l + \frac{8 d_1^2}{3l} = 30 + \frac{8 \times 0.114^2}{3 \times 30}$$
$$= 30 \text{ metres (approximately).}$$

Since the coefficient of linear expansion for copper $= 0.0000168$ per degree centigrade, the length of wire at 40° C.

$$= L = 30\{1 + 0.0000168(30 - 5)\}$$
$$= 30.016 \text{ metres.}$$

Substituting the value of L in the equation

$$L = l + \frac{8d^2}{3l},$$

the sag at the time of erection $= d = \sqrt{\frac{(L_1 - l)3l}{8}} =$

$$\sqrt{\frac{0.016 \times 3 \times 30}{8}} = 0.425 \text{ metre.}$$

Substituting this value in the equation $T_m = \frac{l^2 w}{8d}$, the tension of wire at time of erection has to be adjusted to

$$T_m = \frac{30^2 \times 0.405}{8 \times 0.425} = 105 \text{ kilogrammes.}$$

INSULATION RESISTANCE

Measurement of Cable Insulation Resistance by Substitution Method.—The insulation resistance of standard insulated cables ranges from 300 to 6000 megohms per mile, the 6000 megohm grade being used for high-pressure systems of distribution. Before electrical cables are dispatched from the factory their insulation resistance is determined in the following manner:

Referring to Figure 122, the length of cable A to be tested is coiled up and placed in water contained in a zinc tank D, the ends of the cable projecting about 30 centimetres above the water. The insulation at the ends of the cable is cut away so as to leave about 4 or 6 centimetres of bare copper. The end C is connected to one terminal of a galvanometer G, of which the other terminal is connected to the 2-way switch E through a battery or source of E.M.F. B. The contacts *a* and *b* of the switch E are respectively connected to the tank D, and a standard high resistance R, which is in turn connected to the galvanometer as shown.

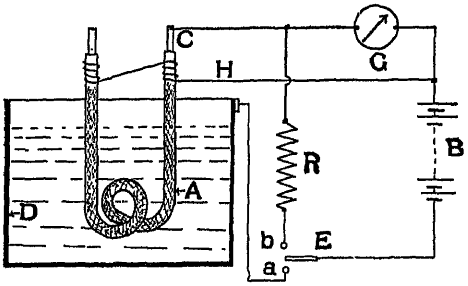


FIG. 122.—Testing the insulation resistance of a cable.

Having made the connections, the insulation resistance of the cable may be determined as follows:

The resistance R is connected in series with the source of E.M.F. B and the galvanometer G by turning the switch K to contact *b*. The deflection d_1 of the latter is then observed. The switch is then turned to *a*, thus substituting the cable insulation for the resistance R. The corresponding deflection d_2 of the galvanometer is then observed. Now the deflections d_1 and d_2 are respectively inversely proportional to the resistance R and the insulation resistance of the cable, so that if R be in megohms

$$\text{Insulation resistance of the cable} = \mathrm{R} \times \frac{d_1}{d_2} \text{ megohms.}$$

The cable insulation varies inversely as its length, so that, knowing the length of cable tested the insulation resistance can be expressed in megohms per mile.

It is found that the value of the insulation resistance varies with the testing E.M.F., the period of electrification, and the period of immersion in water. Hence the standard test is to immerse the cable in water for 24 hours, and to measure the resistance after one minute's electrification at an E.M.F. of 600 volts.

In order to eliminate errors due to surface leakage at the cable ends, 3 or 4 turns of bare copper wire H are wound round the insulation about 4 centimetres away from the bare ends, and connected to the battery side of the galvanometer as shown. Any leakage current which may then be set up passes from the external surface of the insulation to the battery by way of the guard wire H instead of going through the galvanometer.

Measurement of Insulation Resistance by an Ohmmeter.—For testing the insulation resistance of electric-light wiring, street cables, and electrical machinery an instrument is necessary which, by the deflection of a pointer over a graduated scale, directly indicates the insulation or other resistance connected to its terminals. Several types of instruments have been devised for this purpose, but the one designed by Sidney Evershed is probably the most widely used.

The principle and construction of Evershed's ohmmeter, or "Megger," as it is sometimes called, is illustrated in Figures 123 and 124. It consists of a moving-coil instrument combined in one box with a hand-driven direct-current generator to provide current for the test. The armature G of the generator, wound for a pressure of from 100 to 1000 volts, is centred in bearings between the poles N_1 and S_1 of the permanent magnets MM, and coupled through gearing to the driving handle.

The magnets MM are magnetised to a strength of about 2000 lines per square centimetre, and to those ends remote from N_1 and S_1 are fitted the poles N and S. Between the latter is fixed a specially shaped iron core C. The magnetic flux passes from N to S by way of C, and thereby sets up a uniform magnetic field in the air-gaps under the poles. Pivoted on the same spindle, between N and S, are two coils, A and B, fixed at an angle of about 70 degrees with each other. The coil A is wound so as to embrace the core C, while B extends from the centre to one side of C, as shown. To the spindle supporting the coils is attached the pointer P, which moves over a scale graduated from zero to infinity.

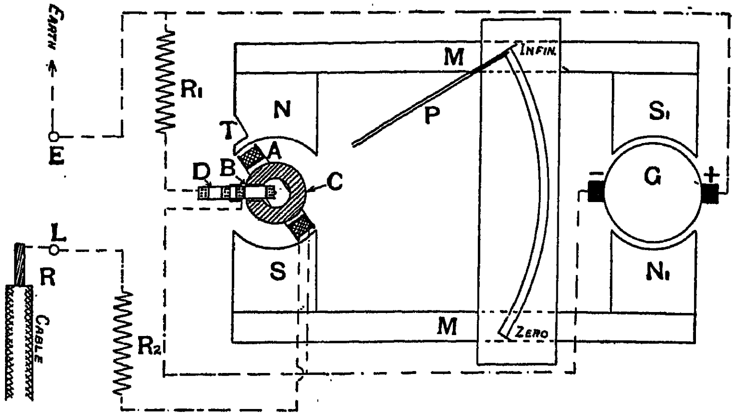


FIG. 123.—Evershed's ohmmeter.

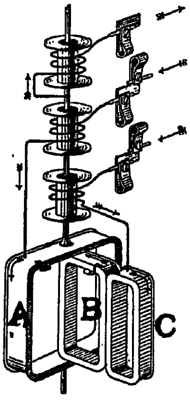


FIG. 124.—Moving system of Evershed's ohmmeter.

The coil B, known as the "pressure coil," is connected in series with a high resistance R_1 across the generator terminals. The "current coil" A is connected, in series with the resistance R_2, between the terminals L and the negative brush of the generator, the positive of the latter being connected to the terminal E.

The resistance R to be measured is connected between the terminals E and L, and in testing the insulation resistance between the conductor of a cable and earth, terminals L and E are respectively connected to the conductor and to earth,—a water pipe forms a good connection with earth.

When the resistance R is "infinite" no current flows in the series coil, and the pressure coil will take up the position shown in Figure 123, the pointer P at the same time indicating "infinity." When a current does flow through the series coil, the latter will have an opposing turning moment, so that the moving system rotates clockwise. The angle made with the infinity position is a function of the resistance in circuit with the series coil and independent of the generator voltage. The instrument is made dead-beat by winding the coil A on an aluminium frame.

In order to protect the instrument from the influence of stray magnetic fields, a compensating coil D is fixed to the outside of the pressure coil. The coil D is connected in series with the former, and wound in the opposite direction. The coils B and D therefore form an *astatic* electro-magnet which will be uninfluenced by stray magnetic fields. The pole piece N is at one tip cut away to form a tooth T, and when the moving system is in a position corresponding to zero insulation resistance the compensating coil lies over T.

It is essential that there be no controlling force, otherwise the law of the instrument will be altered, and in order to effect this phosphor-bronze wire of .01 millimetre diameter is used for leading the current into and out of the moving coils. Such fine wire is very springy, and to prevent any danger of its becoming entangled with the instrument in transport, it is wound round the comparatively large aluminium drums shown in Figure 124, the latter indicating the arrangement of the moving system.

Instruments working on the above principles are constructed to measure resistances up to 2000 megohms.

CHAPTER VIII

THE DYNAMO—ARMATURE, COMMUTATOR, AND BRUSH GEAR

PRINCIPLE OF THE DYNAMO

THE construction of the dynamo is based upon the principle of electro-magnetic induction discovered by Faraday. This principle affirms that, when a conductor is moved so as to cut magnetic lines of force, an electromotive force is induced in the conductor, which, if it form part of a closed circuit, will cause a current to flow. The direction in which the current will flow will depend upon the direction of motion of the conductor, and the value of the induced E.M.F. will be proportional to the rate at which the conductor cuts the lines of force.

In a commercial machine the electrical conductors are fixed around the periphery of an iron drum, and so connected that when the drum is centred in bearings between the poles of an electro-magnet and mechanically rotated, an E.M.F. is induced in the conductors as they cut the lines of force, and if the conductors form part of a closed circuit a current flows. A dynamo is thus a transformer of energy. The rotary part carrying the conductors is called the *armature*, and the stationary electro-magnet the *field magnet*.

The simplest mode of generating an E.M.F. by electro-magnetic induction is illustrated in Figure 125. The conductor AB is placed in a magnetic field between the poles N and S, and is connected to the external circuit R. If AB be moved in an upward direction, an E.M.F. will be induced in it and current will flow through the external circuit in the direction indicated by the arrow; but when the direction of motion is reversed, the current will flow through R in the opposite direction.

Next consider a rectangular loop of wire revolving in a magnetic field about an axis perpendicular to the lines of force. The arrangement is shown in Figure 126, where the simple armature consists of two conductors A and B supported on the spindle D, and centred between the poles N and S of an electro-magnet. The ends of the loop are electrically connected to two insulated metal slip rings C_1 and C_2, and a circuit is completed through the external circuit R by means of metal contacts or brushes bearing on the

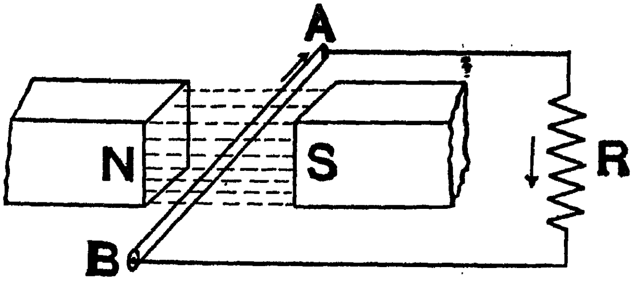


FIG. 125.

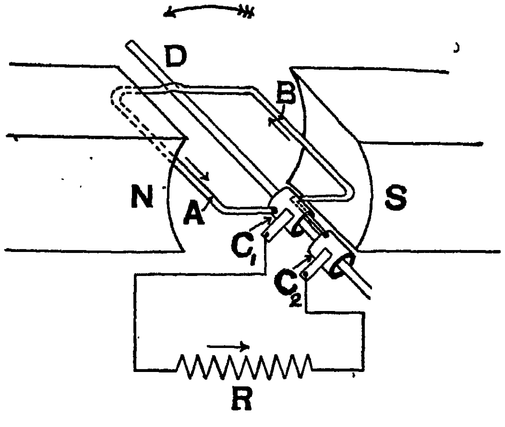


FIG. 126.

rings. When the armature is rotated uniformly in the direction indicated, the sides A and B of the loop have E.M.Fs. induced in them as they cut through the magnetic field.

It is of importance to remember that the E.M.F. is only induced in that portion of the conductor which lies parallel to the axis of rotation and is under the influence of the magnetic field. Any portion of an armature conductor which is the seat of an induced E.M.F., will hereafter be referred to as an *inductor*.

Referring to Figure 126, at the instant the loop AB is in a vertical plane, the inductors A and B are moving parallel to the lines of force (*i.e.* no lines of force are cut), so that the value of the induced E.M.F. will be zero. At the instant the loop has been rotated through 90 degrees, A and B will be passing under the centres [illegible] the N and S poles respectively, and since they cut the lines of force at right angles, the maximum E.M.F. will be induced in each inductor. Now the inductors are moving across the field in opposite directions (*i.e.* inductor B moves up while A moves down), so that the induced E.M.Fs. on each side of the loop will be in opposite directions, as shown by the arrows alongside of the inductors; but A and B are so connected that the E.M.Fs. are cumulative, therefore the E.M.F. across the slip rings will be twice that set up in each inductor, and a current flows through the circuit R from C_1 to C_2. Continuing the rotation, by the time the loop has moved through 180 degrees, the E.M.F. will again be zero, and at the instant the inductors A and B are passing under the centres of the S and N poles respectively, the induced E.M.F. will again be a maximum, but its direction in each inductor will have reversed, so that the current flows through the external circuit R from C_2 to C_1, *i.e.* the direction of the current leaving the armature is also reversed.

During each complete revolution the current reverses in direction twice, and if the inductors rotate with a uniform velocity V in a uniform magnetic field of strength M, the value of the induced E.M.F. at any instant will be equal to the product of the field strength

M, the number of inductors C in series, and the vertical component of the velocity at that instant. The latter is equal to V Sin θ, where θ is the angle through which the coil has rotated from the vertical plane. The electromotive force induced in a coil at any instant will therefore be given by the expression :

$$e = \text{M.C.V Sin } \theta \quad \text{C.G.S. units.}$$

In this equation, when Sin $\theta = 1$, the value of e is a maximum and equal to MCV, so that denoting this value by E_{max}, the instantaneous value of the induced E.M.F. is given by $e = E_{max}$ Sin θ. That is the curve connecting E.M.F., and angular displacement of the loop is a simple sine curve, as shown in Figure 127. The change in the direction of the E.M.F. which occurs when the inductors begin to cut the lines of force in the opposite direction, is shown by the curve crossing the zero line. The current maintained by such an E.M.F. is known as an alternating current, and the arrangement shown for producing it as a simple *alternating current generator* or *alternator*.

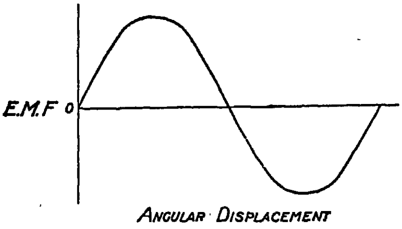
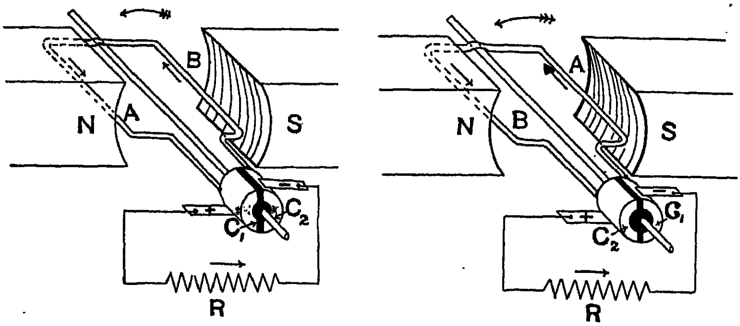


FIG. 127.

The Commutator.—Next suppose the slip rings to be removed and the ends of the loop AB connected one to each half C_1 and C_2 of a split metal ring as shown in Figure 128, the two halves of the ring being insulated from each other. Such an arrangement constitutes a simple *commutator* having two *segments* C_1 and C_2. If the brushes are so placed that at the instant the induced E.M.F. is zero they slide across from one segment to the other, the current, while reversed in the loop, flows through the external circuit in the same direction. That this is the case will be clear from Figures 128 and 129. In Figure 128 the inductors A and B are under an N and S pole respectively, and the current flows from segment C_1 by way of the positive brush, external circuit R, and negative brush to the segment C_2, where it enters the loop and completes its circuit through inductors B and A, as shown by the arrows alongside the latter. When the loop has been rotated so that the inductors A and B are under an S and N pole respectively, the direction of the induced E.M.F. is as shown in Figure 129.



FIGS. 128, 129.—Principle of commutator.

The current now flows through A and B in the reverse direction, but as the segments C_1 and C_2 have also moved through 180 degrees, the current will flow from C_2 to C_1 through the external circuit in the same direction as before. The current in the external circuit is therefore uni-directional, although its numerical value fluctuates from zero to a maximum as the loop moves from a position midway between the poles to a position under the centre of the poles.

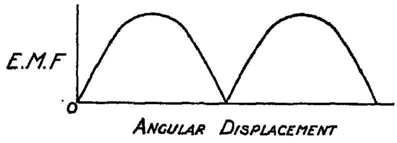


FIG. 130.

The variation of the E.M.F. across the brushes with the angular displacement of the loop will be as shown in Figure 130. The arrangement of inductors shown in Figure 129 sends a *uni-directional* or *direct current* through the external circuit, and is therefore known as a *direct current generator*, or simply a *dynamo*.

The method by which the E.M.F. generated by a dynamo is increased, and the fluctuations eliminated, will now be explained. In a ring armature the winding consists of a number of convolutions of wire on an iron ring mounted on a shaft. Figure 131 illustrates the principle of an armature of this type. The coils A and B are wound on opposite sides of the iron ring C and connected in parallel at the two commutator segments. The lines of force emanating from the N pole, cross the air-gaps, pass through the iron ring, and enter the S pole in the manner indicated in the figure.

As the armature rotates only those parts of a coil which lie along the outer surface of the core cut the lines of force, so that the sides of the coils passing along the inner surface of the ring are inactive, serving only to connect the inductor portions of the coils in series. Suppose the armature to be rotating in the direction indicated, then an E.M.F. will be induced in each coil, causing a current to flow through the external circuit R in the direction shown by the arrow-heads. Both coils have the same number of convolutions, and being connected in parallel, each coil carries one-half of the total current entering the external circuit: that this is so will be apparent by tracing out the path taken by the current from the negative to the positive brush.

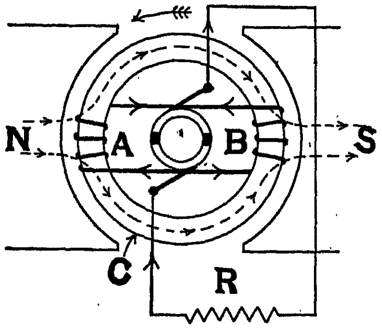


FIG. 131.—Two-coil ring armature.

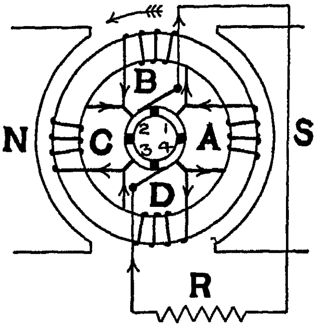


FIG. 132.—Four-coil ring armature.

A ring armature wound with four coils A, B, C, and D, each placed 90 degrees apart and connected to four commutator segments, is shown in Figure 132. Between the negative and positive brushes there are two circuits as before, but each circuit consists of two coils connected in series. On entering the armature at the negative brush the current divides, one-half flowing through coil A to segment No. 1, and thence to the positive brush by way of coil B and segment No. 2, where it is rejoined by the other half of the current flowing through the coils D and C.

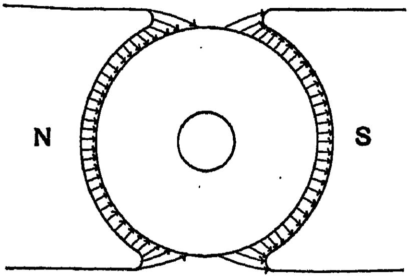


FIG. 133.

In Figure 129 the lines of force pass horizontally from pole face to pole face, with the result that the E.M.F. induced in the inductors is a sine function. By the insertion of an iron core (Figures 131 and 132), the distribution of the magnetic flux as cut by the inductors is considerably altered. Referring to Figure 133, the lines of force now pass through the air-gap and enter or leave the core much more radially, and are distributed almost uniformly over the arc embraced by the pole face, the result being that the curve of instantaneous E.M.F. is considerably altered from that of a sine wave. The actual form of the E.M.F. wave is shown by the curves A and B (Figure 134). The curves have a flat top caused by the inductors cutting the uniform magnetic field under the pole face, and the existence of a weak field or fringe at the edge of the pole face causes the curve to pass gradually from a maximum value to zero.

The instantaneous values of the total E.M.F. in a circuit consisting of two or more coils in series will be equal to the sum of the instantaneous E.M.Fs. induced in each coil. Taking the circuit formed by the coils A and B (Figure 132), the curves A and B in Figure 134 represent the variation in value of the E.M.F. in the two coils respectively. It will be observed that at the instant one E.M.F. is zero the other is at a maximum, the coils being displaced relative to each other by 90 degrees, On adding the ordinates of the two curves together a third curve C is obtained, which shows the variation in E.M.F. across the brushes with the angular displacement of the armature. From this curve it will be seen that (1) the maximum E.M.F. of the two coils in series is greater than that of a single coil, and (2) the percentage fluctuations of the combined E.M.Fs. are very much less than those of each coil when acting separately. If the fluctuation on either side of the mean value in the case of a single coil be taken as 100 per cent., then the fluctuations due to the two coils in series are reduced to about 30 per cent.

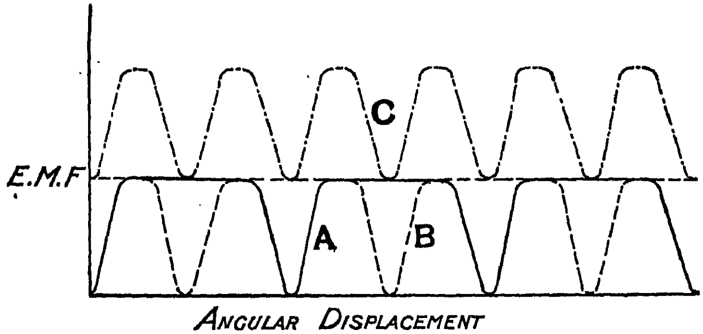


FIG. 134.

The fluctuations of the E.M.F. across the brushes can be still further reduced by winding the armature with more coils and providing a correspondingly increased number of commutator segments. An armature having about 30 coils in series per circuit generates an E.M.F. which is practically free from fluctuations. The current set up by such an E.M.F. is sometimes referred to as a *continuous current.*

The ring armature was first constructed by Gramme, and in the early days of electrical engineering was the only type employed, but having now become obsolete its construction need not be dealt with in further detail here. The armatures of modern dynamos have their windings placed in slots in the periphery of a cylindrical iron core or drum, and are therefore known as *drum armatures.* The principles of construction discussed in the next portion of this chapter refer only to armatures of this type.

THE ARMATURE

Construction of Core.—As the iron core carrying the windings rotates, it becomes the seat of induced E.M.Fs., and Foucalt or eddy currents are set up. These currents, according to Lenz's law, flow at right angles to the direction of the lines of force and the planes of the direction of rotation. In Figure 135, if A be the armature core supporting the inductors, then as the armature revolves between the poles N and S, the eddy currents induced in the core will take the paths indicated by the arrows. Although the induced E.M.F. may be low, yet, owing to the low resistance of the numerous metallic paths, these eddy currents may assume large proportions, with the result that the core becomes heated and energy wasted.

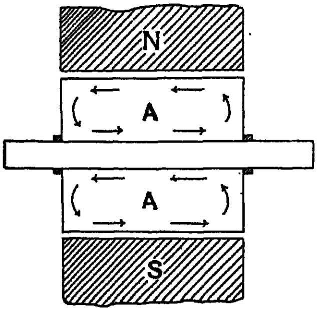


FIG. 135.—Eddy currents in armature core.

So long as the iron core rotates it will be impossible to prevent the induction of E.M.Fs. in it, but the resulting eddy currents can be prevented from attaining any appreciable strength by interposing a high resistance in their paths. This is effected by constructing the core of thin discs of iron, each of which is more or less isolated from its neighbour by a thin layer of insulating material, such as paper or varnish. A core built up in this manner is known as a *laminated core*, and the planes of lamination are at right angles to the axis of rotation and to the inductors.

Although the core be laminated as described, small E.M.Fs. will still be induced in each section, but they will be prevented by the intervening insulation from acting together, so that the eddy currents are reduced. The thinner the laminations the less the quantity of energy wasted.

The material used for the core of armatures is soft annealed wrought iron or mild steel. The discs are stamped out of sheets the thickness of which ranges between 0·5 and 0·6 millimetre. Up to 1 metre in diameter the discs are cut out in complete rings, each forming one stamping; for larger armatures segments are employed. The thickness of the insulation between the laminations is about 0.05 millimetre, so that, neglecting ventilating ducts, the net length of iron in the core is about 90 per cent. of the gross length.

In armatures of small diameter the sheet-iron stampings are usually threaded directly on the shaft and held in position by a feather running along the length of the core, as shown in Figure 136. At each end of the core are cast-iron end flanges F_1 and F_2: the flange F_1 is first driven on the shaft and held in position against a collar C_1. The iron stampings A are then assembled, and after compression the second flange F_2 is fitted on to the shaft and held in position by the screwed collar C_2, which is prevented from moving by the lock-nut N. Frequently sufficient cross-section of iron is obtained without utilising the full radial depth of the core, and in such cases the laminations are pierced with holes H so as to provide ventilating tunnels for the passage of air through the interior of the armature.

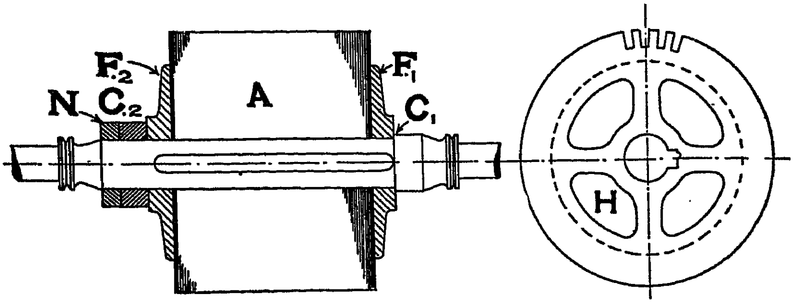


FIG. 136.—Core for small armatures.

Figure 137 shows the construction of core suitable for armatures of machines up to 200 k.w. The laminations are of sufficient radial depth to give the desired magnetic induction, and are mounted on a cast-iron spider A, which in turn is keyed to the shaft B. Keyways C are stamped out of the inner periphery of the laminations to fit the arms D which project from the boss of the spider. The latter abuts against a collar E on the shaft and has an end flange F_1, either cast solid or bolted to it. The laminations after being assembled are compressed by the second end flange F_2, which is bolted to the spider, thus making the core entirely self-contained. To the end flanges are cast rings G, which serve to support the end connections of the armature winding.

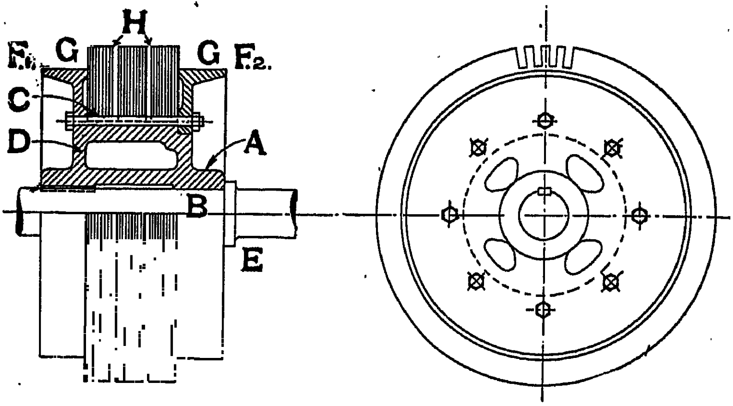


FIG. 137.—Armature core for 100–200 k.w. machines.

In armatures of still larger diameter it has become the practice, as a matter of convenience, to build the laminations in segments, and the latter are provided with wedge-shaped lugs so that they can be dovetailed to the arms of the spider, thus preventing them from moving radially, due to the action of centrifugal force. The construction of an armature core of this type is shown in Figure 138, where the laminations A are mounted on the arms of the spider B. The segments in any one layer are built up with butt joints, and so arranged that the joints of neighbouring layers do not coincide. The segments are clamped between the end flanges F_1 and F_2 by bolts C passing through holes near the inner edge. The end flanges are extended to form a support for the

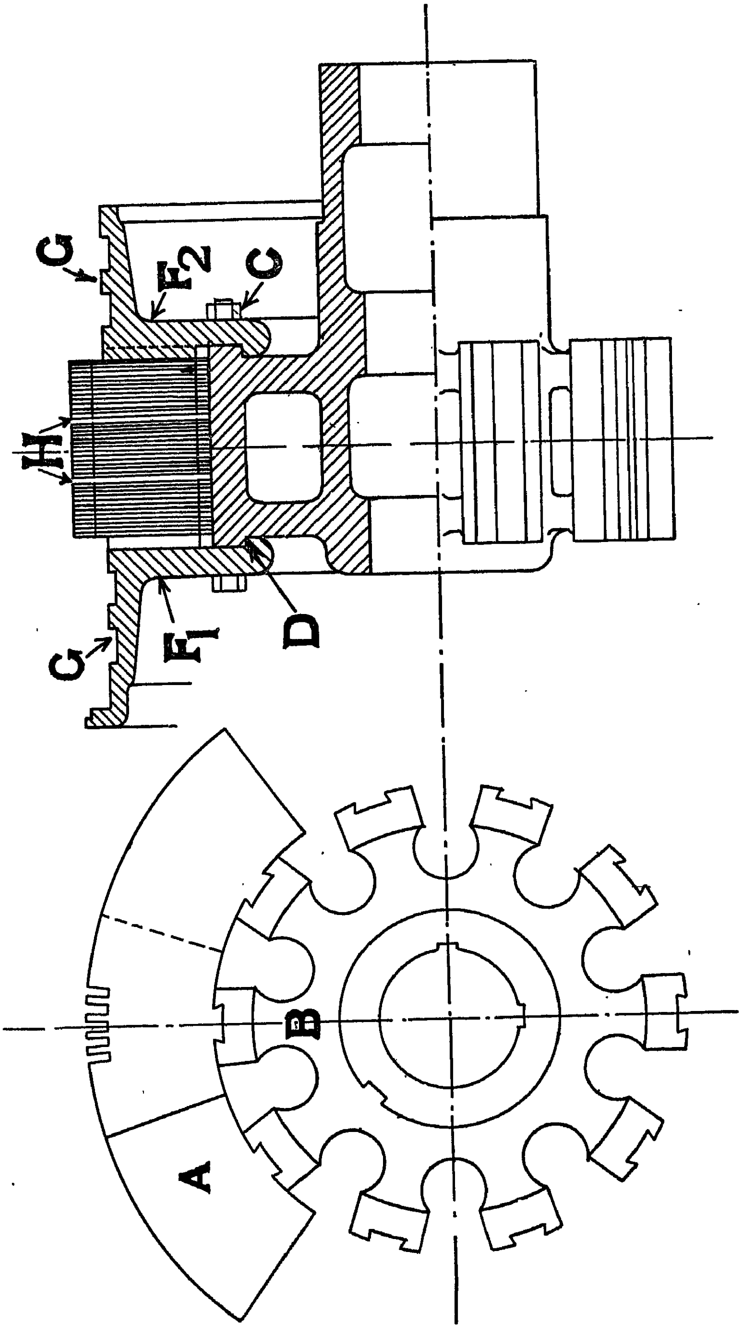


FIG. 138.—Armature core for 200–500 k.w. machines.

end connections of the winding, as shown at G, and are in some cases made in several pieces, each being secured against centrifugal force by engaging beneath a ring on the armature spider as indicated at D.

Figure 139 shows another form of spider suitable for traction generators of from 700 to 1500 k.w.

In order to provide sufficient ventilation, armatures require to be designed with channels or tunnels to permit of the free passage of air into and through the core. For this purpose the laminations are separated at intervals with distance pieces, so as to form radial air-ducts from the interior to the exterior of the armature core, as shown at H in Figures 137 and 138. In order that the distance pieces may obstruct as little as possible the circulation of the air, they should be made of as skeleton a construction as is consistent with the necessary mechanical strength. The width of the ducts ranges from 10 to 15 millimetres, and the distance between them from 4 to 7 centimetres, depending upon the degree of ventilation required. Figures 140 and 141 illustrate two types of ventilating distance pieces.

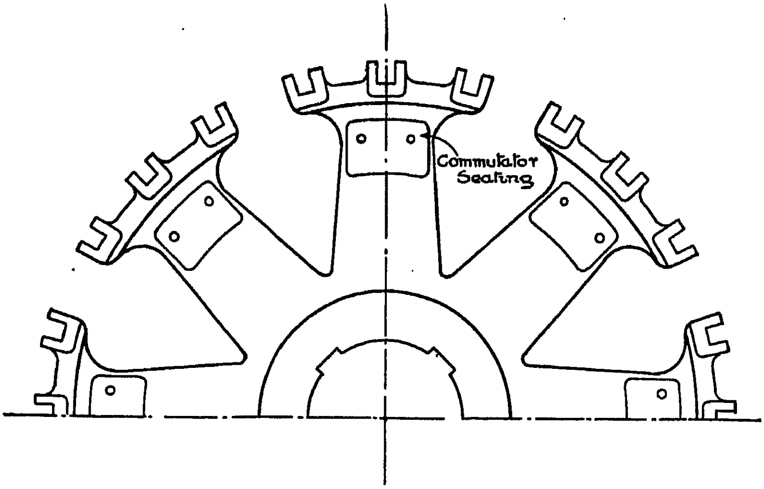


FIG. 139.—Armature spider for 700–1400 k.w. generators.

In Figure 140 the distance piece consists of small metal projections A fitted into slots punched in the plate

B and riveted over at the back of the plate, thus rendering them quite firm. The plate B usually consists of two or three laminations riveted together. In the type shown in Figure 141 one lamination is used for the space block, and small pieces A of the latter are nicked out and turned up at right angles. The first type illustrates the best construction, as it gives support right up to the top of alternate teeth, and so prevents the latter from falling over and obstructing the passage of air coming from the interior of the armature.

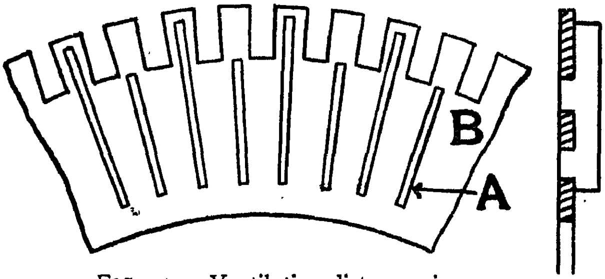


FIG. 140.—Ventilating distance piece.

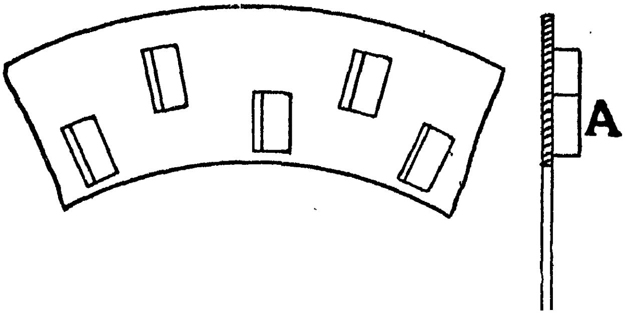


FIG. 141.—Ventilating distance piece.

It is desirable that all end flanges, supporting spiders, etc., should as far as possible be kept outside the influence of the magnetic field, otherwise they may become the seat of eddy currents: also all bolts passing through the body of the armature laminations must be either so placed that they cut as few lines of force as possible, or be insulated from the core by fibre tubes, so as to obstruct the paths that eddy currents generated in them would follow.

Pull on Inductors.—Of the total power expended in driving a dynamo, from 80 to 96 per cent. is absorbed in overcoming the magnetic pull which the inductors experience as they move through the magnetic field. The driving power is transmitted from the shaft through the spider and core to the inductors, where the actual conversion of the mechanical energy supplied to the dynamo into electrical energy takes place.

The force acting on one inductor

$= B \times i$ dynes per centimetre of length, where—

$B =$ induction density of the field in which the inductor is placed

$i =$ the current in C.G.S. units flowing in the inductor. If the inductor be l centimetres in length and carry a current of C ampères, then the force acting on each inductor and opposing the direction of rotation of the armature is expressed by

$$F = B \cdot \frac{C}{10} \cdot l \cdot \frac{1}{981} \text{ grammes}$$

$$= \frac{B \cdot C \cdot l}{981 \times 10^4} \text{ kilogrammes.}$$

For example, suppose an inductor having a length of 40 centimetres and carrying a current of 100 ampères is rotated in a magnetic field having an induction density of 8000 lines per square centimetre. The force or pull on the inductor

$$= \frac{8000 \times 100 \times 40}{981 \times 10^4} = 3.2 \text{ kilogrammes.}$$

It will be clear from this example that the inductors of an armature must be supported on the core in such a manner that they are effectively driven through the magnetic field without any possibility of their slipping round the surface of the core. This is, in most cases, effected by the use of slotted armatures, as originally suggested by Pacinotti. The laminations of such armatures have a number of projecting teeth stamped out round the periphery, so that when assembled the surface of the armature has a series of grooves or slots running from end to end, as shown in Figures 136 to 138. The inductors are well protected with insulation and tightly fitted into the slots, one slot holding two or more inductors.

In some armatures the inductors are placed around the periphery of a smooth core, as shown in Figure 142. In such constructions shallow grooves are milled out longitudinally along the core B, and into these hard wood or fibre strips C are driven, the tops of which are flush with the inductors. These strips or *drivers* thus oppose any slip of the inductors relative to the core, and are placed at intervals of from 8 to 10 centimetres along the circumference. Their width should be sufficient to withstand the pull of the inductors acting on any one strip, but at the same time they should not be too wide, otherwise the symmetry of spacing of the inductors is interfered with unnecessarily.

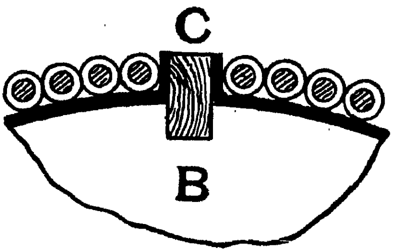


FIG. 142.—Driving strip for smooth core armature.

E.M.F. Equation for a Dynamo.—As already stated, the electromotive force induced in an armature inductor is directly proportional to the rate of cutting lines of force. In Chapter I. the absolute unit of pressure was defined as that generated in an inductor cutting one line of force per second. From this definition the E.M.F generated in the armature of a dynamo can be expressed in terms of the armature winding, armature speed, number of poles, and the flux per pole cut by the armature inductors. Consider a two-pole dynamo: let M denote the flux per pole entering the armature, then the E.M.F. generated in each inductor when the armature makes one revolution per second $= 2$ M, C.G.S. units, the constant 2 being introduced since each inductor cuts the total flux twice in one revolution. Further, let R denote the speed of the armature in revolutions per minute (R.P.M.) and C_1 the number of inductors in series per circuit, then the E.M.F. generated in each circuit $= 2\ \mathrm{M} \times \frac{\mathrm{R}}{60} \times \mathrm{C}_1$ C.G.S. units.

Since two inductors constitute one armature turn, $\mathrm{C}_1 = 2\ \mathrm{T}$, where T is the number of turns in series per circuit; so that for a two-pole dynamo the E.M.F. in volts is expressed by

$$\mathrm{E} = 2\ \mathrm{M} \cdot \frac{\mathrm{R}}{60} \cdot 2\ \mathrm{T} \cdot 10^{-8}.$$

This formula holds good only for 2-pole machines: but if p denote the number of pairs of field poles, the voltage of any direct-current dynamo is expressed by

$$\mathrm{E} = 2\ \mathrm{M} \cdot \frac{\mathrm{R}}{60} \cdot p \cdot 2\ \mathrm{T} \cdot 10^{-8}.$$

$\frac{\mathrm{R}}{60} \cdot p$ may conveniently be considered as one term, since it represents the number of magnetic cycles through which the armature iron passes per second. The letter N is used to denote the quantity $\frac{\mathrm{R} \cdot p}{60}$. The general equation for obtaining the voltage of dynamos is

$$\mathrm{E}\ (\text{volts}) = 4\ \mathrm{T}\ \mathrm{N}\ \mathrm{M}\ 10^{-8}.$$

This formula, if correctly interpreted, is applicable to any type of dynamo. It is important to remember that *T is the number of turns in series per circuit*, and M is *not* the flux generated in one pole, but that which, after deducting leakage, finally becomes linked with the armature inductors.

Example.—The armature of an 8-pole dynamo is required to generate 550 volts when revolving at a speed of 240 R.P.M. Calculate the magnetic flux M per pole passing through the armature if the latter has 272 slots and 6 inductors per slot. The armature winding has 8 circuits in parallel.

The total number of armature inductors $= 272 \times 6 = 1632$.

Total number of armature turns $= \frac{1632}{2} = 816.$

The number of turns per circuit $(T) = \frac{816}{8} = 102$.

The number of magnetic cycles per second $(N) =$

$$\frac{R \cdot p}{60} = \frac{240 \times 4}{60} = 16.$$

From the E.M.F. equation

$$M = \frac{E \times 10^8}{4 \times T \times N} = \frac{550 \times 10^8}{4 \times 102 \times 16} = 8{,}400{,}000 \text{ lines}$$

$= 8.4$ megalines, where 1 megaline equals 10^6 lines.

Windings.—The number of inductors having been determined by the preceding formula, it remains to consider the different methods of connecting the ends of the inductors of an armature so as to form a symmetrical closed coil winding. According to the method of inter-connecting the inductors windings may be divided into

1. Two-circuit or wave-winding.
2. Multiple-circuit or lap-winding.

Wave-windings are employed in machines of low output, and are also well adapted for high-speed machines of intermediate output. A characteristic of this type of winding is that there are only two circuits through the armature from positive to negative brushes, independent of the number of pairs of poles or sets of brushes. Each of the two circuits supplies one-half of the total current output.

Multiple-circuit windings are adopted for machines of large output, and are also preferable for slow-speed machines of intermediate output. This type of winding differs from the wave-winding, in that there are as many circuits from positive to negative brushes as the machine has poles: each circuit being in parallel, the current will divide equally between them. Thus in a 6-pole machine giving 600 ampères with a multiple-circuit winding, each circuit will carry 100 ampères.

The inductors of an armature may be considered as arranged in a number of groups or *elements* distributed at equal distances round the periphery of the core, as shown by the numbered circles in Figures 143 and 144, each circle representing one element. A *coil* or *winding unit* is formed by connecting two suitable elements together, and will herein be referred to as that part of the winding which terminates in two commutator segments. In practice an armature coil consists of one to six turns, but so far as the foregoing diagrams are concerned it is immaterial as to how many turns there are in a coil. Each coil will therefore for simplicity be considered as having one turn only, and each side of the coil considered as a single inductor.

The *pitch* or *spacing* of an armature winding is the term which denotes the distance between two separate elements connected together. It is usual to express the pitch of a winding in terms of the number of elements or inductors passed over. For instance, in Figure 143 the inductors are numbered consecutively round the armature, and at the rear end of the winding inductor No. 1 is joined to inductor No. 6, thus forming a loop; the pitch of the winding is therefore 5. At the front or commutator end inductor No. 6 is joined to inductor No. 11, so that the pitch is again 5. Sometimes the front and rear pitches differ by an even number, in which case if y_F and y_R denote the front and rear pitches respectively, the mean pitch is expressed by $y = \frac{y_F + y_R}{2}$.

In the case of drum windings all connections from inductor to inductor must be made at the front and rear ends exclusively. It therefore follows that the inductors forming the sides of any one turn must be situated in fields of opposite polarity, so that the electromotive forces generated in the inductors constituting a turn shall act in the same direction round the turn. The width of a coil or winding element should therefore be approximately equal to the polar pitch, *i.e.* the distance between the centres of two consecutive poles. Both front and rear pitches will be approximately equal to $\frac{C}{2p}$, where C is the total number of inductors and $2p$ the number of poles: but the exact values of y_F and y_R must be such that the winding closes on itself, and that starting from any one inductor and tracing through the successive inductors in the order of the winding, each inductor should be traversed once, and once only, before the original inductor is reached. Such a winding is said to be *re-entrant.*

Two-Circuit or Wave-Winding.—Let C denote the number of inductors forming a winding, and $2p$ the number of poles, then the mean pitch is given by

$$y=\frac{C\pm 2}{2p}.$$

Owing to the mechanical arrangement of the end connections both pitches must be odd numbers, so that if y, determined by the above equation, be also an odd number, the front and rear pitches are both made equal to the mean pitch, *i.e.* $y=y_F=y_R$. On the other hand, should y be an even number, the front pitch requires to be less or greater than the mean pitch by 1, according as the rear pitch is greater or less than the mean pitch by 1.

$$\text{i.e. } y_F=y\pm 1$$
$$\text{and } y_R=y\mp 1.$$

As a demonstration of the first case the winding diagram of a 4-pole armature having 22 inductors will be examined. Figure 143 shows the position of the poles and indicates how the 22 inductors are arranged around the armature. To make clear the method of interconnecting the inductors, the winding is represented as being laid out flat, and the eleven commutator segments indicated by the letters *a* to *k*.

The mean pitch of the winding $(y)=\frac{22\pm 2}{4}=5$ or 6.

Taking $y=5$, then $y_F=y_R=5$.

Starting from commutator segment *a*, connection is made to the front end of inductor No. 1, and the rear end of the same inductor is connected to an inductor 5 in advance, *i.e.* to No. 6. Since the front pitch is also 5, the commutator end of inductors Nos. 6 and 11 are connected together at segment *f*. Similarly the winding is completed as follows:

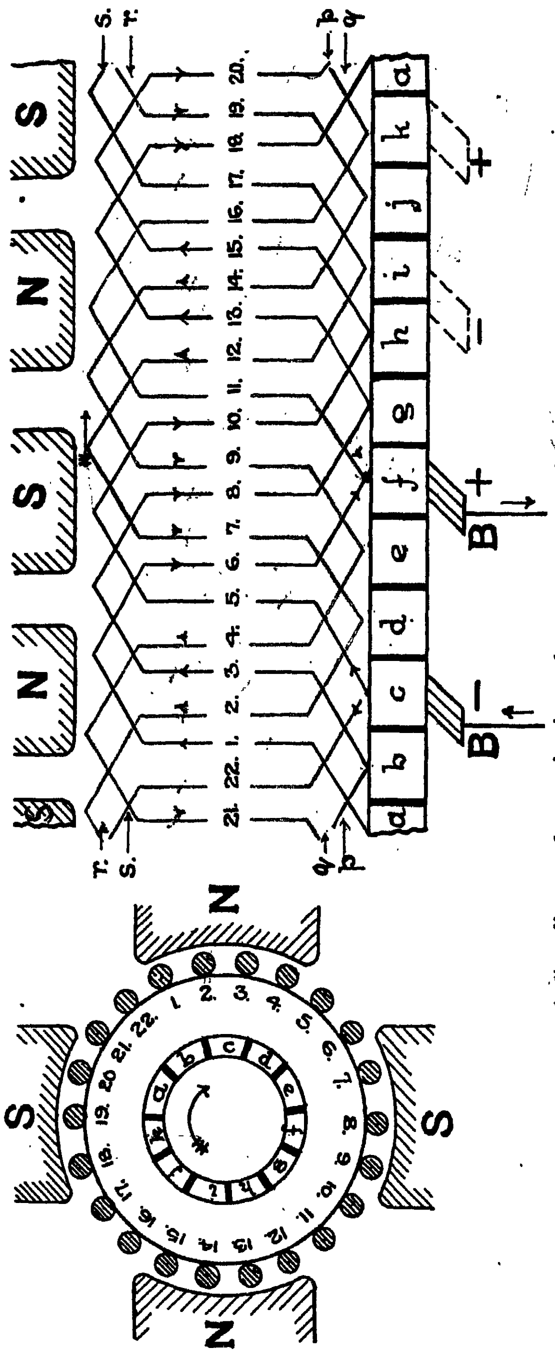


FIG. 143.—Winding diagram for a 2-circuit 4-pole armature, 22 indicators. $y_F = 5$, $y_R = 5$.

Rear end.				*Front end.*			
11	connected to	16	and	16	connected to	21	at segment *k*
21	"	4	"	4	"	9	" *e*
9	"	14	"	14	"	19	" *j*
19	"	2	"	2	"	7	" *d*
7	"	12	"	12	"	17	" *i*
17	"	22	"	22	"	5	" *c*
5	"	10	"	10	"	15	" *h*
15	"	20	"	20	"	3	" *b*
3	"	8	"	8	"	13	" *g*
13	"	18	"	18	"	1	" *a*

The winding thus forms a closed circuit. By tracing any two consecutive winding units it will be observed that they have a wavy or zigzag appearance, hence the name *wave-winding*.

The direction of the induced E.M.F. in each inductor as it passes through its respective field is indicated by the arrows. The current in the inductors under an N pole flows from front to rear, and in the opposite direction when under an S pole.

The positions on the commutator where it is necessary to place collecting brushes can be ascertained from the winding diagram. A positive brush should be placed at any segment of the commutator where two arrow-heads meet; and a negative brush should be placed at those segments where two arrow-heads go in opposite directions through the winding. Thus in Figure 143 the positive and negative brushes B+ and B− are placed 90 degrees apart, and make contact with segments *f* and *c* respectively.

By tracing through the winding from the negative to the positive brush, it will be found that the two paths through the armature are along the inductors and in the following order:

$$B-\left\{\begin{array}{lcccccccccccccc}\text{(Black circuit)} & . & 5 & 10 & 15 & 20 & 3 & 8 & 13 & 18 & 1 & 6 & & \\ \text{(Red circuit)} & . & 22 & 17 & 12 & 7 & 2 & 19 & 14 & 9 & 4 & 21 & 16 & 11\end{array}\right\}B+$$

With wave-wound armatures only two sets of brushes are necessary so far as the collection of the current is concerned; but in order to reduce the size of the commutator it is usual in machines greater than 15 k.w. to use as many sets of brushes as there are poles.

The fact that two sets of brushes are sufficient for a wave-winding will be evident from Figure 143. Suppose there be four sets of brushes arranged as shown. The positive brushes, being in contact with segments *f* and *k*, are electrically connected through the inductors 11 and 16. Now these two inductors are situated midway between the poles, so that no E.M.F. is induced in them; consequently the above-mentioned inductors simply form a cross-connection between the two positive brushes. Similarly, the two negative brushes are cross-connected by inductors 17 and 22. From this it will be evident that the extra set of brushes in no way alter the distribution of current in the winding, their only function being to increase the brush-contact area with the commutator, and thus assist the first set in collecting current from and delivering current to the commutator segments. When only one set of brushes are used the angle between them will be the same as the angle between any N pole and any S pole. Thus in a 6-pole machine with a wave-winding the positive and negative brushes may be either 60 degrees or 180 degrees apart.

Multiple-Circuit or Lap-Winding.—In a lap-winding the front and rear pitches must both be odd and differ by 2, so that the mean pitch (y) is always an even number, and is made equal to $\frac{C}{2p}$, where C and $2p$ represent the same values as in the formula used for wave-windings. The front and the rear pitches respectively are given by:

$$y_F = y \pm 1$$
$$y_R = y \mp 1.$$

Since the mean pitch $y \left(= \frac{C}{2p}\right)$ requires to be an even number, the inductors C must be so chosen that this condition is fulfilled. The front and rear pitches are in opposite directions, so that if the latter be considered positive the former is considered negative.

Figure 144 shows the winding development of a 4-pole lap-wound armature having 24 inductors and 12 commutator segments lettered from *a* to *l*.

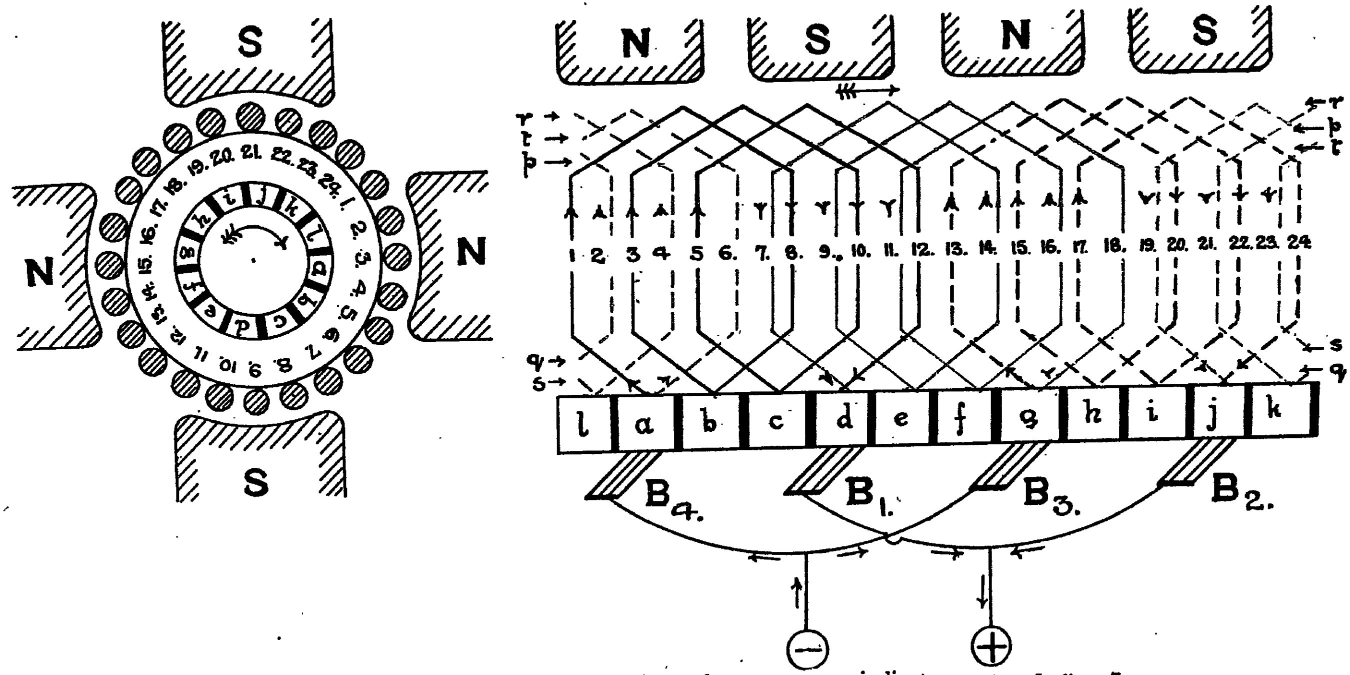


FIG. 144.—Winding diagram for a 4-circuit 4-pole armature, 24 indicators, $y_F = 5$, $y_R = 7$.

The mean pitch $= y = \frac{C}{n} = \frac{24}{4} = 6$, so that $y_F = y - 1 = 5\ (-)$, and $y_R = y + 1 = 7\ (+)$.

Inductor No. 1 is connected at the front to commutator segment *a*, and at the rear to inductor No. 8. Since the front pitch is negative, *i.e.* in the opposite direction to the rear pitch, and equal to 5, inductor No. 8 is connected to No. 3 at segment *b*. Similarly, inductor No. 3 is connected to No. 10 at the rear, and No. 10 connected to No. 5 at commutator segment *c*. The rest of the winding is merely a repetition of the steps described, and after passing over all the inductors the winding re-enters itself at *a*. It will be observed that in tracing any two consecutive winding units the second unit laps over the first; hence on account of this overlapping the winding is called a *lap winding*.

The arrow-head on each inductor indicates the direction of the induced E.M.F., and by adopting the same rule as before, positive brushes B_1 and B_2 require to be placed at segments *d* and *j*, while negative brushes B_3 and B_4 are placed at segments *g* and *a*. Brushes of similar polarity are connected together, and to one or other of the terminals marked + and —.

By tracing through the winding from the negative to the positive terminals, the four paths through the armature will be along the inductors and in the following order:

(−)	B_4	black full line	1	8	3	10	5	12	B_1	(+)
		red full line	6	23	4	21	2	19		
	B_3	black dotted line	13	20	15	22	17	24	B_2	
		red dotted line	18	11	16	9	14	7		

Four sets of brushes, *i.e.* two positives and two negatives, are necessary, because a lap-winding has as many circuits as poles. In a lap-wound armature the angle between each set of brushes is the same as the angle between two consecutive poles. For instance, in the example under consideration there are 4 poles, and the brushes are set 90 degrees apart.

So far as relates to the winding diagrams in Figures 143 and 144, it has been assumed that each winding unit consists of two inductors, *i.e.* constitutes one armature turn. Winding units having 2, 3, or 4 turns are frequently employed for machines generating high voltages, and it will be readily seen that, using the same commutator, the number of inductors can be doubled (and therefore also the E.M.F. of the dynamo doubled) by substituting for each 1-turn winding unit a unit consisting of two turns, *i.e.* four inductors. In Figure 145 is shown the development of a 3-turn winding unit (*i.e.* a winding having three turns per commutator segment) for a wave-winding. In Figure 146 is shown a similar development for a lap-winding having 3 turns per winding unit.

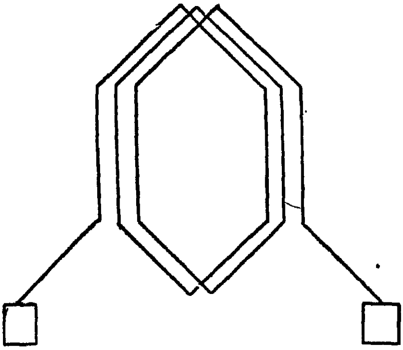


FIG. 145.—Three-turn wave-winding unit.

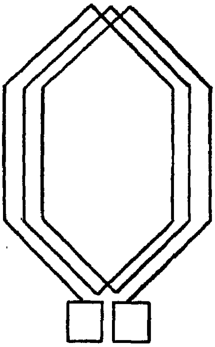


FIG. 146.—Three-turn lap-winding unit.

The method of developing such a winding would be similar to those already described, but instead of assigning numbers to individual inductors, numbers would be assigned to the group of inductors forming the sides of the winding units.

Examples of Drum-Windings.—Figures 147 and 148 show typical drum-winding diagrams. In these figures the armature inductors are represented by short radial lines, and the connecting lines inside represent the end connections at the commutator end, and those on the outside the connections at the rear end. The commutator is placed in the centre, and for the sake of convenience the brushes are drawn inside the commutator. Such diagrams form the most convenient method for the study of drum-windings.

Figure 147 shows a 6-pole wave-winding with 62 inductors. In this example $y = \frac{62-2}{6} = 10$. Since y is even

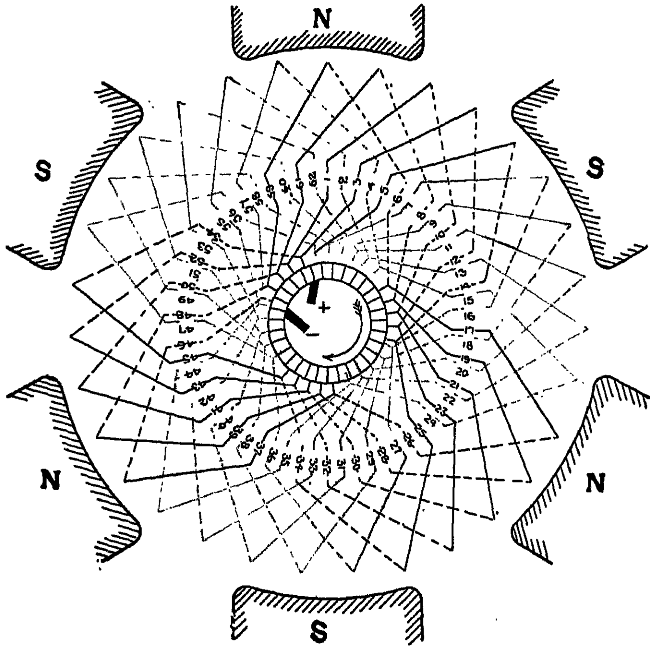


FIG. 147.—Winding diagram for 2-circuit 6-pole armature, 62 inductors. $y_F = 11$, $y_R = 9$.

$$y_F = y + 1 = 11 \text{ and}$$
$$y_R = y - 1 = 9.$$

The winding diagram shows two paths in parallel between the brushes.

Figure 148 is an example of a 6-pole lap-winding having 60 inductors. The mean pitch $(y) = \frac{60}{6} = 10$.

so that $y_F = 10 + 1 = 11$, and $y_R = 10 - 1 = 9$. There are six paths in parallel between the brushes, of which there are therefore six sets.

Multiplex Windings.—Armature windings, so far discussed, have had a single or simplex winding, but they may also be wound with two or more independent windings. Figure 149 shows a 6-pole lap-wound armature having 60 inductors and two independent windings. Each winding has 30 inductors, and the armature might have been provided with two commutators, say one at each end of the armature. Instead of this, the number of commutator segments is doubled, and the two sets of inductors intercalated with one another.

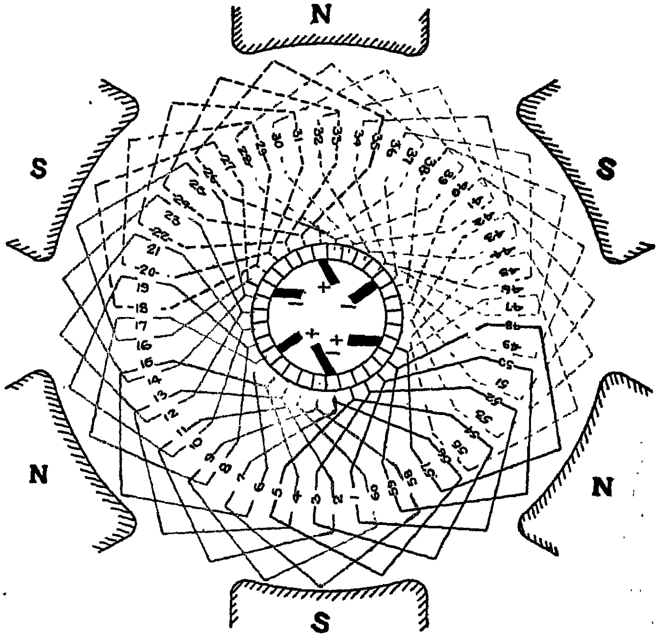


FIG. 148.—Winding diagram for 6-circuit 6-pole armature, 60 inductors. $y_F = 11, y_R = 9$.

Referring to the figure, odd-numbered commutator segments belong to one winding, while even-numbered segments belong to the second or red winding. With such a winding the brushes must be of sufficient width

5 commutator segments, so as to
he two windings simultaneously.
own as a *duplex winding*, and if
would be four circuits in parallel
es. The 6-pole duplex lap-winding

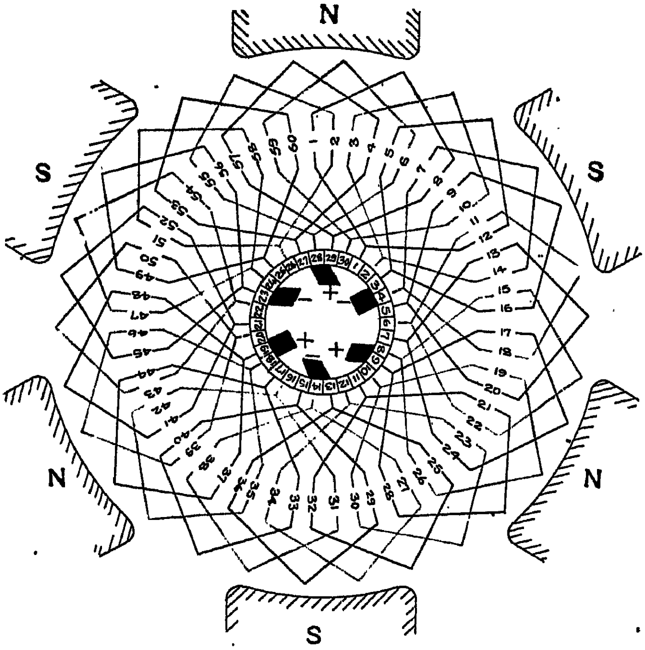


FIG. 149.—Winding diagram for a 6-pole duplex lap-winding, 60 inductors.

shown in the figure has 12 circuits in parallel from negative to positive brushes.

A *triplex winding* consists of three independent windings with three sets of commutator segments similarly intercalated, the three windings being put in parallel by using brushes of sufficient width to bridge at least 3.5 segments.

Multiplex windings are extensively used on the Continent, but are only used to a limited extent in Britain and America. These windings are of an advantage where large currents are to be collected at the commutator, because, as will be explained later when discussing commutation, they materially conduce to effecting this operation sparklessly. The formulæ for such windings is necessarily more intricate than that for simplex windings, and their treatment is beyond the intended scope of this book.*

✓*Equalising Connections.*—With lap-wound armatures of large output it becomes practically impossible to avoid small differences of pressure between the various circuits that are in parallel, with the result that the current does not divide equally between them. Local currents are thus caused to flow through certain parts of the winding and the conductors which connect brushes of similar polarity. As a result of these local currents the armature becomes excessively heated and serious sparking ensues. Small differences of pressure between the various circuits may be produced by inaccurate centring of the armature or unequal quality of iron in each pole, with the result that the flux entering or leaving certain poles may be less than at others. When the sections of the armature winding pass these poles, they will not generate equal E M.Fs., with the result that abnormal currents flow in the other sections.

It is now customary in lap-wound armatures of large output to use *equalising connections.* These are of low resistance, and form cross-connections between those parts of the armature circuits which should be at the same potential, so that any local currents due to the above-mentioned differences of pressure are prevented from flowing through the brushes and causing sparking.

The equalising connections are of copper, and generally made in the form of rings fixed on insulated supports to the core of the armature. To be effective, eight or ten independent rings are necessary, and each is in connection with the armature winding, at points spaced out at distances apart equal to twice the polar pitch.

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* Those wishing further information on this subject are referred to the following books:

Armature Construction, by Hobart and Ellis (Whittaker & Co.); *Design of Dynamos*, by S. P. Thompson (E. & F. N. Spon).

In Figure 150 is shown the development of a 6-pole lap-wound armature having 36 inductors, and for simplicity only three equalising rings are shown, the latter, and their connections to the armature winding, being indicated by red lines. Ring No. 1 is connected to the three points A spaced 120 degrees apart, *i.e.* twice the polar pitch. Each part of the winding embraced between the 120 degrees is divided into three equal parts, as indicated by B and C. The three points marked B should be at the same potential, and are connected to ring No. 2. Similarly the other points marked C are connected to ring No. 3. In practice each part of the winding embraced between the points A A would be divided into eight or ten equal parts, and the points spaced equi-distantly apart would be connected to eight or ten rings in a similar manner as indicated above.

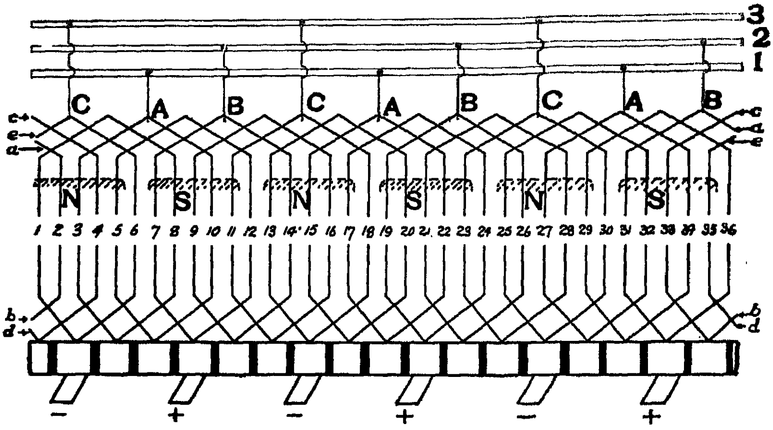


FIG. 150.—Six-pole lap-winding showing connections to equalising rings, 36 inductors.

In the figure, connection is made to the equalising rings at the rear end of the winding, but the more usual practice is to make connection, between the equalising rings and the winding, at commutator segments.

In a wave-winding the inductors belonging to any one circuit are distributed under all the poles, so that equal electromotive forces are induced in each armature circuit, and there is no appreciable tendency to produce unequal distribution of current in the winding.

Arrangement of the Inductors in Slots.—In the preceding winding diagrams it is more convenient to represent the inductors as being spaced equi-distantly around the periphery of the core. In the actual application of such diagrams the winding is generally arranged in two or four layers, with two, four, or more pairs of inductors in each slot, the usual convention being that odd numbers represent upper and even numbers lower inductors. Thus if the inductors in the winding diagram in Figure 144 be arranged four in a slot, the actual location of each inductor will be as shown in Figure 151. The best mechanical design in the arrangement of the end connections is secured by always connecting an upper to a lower inductor. Such an arrangement makes it necessary that the front and rear pitches, in any class of drum-winding, should be odd numbers. In the latter figure, inductor No. 1 placed in the top of the first slot is connected at the rear to inductor No. 8 in the bottom of the second slot, which in turn is connected at the front to inductor No. 3 in the top of the first slot.

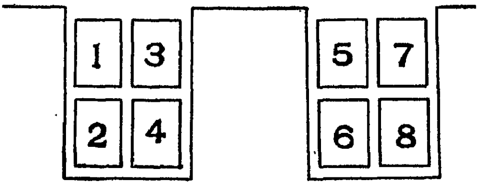


FIG. 151.—Conventional method of numbering armature inductors.

Cross-Section of, and Method of insulating Inductors.—Inductors and end connections should be of high-conductivity annealed copper, prepared electrolytically. Where the sectional area of each inductor does not exceed 0.15 square centimetre, solid wire of circular cross-section may be used. Wire of larger sectional area is difficult to bend, so that an armature turn requires to be made of two or more loops connected in parallel, the sum of the areas of cross-section giving the requisite equivalent sectional area of a single inductor. By this arrangement a large amount of slot space is wasted by interstices between individual wires, so that in order to reduce the size of slots to a minimum the general practice is to use inductors of rectangular cross-section when the area of any one inductor exceeds 0.3 square centimetre.

Wires of circular cross-section are insulated with a *double cotton covering* when used for low-voltage windings, but for pressures greater than about 230 volts the insulation is in the form of a *braided cotton covering.* The thickness of insulation ranges from 0.2 to 0.3 millimetre, depending upon the voltage. With inductors of rectangular cross-section the insulation takes the form of a single or double cotton covering with an external covering of braided cotton, the total thickness varying from 0.4 to 0.6 millimetre. In order to enhance the insulation all insulated conductors used for armature windings are varnished or soaked in shellac and then baked in a hot chamber.

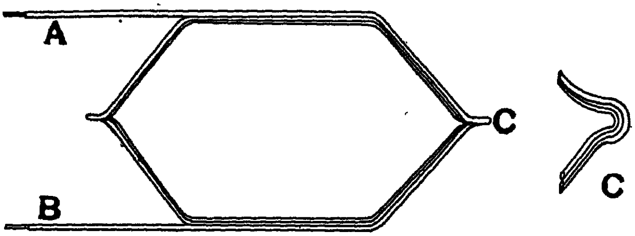


FIG. 152.—Former-wound coil of three turns.

Former-wound Drum Armatures. — When the inductors of an armature consist of wire of circular cross-section *former-wound coils* are invariably used. These have been adopted for—(1) small machines where a number of winding units can be conveniently grouped together; and (2) high voltage machines in which a winding unit consists of several turns. The wires forming such a coil are wound together on a *former* of special shape, and each coil is efficiently insulated before being placed on the armature core. The coils are perfectly symmetrical and interchangeable, and are specially suited for slotted armatures.

The shapes into which the coils are formed are various. Coils of the shape shown in Figure 152 are more frequently used, and windings composed of such coils are technically known as *barrel windings.* The coils are wound on a former shaped to the exact dimensions required: the wire at the top of one side leads at the bend C on to the wire at the bottom of the other, thus allowing the ends A and B to project from the tops of each side. For a lap-wound armature both of the ends project from the centre of the widest portion of the coil, while in a wave-winding (Figure 153) the ends are led away in opposite directions for connection to the respective commutator segments.

If there be more than one winding unit per slot, two or more may be wound together. The coil in Figure 153 consists of three winding units. After being formed the coil is wrapped round with special varnished tape,

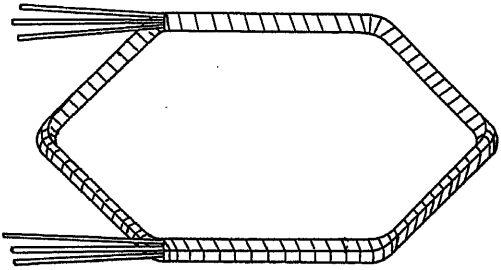


FIG. 153.—Former-wound coil consisting of three winding units.

soaked in shellac and finally baked. Figure 154 shows two former-wound coils *in situ.*

Bar-wound Drum Armatures.—In this style of winding, a winding unit is invariably a complete loop consisting of two inductors and end connections of rectangular cross-section. Figure 155 shows the shape of such a loop used for a lap-winding. The two sides A and B are bent in different planes, so that the side B can be placed in the bottom of a slot and the side A in the top of another. After shaping, each loop should be well taped, especially at the bend C, and the whole treated with shellac varnish.

Where the inductors are of large sectional area each loop is composed of two separate bars, A and B (Figure 156). After each part is placed in position copper clips C are passed at the rear, over the bare ends E. The electrical contact is then completed by soldering.

Insulation of Slots.—The core slots are lined with insulating material before placing the inductors in position. The slot lining requires to be relatively thick, compared with the insulation round each inductor. This is because the core must be insulated from the full voltage of the machine, while the voltage between any one inductor and its neighbour is only a small fraction of the full voltage.

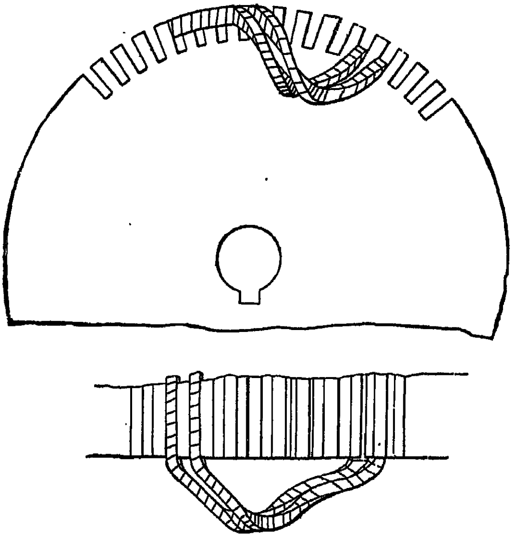
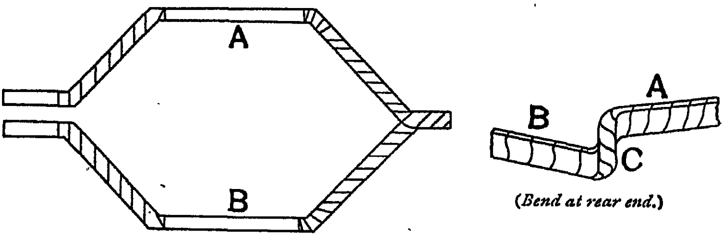


FIG. 154.—Former-wound coils on core.



(*Bend at rear end.*)

FIG. 155.—Loop for bar-wound armature.

Press-spahn and micanite are the insulators generally used, and the former requires to be specially treated to render it moisture-proof. These materials can be obtained in almost any thickness, but it is preferable to use two or three layers of thin material in making up the slot lining to the required thickness. Thus suppose a slot requires to be lined with press-spahn to a thickness of 0.75 millimetre, then three layers, each 0.25 millimetre thick, would be used, as it is more flexible and has a somewhat higher disruptive strength than single sheets of equivalent thickness. Figures 157 and 158 illustrate two methods, according to the cross-section of the inductors, of insulating the slots.

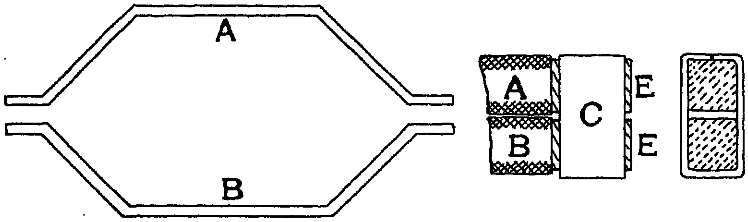


FIG. 156.—Loop for bar-wound armature.

The sectional area of copper only forms a portion of the total sectional area of a slot, and the ratio of the former to the latter is known as the *space factor*. The higher the voltage generated the lower the space factor, and in present-day machines it ranges from 0.5 or 0.4 in 550-volt generators to 0.6 or 0.7 in 230-volt and 100-volt generators.

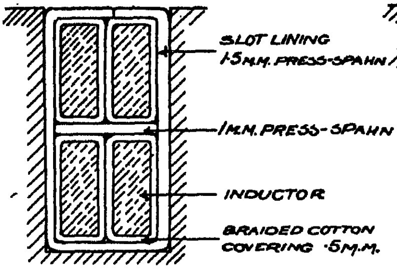


FIG. 157.—Slot insulation for 500 volts.

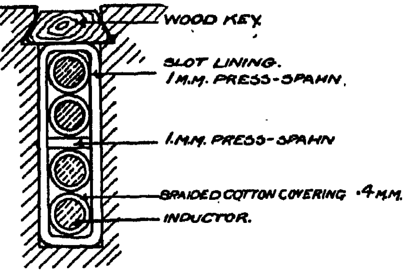

WOOD KEY
SLOT LINING. 1M.M. PRESS-SPAHN.
1.M.M. PRESS-SPAHN
BRAIDED COTTON COVERING ·4M.M.
INDUCTOR.

FIG. 158.—Slot insulation for 250 volts.

Methods of Retaining the Inductors against Centri-

fugal Force: (1) *Low and Medium Speed Machines.*—When the winding of an armature has been completed several bands of *binding wire* require to be wound circumferentially round it to hold the inductors securely in place. The tendency for them to fly out of the slots, due to the action of centrifugal force, is thus counteracted. Non-magnetic steel wire of Nos. 18 to 22 S.W.G. and well tinned, is the material generally used for binding. Referring to Figure 159, a band M of mica strip is first fastened round the armature, and over this the binding wire B is wound under considerable tension to form a belt varying in width from 1 to 3 centimetres. Bands wider than this are liable to become heated, due to the eddy currents induced in them. Along the circumference of the band, and at intervals of about 20 centimetres, small strips C of sheet copper about 1.5 centimetres wide are introduced. The ends of the strips, after the band has been wound, are turned over so as to form clips, as shown in the figure. The convolutions of the band thus formed are finally soldered together throughout their entire length. Such bands are placed at intervals of about 7 centimetres along the length of the armature.

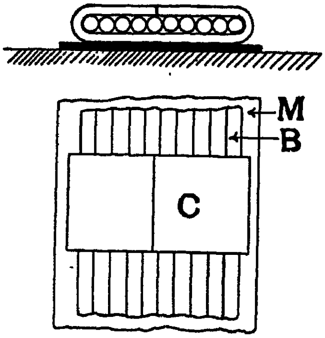


FIG. 159.—Binding wire band.

The end connections of drum armatures are supported on rings or projections cast with the end flanges as shown at G in Figures 137 and 138, and the end bands firmly compress the end connections on to the supporting rings. The latter should be covered with one or two layers of press-spahn held on with tape, thus protecting the insulation on the conductors from mechanical injury.

At places where strips are used the binding wire adds 1.5 to 2.0 millimetres to the radius of the armature, and the mechanical clearance between the armature and the pole face must be reckoned from the overall radius. In certain constructions, during the building of the core batches of laminations of slightly smaller diameter are interposed at intervals along the core length; these form hollow circumferential grooves of sufficient depth to allow of the binding wire and clips being finished flush.

In armatures of over 1.5 metres diameter the eddy currents which would be induced in the bands necessary to secure the winding would produce excessive heating. In such cases the inductors are held in place by driving beech-wood keys into grooves stamped in the tops of the slots, as shown in Figure 158. This arrangement has the disadvantage that the slots, for the same number and size of inductors, require to be deeper, and the emission of heat from the inductors is obstructed. Except for very large machines, the use of wooden keys should be avoided; but when used they must be thoroughly dried, and made waterproof by suitable treatment.

(2) *High-Speed Machines.*—The construction of armatures for high speeds, such as occur in turbine-driven dynamos, presents certain mechanical difficulties owing to the high stresses set up by the centrifugal forces of the rotating parts. To retain the inductors in position against the action of centrifugal force the best mechanical design would be obtained by placing them in completely closed slots. This, however, is impossible, owing to the increased inductance of the winding being very conducive to sparking. The majority of high-speed dynamos have semi-enclosed slots, as shown in Figure 160, the inductors being held in position by beech-wood keys K.

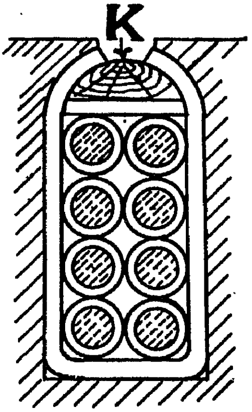


FIG. 160.—Semi-enclosed slot for high-speed armature.

In some cases, where the speed is not excessive, open slots of the type shown in Figure 158 are sufficient. Where the inductors are held in place by wood-keys, the latter communicate the centrifugal force of the inductors to the teeth, which therefore must be of sufficient mechanical strength to withstand the consequent stresses.

An armature tooth is subjected to a radial stress due to (1) its own centrifugal force, (2) the centrifugal force of the inductors in any one slot, and (3) the magnetic pull on the tooth.

The centrifugal force of the inductors in any one slot is expressed by

$$F_1 = 0.0000112\ M_1\ R_1\ N^2 \text{ kilogrammes}$$

where

M_1 = Weight, in kilogrammes, of inductors lying in a slot.

R_1 = Mean radius of gyration of mass M_1.

N = Speed of armature in revolutions per minute.

Similarly, the centrifugal force of a tooth of mass M_2 kilogrammes, and revolving at N revolutions per minute with a mean radius of gyration of R_2 centimetres, is expressed by

$$F_2 = 0.0000112\ M_2\ R_2\ N^2 \text{ kilogrammes.}$$

If B denote the pole face density in lines per square centimetre, and A the area (in square centimetres) of iron per tooth at armature surface, then the magnetic pull on a tooth is expressed by

$$F_3 = \frac{B^2A}{8\pi \times 981 \times 1000} \text{ kilogrammes}$$

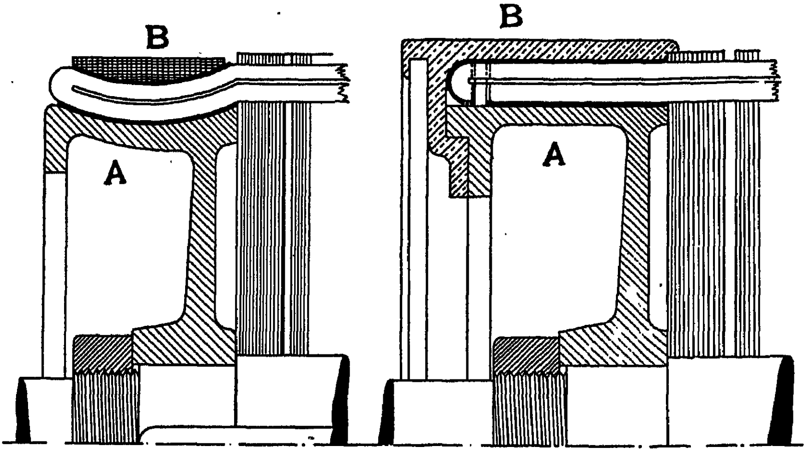
$$\text{or } F_3 \approx B^2A \times 4 \times 10^{-8} \text{ kilogrammes.}$$

The total radial stress on a tooth therefore $= F = F_1 + F_2 + F_3$ kilogrammes. This force F has to be resisted in tension at the root of the tooth. If a denote the width of a tooth at root, and l_n the net length of iron parallel to the armature shaft, then the stress at the tooth root is expressed by

$$S = \frac{F}{a \times l_n} \text{ kilogrammes per square centimetre.}$$

The value of S should not exceed 600, this being the average working value for wrought iron in tension, after allowing for a factor of safety of 5.

The end connections of high-speed armatures must also be specially supported against the action of centrifugal force: the two more prevalent methods are shown in Figures 161 and 162. In Figure 161 the end connections are supported on a special shaped end flange A, which is curved on its surface. The surface of the support is insulated with micanite, press-spahn, and tape: the end connections are firmly compressed on it by an end band B of steel wire wound over a thin mica ring, which prevents the binding wire cutting through the insulation of the end connections. The maximum stress on the binding wire occurs at the centre, and the curved surface of the support enables the thickness of the steel band to be greatest at this part, so as to counteract the increased strain.



FIGS. 161, 162.—Methods of supporting the armature end connections of turbine-driven dynamos.

The second method of supporting the end connections is shown in Figure 162, where they are kept in position by a cylindrical metal end cover B, which is bolted to the supporting flange A. The end connections are insulated from the latter and the end cover by means of micanite and press-spahn. The end covers are either of phosphor-bronze, manganese-bronze, or nickel steel.

The thickness of the end cover B ranges from 2 to 4 centimetres.

If F_1 and F_2 denote the centrifugal forces due to the end connections and covers respectively, then the total internal load acting on one half of the ring $= \frac{F_1+F_2}{2}$. By a well-known law of mechanics, the force tending to burst the covers at two sections on the same diameter

$$= \frac{F_1+F_2}{2} \times \frac{\text{mean diameter}}{\text{mean semi-circumference}}$$
$$= \frac{F_1+F_2}{2} \times \frac{2}{\pi} = \frac{F_1+F_2}{\pi}.$$

If a denote the area of cross-section of the cover and S the stress set up in it, then the moment of resistance $= S \times 2a$. Since the latter is equal to $\frac{F_1+F_2}{\pi}$, the value of S is expressed by

$$S = \frac{F_1+F_2}{2\,\pi\,a}.$$

The thickness of the cover should be such that the value of S does not exceed the safe working stress of the metal. For phosphor-bronze and manganese-bronze the safe working stress is usually taken at about 650 kilogrammes per square centimetre; this allows for a factor of safety of 5.

Example.—From the following data of a 33-k.w. dynamo calculate the number of inductors required for an armature having a wave-winding.

Number of poles = 4.
Voltage at terminals = 550 volts.
Full load current = 60 ampères.
Magnetic flux in armature per pole at no load = 3.45 megalines.
Armature speed = 660 R.P.M.

$$N = \frac{Rp}{60} = \frac{660 \times 2}{60} = 22.$$

The turns per circuit are given by

$$T = \frac{E \times 10^8}{4 \,.\, N \,.\, M} = \frac{550 \times 10^8}{4 \times 22 \times 3.45 \times 10^6} = 180.5.$$

Inductors per circuit $= 180.5 \times 2 = 361$.

The armature is wave-wound, so that there will be two circuits in parallel, and the total number of armature inductors = 361 × 2 = 722.

In this particular armature there were 121 slots; 117 containing 6 inductors, and 4 containing 5 inductors, *i.e.* 722 = (117 × 6) + (4 × 5).

In the four slots containing 5 inductors there would be a vacant space. This could be filled up with wood strips of the same section as the inductors; but the usual practice is to fill the slots with the full number of inductors, and, when making connection to the commutator segments, to join two loops in parallel. The loops thus joined only form two inductors, so far as the winding is concerned. The loop in which the sides are only used to fill up the slots is known as a *dummy coil.*

In the example under consideration the commutator had 361 segments, so that there is one armature turn per winding unit. The mean pitch $= \frac{722 + 2}{4} = 181$, and since y is odd, $y_F = y_R = 181$.

Figure 163 indicates how the inductors are arranged in the slots. Since $y_R = 181$, inductor No. 1, in the top layer of the first slot, is connected at the rear to inductor No. 182 placed in the bottom of the forty-first slot. Inductor No. 182 is joined at the commutator end to inductor No. 363 (*i.e.* 182 + 181) in the top of the eighty-first slot.

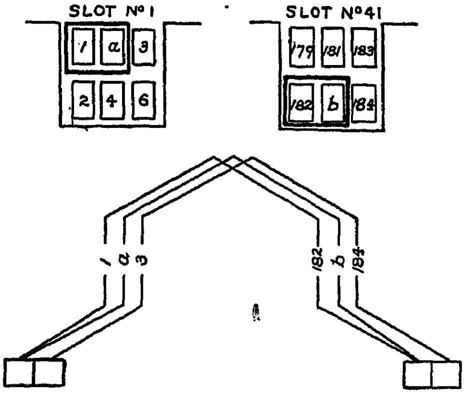


FIG. 163.

One dummy coil is that formed by the middle top inductor *a* in the first slot, and the middle bottom inductor *b* in the forty-first slot. The loop *ab* is connected in parallel, at the respective commutator segments, to the loop formed by inductors 1 and 182. The two left-hand inductors at the top of the first slot are taken together and considered as inductor No. 1. Similarly the two left-hand inductors at the bottom of the forty-first slot are considered as inductor No. 182. This shows clearly the relation of the dummy coil to the rest of the winding.

Example.—As an example of a lap-winding, consider an armature having the following constants:

Number of poles = 4.
Maximum induced E.M.F. = 224 volts.
Magnetic flux in armature per pole = 4.0 megalines.
Armature speed = 600 R.P.M.

$$N = \frac{600 \times 2}{60} = 20.$$

$$\text{Turns per circuit} = T = \frac{224 \times 10^8}{4 \times 20 \times 4.0 \times 10^6} = 70$$

Inductors per circuit = 140

Since the armature is lap-wound there will be four

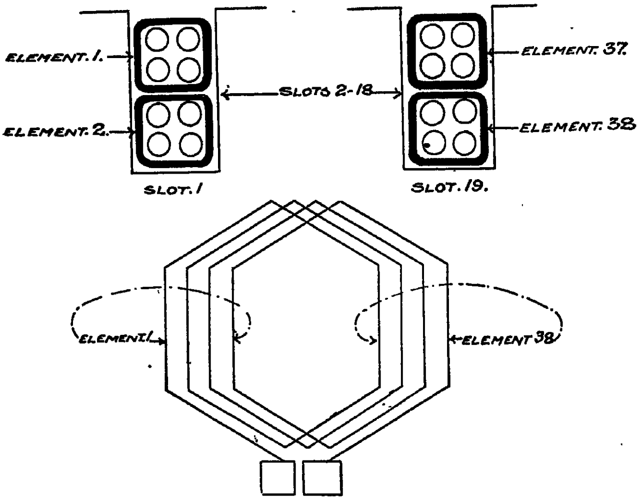


FIG. 164.

circuits in parallel, so that the total number of inductors = 140 × 4 = 560.

The armature had 70 slots, each containing 8 inductors. There were 70 commutator segments, and each winding unit consisted of a former-wound coil of 4 turns, *i.e.* each side or element has 4 inductors, as shown in Figure 164.

For spacing purposes the winding is considered as made up of $\frac{560}{4} = 140$ elements; two elements are placed in each slot, odd-numbered elements at top and even-numbered elements at bottom.

The mean pitch $= \frac{C}{n} = \frac{140}{4} = 35$ so that y_R and y_F are ? 37 + and 35 − respectively. The arrangement of the inductors in the slots is shown in Figure 164, the inductors belonging to each element being grouped together. Referring to the winding diagram, the front end of element 1 is joined to the first commutator segment, and at the rear end is connected to element 38, since $y_R = 37$. Elements 38 and 3 are connected together at the second commutator segment.

THE COMMUTATOR

Commutators are of various designs, but broadly all are of one of two types, namely, those in which the component parts are held together by insulated metal end rings, or those in which the parts are held together by steel rings shrunk on over them.

To the first type belong those in which the peripheral speed does not exceed 20 metres per second, and in most cases is less than 14. Those of the second type are for high-speed dynamos, such as are direct coupled to steam turbines and have peripheral speeds of from 30 to 40 metres per second. All commutators are built up of segments of high-conductivity, hard-drawn, drop-forged copper of tapered section, insulated from each other by mica, the whole forming a cylindrical structure as shown in Figure 165.

Commutators for Low and Medium Speeds.—The structure thus formed is clamped inside a pair of tapered steel rings A and B, the inner one A of which is cut. Pressure is then applied to force the ring A into B, thus drawing the component parts of the commutator tightly together. The pressure is in most cases applied when the segments have been heated to a temperature of about 150° C., so as to exclude any superfluous cementing material which may have been used as an adhesive for the mica laminæ in forming the strips to the requisite thickness. When cold, the grooves V are turned out, and into these correspondingly shaped steel clamping rings are fitted, and insulated from the commutator by mica rings shaped to suit. The V-shaped clamping rings are finally drawn up and permanently held by bolts, incidentally double wedging together the parts of the cylinder forming the commutator, so that on withdrawal of the temporary external clamping rings a self-contained structure remains.

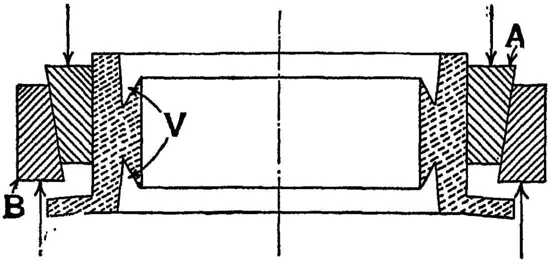


FIG. 165.—Method of building a commutator.

The mechanical strength of the commutator depends greatly upon the design of the clamping rings, and their dimensions must be such that sufficient resistance is offered to prevent the dislocation of the segments, which might be caused by the contraction and expansion of the commutator with varying temperatures. The diameter of a commutator is generally about 0.75 of the diameter of the armature, and for mechanical reasons the breadth of a segment at the periphery should preferably not be less than 4 millimetres. In order to allow ample wearing depth of copper it is customary to increase the radial depth of the segments by about 100 per cent. over that required for mechanical rigidity and electrical conductivity.

When the segments of a commutator pass under a brush sparking is liable to occur, and almost every insulating material except mica would carbonise and so become conductive. For this reason mica only is employed to insulate adjacent segments. Besides being incombustible mica is non-hygroscopic, so that it does not deteriorate when a commutator is exposed to dampness: it is mechanically strong and can withstand the high compression to which it is subjected when it forms part of a commutator. Further, mica has a remarkably fine and uniform cleavage, and is easily divided up into thin sheets of uniform thickness.

Since there is only a very small pressure (equal to that generated in one coil connected between two segments) between adjacent segments, so far as insulation is concerned, the thickness of the intervening mica strip need only be a small fraction of a millimetre; but sparking might then cause small nodules of copper to bridge over the intervening insulation. To prevent this the thickness of the insulation is usually about 0.7 of a millimetre.

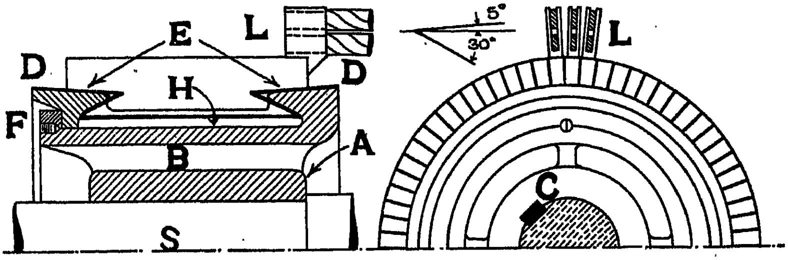


FIG. 166.—Commutator for machines up to 30 k.w.

It is of great importance that the mica used should wear at approximately the same rate as the copper segments, and for fulfilling this condition green-shaded or amber-coloured flexible Canadian mica is found to be the most suitable. As it is commercially almost impracticable to obtain mica sheets of larger dimensions than about 10 centimetres square, the insulating strips for long commutators are formed of a number of laminæ cemented together with shellac in such a manner that the joints in adjacent segments do not coincide.

Figure 166 shows a suitable construction of commutator for machines up to about 30 k.w. The cast-iron spider A is provided with four tunnels B to ventilate the inside of the commutator, bored internally to fit the shaft S, and held in position by a key C fitted into a key-seat on the shaft. The V-shaped clamping end rings D, one of which is cast with the spider A, fit into the V-grooves turned in either side of the commutator. Complete insulation of the segments from the supporting structure is obtained by conical mica rings E moulded or built up to the required taper. The thickness of these rings ranges from 2 millimetres for 100-volt armatures to 4 millimetres for 1000 volts. The pressure on the clamping rings is applied and maintained by the nut F, and the latter, after the rings are finally tightened, is prevented from slacking back by means of a set-pin.

The apices of the clamping rings are rounded off to provide *draw* for the wedging action previously mentioned. Between the inner periphery of the segments and the spider A is a small air-space H. This reduces the possibility of any of the segments coming into contact with the spider. Experience has demonstrated that clamping rings having a double taper best prevent dislocation of the segments. The total angle is generally about 35 degrees, and is divided into 30 degrees below the horizontal and 5 degrees above, as indicated in the figure. By means of the double taper the segments are

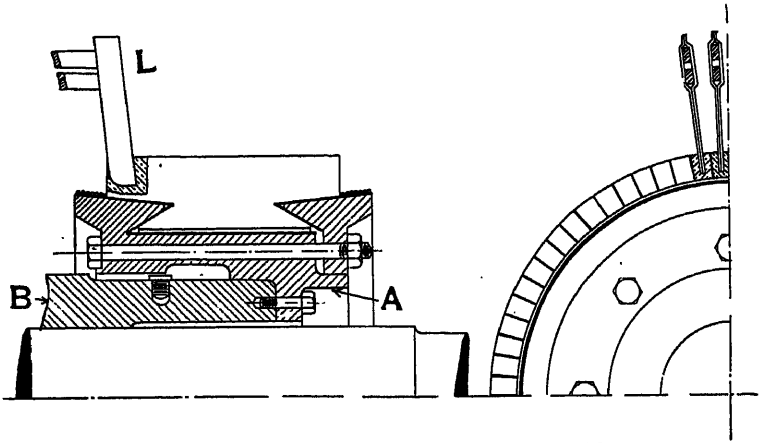


FIG. 167.—Commutator for 30–200 k.w. machine.

held endwise and centred round a circle defined by the diameter of the apices of the rings. As the clamping rings and commutator expand or contract the segments are concentrically maintained.

The commutator shown in Figure 167 is of suitable design for machines of from 30 to 200 k.w. The commutator spider A is bolted direct to the end of the armature spider B, and is therefore free from the shaft, so that the armature and commutator are entirely and separately self-contained. This forms an excellent mechanical arrangement, and no displacement of the armature winding relative to the commutator can possibly take place.

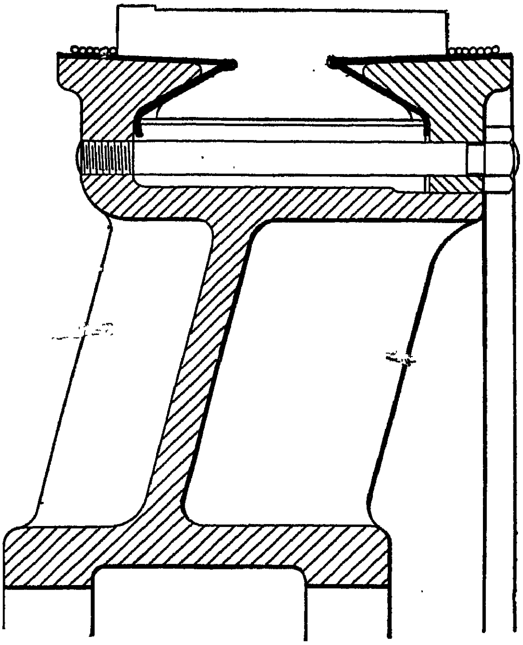


FIG. 168.—Commutator for 200–500 k.w. machines.

In Figure 168 is shown a commutator suitable for machines of from 200 to 500 k.w. The commutator is built on an armed spider, which is subsequently mounted on an extension of the armature spider, the commutator spider being held in position by means of a key sunk into a key-seat on the spider.

For large generators of 500 k.w. and upwards the best mechanical construction and ventilation is obtained by mounting the commutator on a spider which can be bolted to the arms of the armature spider, as shown in Figure 169. Each arm of the former has a projecting seat A, to which, after facing, the commutator spider S is bolted. (The armature spider in Figure 139 shows the projecting seats.) In commutators of this size the V-clamping rings are in some cases constructed in sections, each separately bolted to the spider. This facilitates the removal of a faulty segment without disturbing the rest of the commutator.

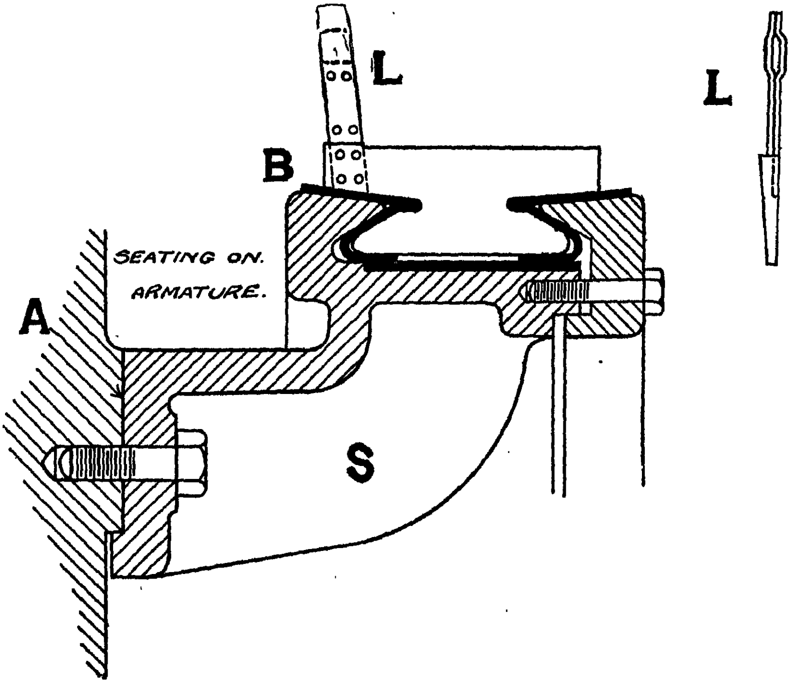


FIG. 169.—Commutator for 500–1400 k.w. generator.

Connections from the armature windings to the commutator segments are made by soldering the armature conductors into lugs projecting radially from the back of the commutator. In small commutators the lug and segment are forged in one piece. At the top of the lug a groove is slotted, into which the conductors are brought out straight and soldered, as shown at L in Figure 166. For large commutators the lug invariably consists of two strips riveted together and fixed to the segment, as shown in Figure 169. The segment has a recess in it equal to the thickness of the lug L, and the latter is riveted as shown at B, and finally brazed or soldered. At the upper end, the strips forming the lug are separated and made to embrace the ends of the armature coils as shown. In Figure 167 the lug L, formed by two strips of copper riveted together, is fixed into a groove cut in the end of the segment and then soldered or brazed.

Commutators for High Speeds.—The ordinary commutator construction, which has just been described, for peripheral speeds not exceeding 20 metres per second, is inadequate when the speed is as high as 30 to 40 metres per second. In order to present sufficient radiating surface, commutators for high-speed dynamos have great axial length compared with their diameter, so that if the ordinary V-shaped clamping rings at each end of the commutator were used the centrifugal forces would tend to cause the long segments to bulge outwards. Hence special constructions are necessary, and the general practice is to provide the segments with retaining members, in the form of steel rings, along their length, usually disposed one at each end and one in the middle.

Figures 170 and 171 show the construction of two high-speed commutators. These may be taken as typical of the best standard practice. The copper and mica segments are assembled in the same manner as before, but instead of the ordinary V-rings the segments S are retained in position by steel rings R shrunk on outside, with an intervening layer of mica insulation M_1 to prevent the rings from short-circuiting the segments. At each end, the interior of the commutator is bored tapered.

The design in Figure 171 is that usually adopted for units of 2000 k.w. and upwards. The commutator is fitted on a cast-iron spider B of the shape shown, and provided with ventilating tunnels V through which air is driven by cup-shaped blades at the outer end of each. At the armature end the spider is cast with a taper, to fit the taper to which the interior of the commutator is bored; at the other end the commutator is fitted with a tapered gun-metal bush G. To insulate the segments

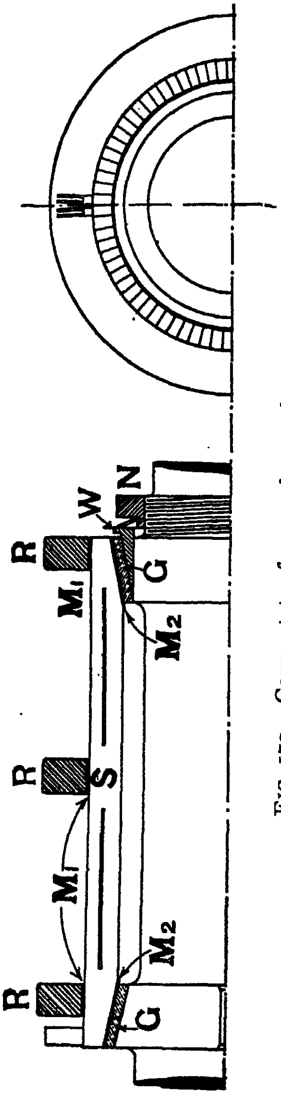


Fig. 170.—Commutator for 2000 k.w. turbo-generator.

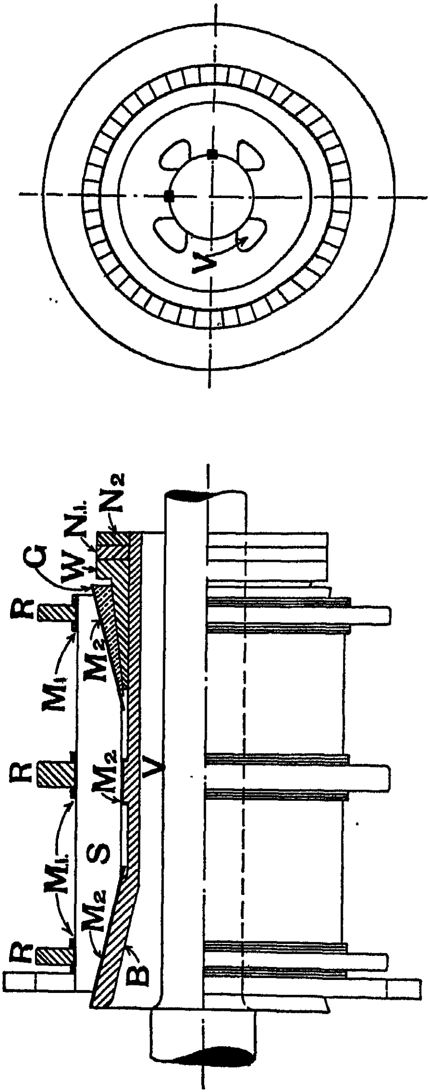


Fig. 171.—Commutator for 2000 k.w. turbo-generator.

from the spider and bush G the surfaces on which the segments would bed are covered with a sleeve of mica M_2. After the commutator is slipped on into position it is forced tight against the tapered seat on the spider by means of the wedge ring W and nut N_1, the latter being secured by the lock nut N_2.

Commutators of comparatively small diameter are usually built up on the lines of design shown in Figure 170. The commutator is fitted at each end with a tapered gun-metal bush G, and the latter is insulated from the segments with a layer of mica M_2. Collars are turned on the shaft at intervals corresponding to the axial pitch of the bushes: the one at the armature end being tapered and that at the other end parallel. The commutator is slipped on into position and forced tightly home on the collar seats by means of the wedge ring W and the spring nut N.

As a further precaution against the action of centrifugal force, commutator segments are sometimes of the cross-sectional formation shown in Figure 172, which ensures an entire circumferential interlocking of the assembled segments.

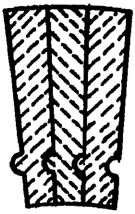


FIG. 172.—Method of locking the segments of high-speed commutators.

Owing to the centrifugal force to which they are subjected the lugs are best forged in one piece with the segments, and connection is made to the armature coils in the manner shown in Figure 166.

The radial wearing depth of commutators provided in good practice has initially 100 per cent. margin over the depth required for mechanical rigidity and electrical conductivity. A limited number of segments with narrow annular slots formed in them, as shown in Figure 170, afford an indication when the radial depth has been worn down to the permissible minimum.

The stress on the commutator shrinking rings is expressed by

$$S = \frac{F_1 + F_2}{2\pi a} \text{ kilogrammes per square centimetre,}$$

where—

F_1 = centrifugal force of all the commutator segments.
F_2 = centrifugal force of the rings due to their own weight.
a = area of cross-section of all the rings.

The value of F_1 can be obtained from the formula

$$F_1 = 0.0000112 \text{ MRN}^2 \text{ kilogramme.}$$

In order to arrive at the value of F_2, let m denote the weight in kilogrammes of the shrinking rings per cubic centimetre; then the centrifugal force of the rings per unit mass $= f = 0.0000112\, m \text{ R N}^2$ kilogramme. Since $2\pi\text{R}a$ denotes the volume of the rings, their total centrifugal force is expressed by

$$\begin{aligned} F_2 &= 0.0000112 \,.\, m \,.\, \text{RN}^2 \times 2\pi\text{R}a \text{ kilogrammes.} \\ &= f \times 2\pi\text{R}a. \end{aligned}$$

The rings being always of cast steel, the safe working stress may be taken as 1100 kilogrammes per square centimetre, so that having calculated F_1 and F_2, the sectional area of all the rings is expressed by

$$a = \frac{F_1 + F_2}{2\pi S} = \frac{F_1 + F_2}{2\pi \times 1100} = \frac{F_1 + F_2}{7000} \text{ square centimetres.}$$

The usual practice is to make the middle ring of 50 per cent. greater sectional area than either of the two end rings.

The bending moment on one commutator segment is expressed by

$$B = \frac{F_3 \times l}{8}, \text{ where—}$$

F_3 = centrifugal force of that portion of a single segment which lies between two shrinking rings.
l = distance between the centres of the rings.

If b denote the average breadth of segment and d the depth, then the modulus of section $= Z = \frac{b \times d^2}{6}$.

The bending stress on one segment =

$$S=\frac{B}{Z}=\frac{F_8\times l}{\frac{8\times bd^2}{6}}=\frac{F_8\times l}{1.33\times bd^2},$$

which for copper should not exceed 600 kilogrammes per square centimetre. The minimum depth of segment should therefore be such that this value is not exceeded.

BRUSH GEAR

Brushes and Brush-holders.—Brushes for collecting current from the commutator were at one time entirely made of copper gauze or metal leaves compressed together. Metal brushes had two inherent faults: first, the soft copper bars of the commutator and the soft metal brushes did not wear well mechanically, so that great difficulty was experienced in maintaining a smooth commutator surface; second, owing to their low-contact resistance sparkless commutation could not be obtained from no-load to full-load with fixed brush position, for reasons which are explained in the next chapter.

Brushes are now invariably made of hard blocks of graphitic carbon, although for turbo-dynamos specially constructed metal brushes are sometimes used. Carbon brushes wear well mechanically and give to the commutator a tough glassy surface, and so reduce wear. In machines having a commutator peripheral speed of more than 15 metres per second, the friction loss due to carbon brushes may assume considerable proportions, and to reduce it to a minimum the carbons are sometimes boiled in paraffin wax.

Besides supporting the brush, the brush-holder has also to press it firmly against the commutator, and at the same time give sufficient flexibility to enable it to ride over any slight eccentricity of the commutator without reducing the efficiency of contact.

Brush-holders for carbon brushes are of two types. The first, or *hammer type*, is shown in Figure 173, and is designed to take a taper-shaped carbon block, which bears radially on the commutator. The frame of the brush-holder consists of two cast brass or aluminium cheeks A, which at the brush end terminate in V-shaped jaws, in which the carbon block C is firmly gripped by a set-screw D. The brass clamping part E of the holder can move through a small angle independent of the cheeks A, and is split at F so that it can be clamped to the spindle G by means of the set-screw H. A copper lead I assists in conducting the current from the brush to the spindle, and the electrical connection between the brush and the jaws is augmented by copper plating the top of the former. The pressure on the brush is uniformly applied by means of a spring J, the tension of which can be adjusted by the angular movement of the clamp E.

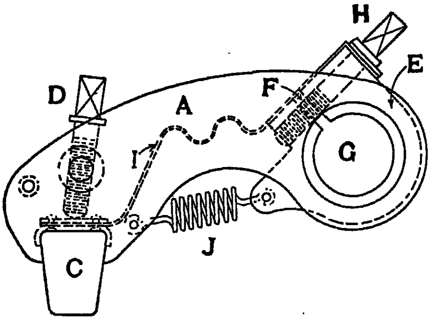


FIG. 173.—Hammer type of brush-holder.

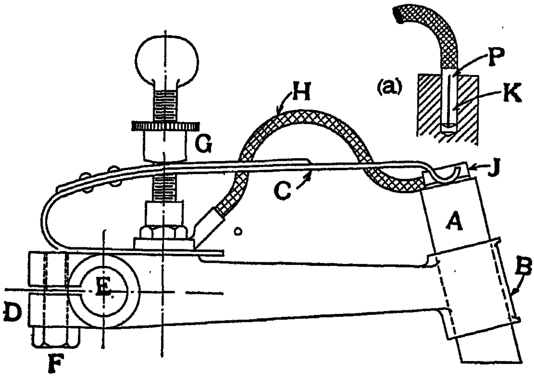


FIG. 174.—Box type of brush-holder.

Figure 174 illustrates the second or *box type* of brush-holder, the frame of which is of brass or steel-bronze. The brush A is a rectangular block free to move radially up and down in the guide-box B, and is pressed down by the spring C bearing on the top of it. The brass clamping part D is split as in the previous type, and is clamped to the brush spindle E by means of the set-screw F. The tension of the spring is adjusted by the milled nut G. Current is led from the brush to the clamp by means of a flexible fine copper wire H, technically known as a *pig-tail.* The pig-tail is soldered to the clamp J which is fixed over the top of the brush, the latter having been previously coppered. In some cases the pig-tail is soldered to a brass split-pin P (shown at *a*), which is driven into a small hole K in the top of the carbon, the sides of the hole being coppered in order to ensure good electrical contact.

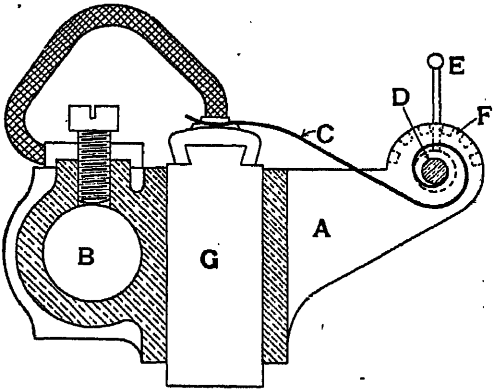


FIG. 175.—Box type of brush-holder.

Figure 175 shows another form of brush-holder of the box type. The brush-holder A is attached to the spindle B at one end, and the tension of the spring C is controlled from the other end. The free end of the spring rests on the brush G as before, but the other end is coiled once round and fixed to a small drum D pivoted between the sides of the brush-holder. The drum can be rotated through a small angle in either direction by means of the lever E attached to the spindle of the drum. There are five notches F on the frame of the holder in which the lever can be held. As the lever is moved over the successive notches the drum either winds up or uncoils the spring, thus adjusting the tension.

Figure 176 shows a brush-holder suitable for the metal brushes used for collecting current from the commutators of turbine-driven dynamos. The brush-holder is entirely of brass. It consists of a movable part A which swings on the brush-spindle E, and a fixed part B which is clamped to E by means of a set-screw. The brush D is composed of fine brass wire enclosed in a brass gauze envelope, and is soldered into a solid brass block K, which in turn is fixed to the feed-screw nut G. Passing through the centre of the movable part of the holder is a feed-screw spindle F, which engages with a worm turned in G so that the brush D can be moved towards or away from the commutator by turning the handle I. When the brush has been moved into the correct position it is clamped to the holder by means of the set-screw L. A copper flexible C assists in carrying current from the part A to the part B. H is a hold-off catch operated by an insulated handle. When in the position shown it engages with a lug M projecting from the movable part A, and so presses the brush against the commutator. When H is moved towards the spring J it disengages the lug M and allows the tension of the spring J to cant the brush-holder, and thereby lift the brush off the commutator. The pressure with which

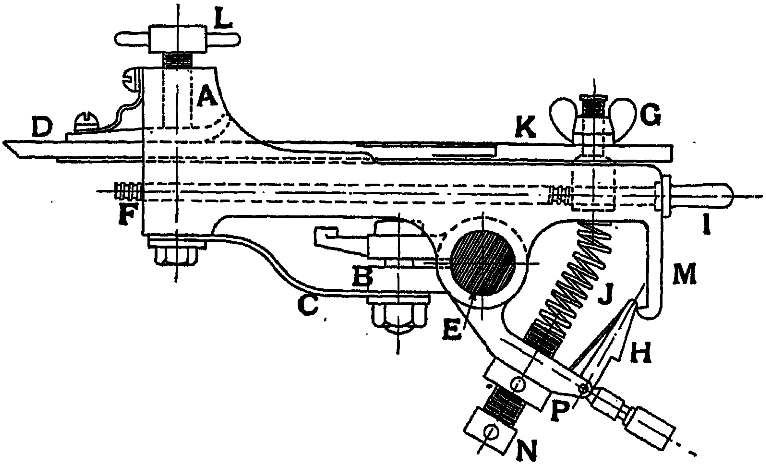


FIG. 176.—Brush-holder for turbo-generator.

the brush bears on the commutator depends on the tension of J, and can be adjusted by means of the set-screw N, which in turn is locked by the nut P.

Brush-holder Rings.—The spindles, to which the brushes are clamped, are fixed to arms projecting from a cast-iron ring which is mounted concentric with the commutator, and so arranged that it can be given a small angular displacement. Where the diameter of commutator does not exceed about 0.8 of a metre the usual practice is to mount the ring (sometimes called the *brush-rocker*) on the bearing next to the commutator. This arrangement is shown in Figure 177, which shows a section and end-view of a four-armed brush-rocker. The ring A is made in two parts, bolted together so as to fit on a path C turned on the surface of the bearing. The ring carries four arms D, placed 90 degrees apart, from each of which there projects over the commutator a brass brush-spindle E. The latter are provided with keyways and bolted to the top of the arms by nuts F, but are insulated therefrom by means of vulcanite bushes and washers G. The brush-holders are firmly clamped on to these spindles. Brush-spindles of similar polarity are

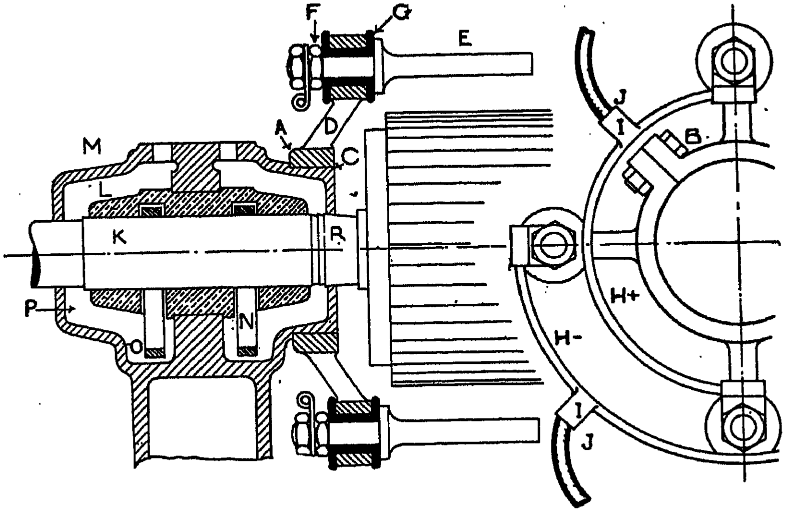


FIG. 177.—Brush gear supported on bearing.

connected together by copper connectors H + and H −, and for effecting the connection the brush-spindles project 1.5 to 3 centimetres beyond the nuts F. To the cross-connectors are cast sockets I, into which the flexible cables J are sweated for connection to the terminals of the machine. Once the correct brush-position has been experimentally determined the rocker is securely clamped.

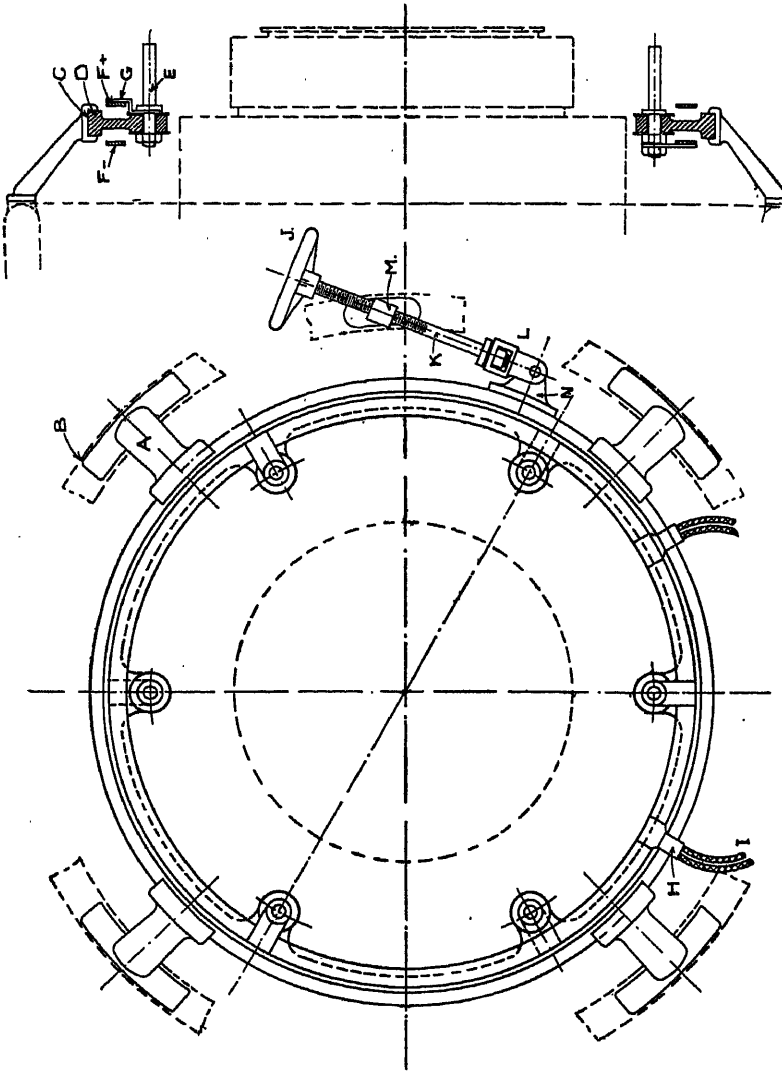


FIG. 178.—Brush gear supported from yoke.

In large machines the rings carrying the brush-gear are supported by brackets bolted to the magnet yoke, as shown in Figure 178. The brackets A are of cast-iron and bolted to machined seats B on the yoke. The under side of the brackets has a turned groove C into which the ring D fits. The latter may be of various sections, and in the case shown is of I section. The brush-spindles E are spaced along the circumference of the ring and bolted in the manner shown, the spindles being insulated from the ring by vulcanite bushes in a similar manner to that shown in the previous figure. On opposite sides of the ring are mounted on insulating supports connectors F + and F − of stout copper strip. These serve to cross-connect brush-spindles of the same polarity, connection with the brush-spindles being made by copper lugs G, which are riveted or brazed to the connectors. To each connector is cast or bolted a terminal H, into which the flexible cables I, connecting the brushes to the terminals of the machine, are soldered.

To the lower part of the right-hand side of the ring is bolted a bracket N, which is connected by the link L to the screwed spindle K. The screw of the spindle engages in a worm M fixed to the magnet frame, so that when the spindle is rotated by operating the handle J the brush-ring is given angular motion. This addition to the brush gear now appears almost unnecessary, as all direct-current machines are required to work with a fixed brush position.

SHAFT AND BEARINGS

Shaft.—Armature shafts are invariably of mild steel and require to be of sufficient stiffness to resist the combined stresses set up by (1) the bending moment due to the weight of the armature and commutator; (2) the twisting strains due to continuous transmission through it of the energy supplied by the prime mover to the armature inductors; and (3) any unbalanced magnetic pull on the armature core produced by the unequal distribution of the magnetic field. The method of calculating the diameter of shafts is discussed in various text-books on mechanics. The following formula is found to give a diameter of shaft of sufficient stiffness to withstand the stresses due to the fluctuating load of a dynamo:

$$d = 18\sqrt[3]{\frac{\text{H.P.}}{\text{R}}}$$

where—

d = The diameter of the shaft in centimetres at any part underneath the armature or commutator.

H.P. = Horse-power transmitted by the shaft.

R = Speed of the shaft in revolutions per minute.

The armature spider considerably increases the stiffness at the centre, and in cases where the armature is coupled to the prime mover by a coupling projecting from the armature core (as shown in Figure 213) the shaft is entirely relieved of the driving strain, and has only to withstand the bending moment due to the weight of the armature and commutator. Every case must, however, be considered on its merits; for instance, in a machine having a very small air-gap the shaft would require to be exceedingly stiff in order to reduce to a minimum the bending liable to occur through unbalanced magnetic pull set up by inequalities in the strength of the several fields. Armature shafts are usually turned of larger diameter at the middle than at the ends, to allow for diminution of cross-section due to the key-ways.

Bearings.—The bearings and pedestals for supporting armature shafts are similar to those employed for other types of machinery, and a typical dynamo bearing is shown in Figure 177. The shaft K revolves in a split brass bush L: the latter is prevented from turning by the cap M. In order to facilitate the removal of the armature, and to provide a means of adjusting for wear, the bearings are generally in two parts, held together by bolts. Two circumferential slots are cut in the upper half of the bush so as to allow two brass rings N to rest upon the shaft. The lower parts of these rings dip into the oil well O, and as the shaft revolves it carries the rings with it, and in this way a continuous automatic lubrication of the bearing is effected.

Each end of the bearing is provided with a hooded chamber P, in which the oil oozing from the ends of the brasses is caught and returned to the oil well. At the end nearest the armature the shaft is provided with an *oil thrower* R, by turning buttress-shaped recesses in that part of the shaft covered by the hood. The oil which oozes out from the end of the brass is thus thrown off the shaft by the action of centrifugal force, and so prevented from creeping along to the armature or commutator.

CHAPTER IX

THE DYNAMO—FIELD MAGNETS, MAGNETIC CIRCUIT, ARMATURE REACTION, COMMUTATION, COMMUTATION POLE MACHINES, AND EXAMPLES OF COMPLETE MACHINES

FIELD MAGNETS

Types of Field Magnets.—For 2-pole dynamos the field magnets are generally in the form of a horseshoe with straight limbs and two exciting coils, as shown in Figure 179. The composite magnet has forged iron or steel limbs L and a cast-iron yoke Y, both limbs and yoke having a rectangular cross-section. The whole machine, *i.e.* armature and field magnets, is supported on a cast-iron bed-plate, so that it becomes necessary to attach, to the magnet poles, feet or brackets F made of some non-magnetic material such as gun-metal. Should

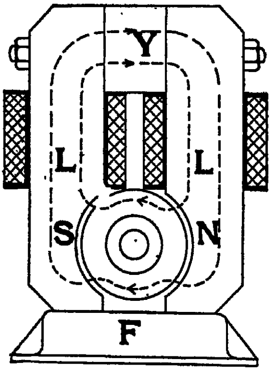


FIG. 179.—Magnetic circuit of a 2-pole dynamo.

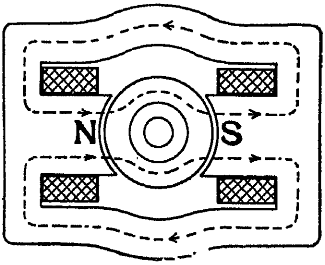


FIG. 180.—Magnet circuit of a 2-pole iron-clad dynamo.

the non-magnetic feet be omitted a closed magnetic circuit would be formed through the bed-plate of the machine, and the bulk of the lines of force would pass from pole to pole by way of the bed-plate, instead of through the armature as required. The vertical depth of the feet must be such as to avoid excessive leakage of lines of force into the bed-plate, and should therefore be commensurable with the total radial length of air-gap. Figure 180 shows the magnetic circuit of an iron-clad 2-pole dynamo. The poles are placed horizontally, and the yoke is divided and passes above and below the armature.

The 2-pole type of field magnet is well suited for high-speed generators, but for lower speeds commercial considerations preclude its general use for large sizes owing to the costliness of the necessarily massive magnet poles. To meet this contingency, when the speed of the armature is low, its diameter is increased, and instead of a single horseshoe field magnet, an iron ring, with many pairs of poles grouped round its periphery, is employed. This is the form of the field magnet of a modern multipolar machine, in which each pair of poles, with its portion of the armature, constitutes a unit. By increasing the number of these units the output of the machine is increased.

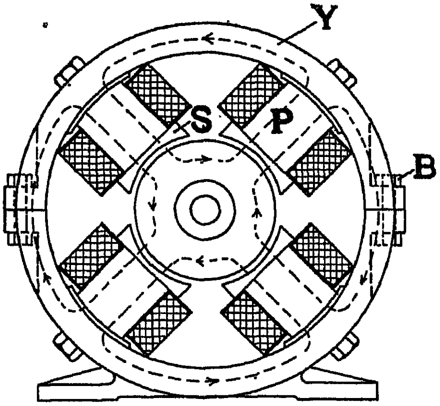


FIG. 181.—Magnetic circuit of a 4-pole dynamo.

The magnetic circuit of a 4-pole machine is shown in Figure 181. This, it will be observed, is in reality only a multiplication or development of the principle of the 2-pole iron-clad machine in the previous figure. The circular yoke Y of the field magnet frame is usually divided on a horizontal diameter, the faces of the upper and lower halves being accurately machined and held together by bolts B. The inward projecting poles P are either bolted or cast to the inner surface of the magnet ring. To prevent too sudden a variation of magnetic flux in the armature teeth when approaching or receding from the poles, the latter are fitted with extensions S known as pole shoes.

In Figures 179 to 181 the path taken by the main magnetic flux is indicated by the dotted lines and its direction by the arrow-heads.

Excitation of Dynamos.—According to the method of exciting their field magnets, dynamos are classified as: (1) Separately excited; (2) shunt-wound; (3) series-wound; and (4) compound-wound.

Separately excited Dynamos.—As the name implies, these are dynamos in which the magnet coils are supplied with current from an independent source, to distinguish them from *self-excited* machines, in which the exciting current is obtained from the armature. Separately excited dynamos are only employed for special purposes; as, for example, the low-voltage generators used for electro-plating, and dynamos used as boosters.

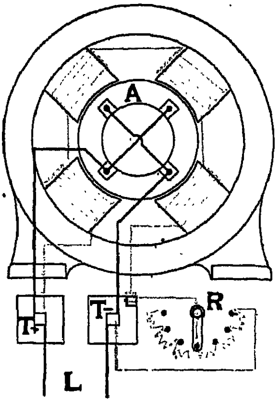


FIG. 182.—Shunt-wound dynamo.

Shunt-wound Dynamos.—The field coils of a shunt machine consist of a large number of turns of comparatively fine wire connected across the armature terminals, the resistance of the field coils being such that the excitation current ranges, according to the output, from 1 to 6 per cent. of the total current. In Figure 182 is a diagram for the connections for a shunt dynamo. The brushes for collecting the current from the armature A are connected to the main terminals T + and T –, whence connection is made by the leads L to the external circuit. The shunt circuit is shown in red, and the four magnet coils are connected in series: one end of the winding is connected to T+ and the other to T− through the regulating resistance R, which is used for regulating the field current. It will be observed that the field winding and the external circuit are connected in parallel or shunt at the terminals T+ and T− : hence the name shunt-wound dynamo.

The ready building up of the voltage of a self-exciting dynamo depends upon the property of *residual magnetism*; that is, the field magnets after being once excited retain an appreciable amount of magnetism. This residual magnetism ensures a small number of lines of force always being linked with the armature inductors, so that when the armature is rotated a small E.M.F. is induced in it.

In the case of a shunt dynamo (Figure 182) the magnet coils are connected across the armature terminals T+ and T−, so that the small E.M.F. due to residual magnetism sets up a small current, which leaves the armature at the positive brush, flows through the magnet coils, and enters the armature at the negative brush. The current sent through the field coils in this way increases the strength of the field in which the armature rotates. The E.M.F. consequently induced will send a still larger current through the field coils. Action and reaction thus succeed one another in this manner until the field magnets become saturated, beyond which an increase in the exciting current does not produce an appreciable increase in the magnetic field. The resistance of the shunt circuit may, however, be so high that the E.M.F. ceases to increase before the saturation point is reached; that this is so will be made clear after studying the characteristic curves of dynamos which are dealt with in the next chapter.

Series-wound Dynamos.—In a series dynamo the magnet coils are connected in series with the external circuit, so that the exciting current is the same as that which passes through the external circuit. The required number of ampère-turns is obtained by winding the coils with comparatively few turns of wire of sufficient cross-sectional area to carry continuously the main current. A diagram of the connections for a series dynamo is shown in Figure 183. The field winding and armature A are connected in series at the common terminal T, while the other end of the field winding and the other armature lead are connected to T – and T + respectively. The latter form the main terminals of the machine, and through them connection is made by the leads L to the external circuit. From the diagram it will be clear that before a current can pass through the magnet coils for excitation purposes the external circuit must be closed.

Compound-wound Dynamos.—A machine of this class is simply an ordinary shunt dynamo having an additional magnet winding connected in series with the external circuit for regulating, under varying loads, the pressure on the armature terminals. A diagram of the connections is shown in Figure 184: the shunt winding Sh is connected, through a variable resistance R, across the armature A at the terminals T + and T –. The series winding Se is in series with the external circuit. T + and T – are the main terminals of the machine, through which connection is made to the external circuit by the leads L.

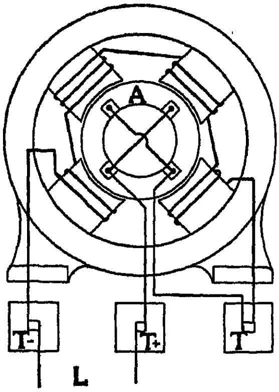


FIG. 183.—Series-wound dynamo.

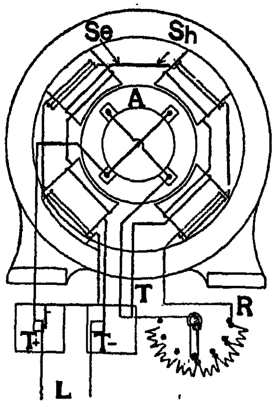


FIG. 184.—Compound-wound dynamo.

For constant potential systems of distribution, shunt or compound dynamos are always employed. Series dynamos were at one time largely used for arc-lighting, but have now very limited applications.

Field-magnet Coils.—Field-magnet coils are either *spool wound* or *former wound.* The first are wound on a flanged spool of metal or fibre, the shape being such that the spool fits tightly on to the magnet core and is held in position by the pole shoes E, as shown in Figure 190. When of metal the spool or bobbin requires to be insulated. Press-spahn or thin sheets of fibre are usually applied before winding operations are commenced. The thickness of the insulation depends upon the voltage for which the machine is designed, and for 600-volt machines is from 2 to 3 millimetres. In compound dynamos this method of winding the magnet coils is invariably adopted; the series winding occupies about 30 per cent. of the axial length of the spool, and is preferably placed below the shunt coils, as shown in Figure 190, the two coils being well insulated from each other. In small machines the series winding is usually wound over the shunt coils, but this retards considerably the radiation of heat from the latter.

In Figure 188 is shown a former-wound coil, which is first wound on a flanged wooden former to hold the wires together during winding. Such coils have strong pieces of tape laced in between the layers to bind them together, and, after winding, the coils are soaked in an insulating varnish and finally covered with strong cotton tapes. The coils must be proportioned so as not to heat beyond a safe limit, and with coils of short axial length it becomes necessary to pile up the winding conically, as shown in Figure 189, thus affording an increased radiating surface. In order to facilitate ventilation, the series coils of compound-wound generators are often supported in such a manner as to leave a space or air-way between them and the shunt coils.

Space Factor.—The value of the space factor of a magnet coil depends upon the shape of the cross section of the wires. The general practice is to employ wires of circular cross-section for shunt windings, in which case the space factor is determined chiefly by the relative thickness of the wires and their insulating covering. The amount of bedding of the wires of adjacent layers also affects the space factor to a small extent. Suppose a coil to be wound so that there is no bedding of the wires, as shown in Figure 185. Let d_1 and d_2 denote the diameter of the wire when bare and when insulated. The winding space allotted to each wire $= d_2^2$, and that actually occupied by the copper $= \frac{\pi d_1^2}{4}$, so that the space factor

$$= \frac{\text{section of copper per wire}}{\text{section available per wire}} = \frac{\pi d_1^2}{4 d_2^2} = 0.78 \frac{d_1^2}{d_2^2}.$$

For example, suppose a wire of No. 19 S.W.G., having a double cotton covering (D.C.C) insulation, in which the diameter of the wire when bare and when insulated is 1.017 millimetres and 1.22 millimetres respectively, be wound as shown in Figure 185, then the space factor

$$= 0.78 \times \frac{1.017^2}{1.22^2} = 0.54.$$

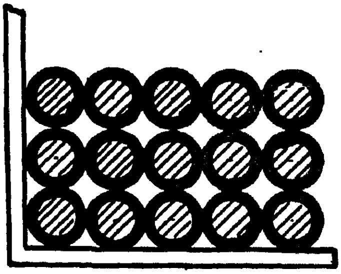


FIG. 185.—Shunt-winding with no bedding.

It has been found by experience that in wires of small diameter, such as are used for shunt coils, the bedding of the wire rarely increases the space factor by more than 2 or 3 per cent., so that the value given by the above formula, in the absence of a value obtained from actual experience, should be used. The shunt coils of standard dynamos up to 600 volts have space factors ranging from 0.45 to 0.55, and in subsequent calculations 0.5 is taken as being an average value.

MAGNETIC CIRCUIT

For convenience in calculating the ampère-turns required to set up a definite magnetic flux through each unit of the magnetic circuit, the latter is divided into the following parts:

1. Armature Core.
2. Armature Teeth.
3. Air-gap.
4. Magnet Core and Pole Shoes.
5. Yoke.

Armature Core.—As already stated, the armature core is constructed of sheet iron or steel laminations, the radial depth of which should be such that the average flux density has a value of between 9000 and 12,000 lines per square centimetre. As seen from the curve in Figure 191, armature laminations can be economically worked at a flux density of 16,000 lines per square centimetre; but in order to keep the iron losses (*i.e.* hysteresis and eddy currents) within certain limits the flux density in armature cores is usually only from 60 to 75 per cent. of that which would be adopted if the armature iron were not subject to these losses. With multipolar machines it is important to bear in mind that after crossing the air-gap and entering the armature the flux from an N pole passes along two diverging paths towards adjacent S poles, as indicated in Figure 181.

The magnetic length of the core is the length of the mean magnetic path which lies between the roots of the teeth and the inner periphery of the stampings. Since the armature stampings are insulated from each other by paper or varnish, the magnetic sectional area is less than the gross sectional area by an amount varying from 15 to 30 per cent., depending upon the thickness of the said insulation and the number of ventilating ducts.

Armature Teeth.—The teeth of armatures require to be worked at a flux density of from 19,000 to 23,000 lines per square centimetre. The lines of force do not pass radially from the pole face to the armature surface, but spread out at the ends of the pole face, as shown in Figure 186. The result of this spreading is that the number of teeth forming the path by which the lines of force pass from the pole face to the armature, is increased by about 10 per cent. The magnetic area of a tooth is the product of the mean width and the net length of armature core.

In calculating the flux density in the teeth it may be assumed that all the flux enters the armature by way of the teeth. The flux density calculated on this assumption is known as the *apparent flux density.* The permeability of sheet iron at the high induction densities occurring in the teeth of armatures is not many times greater than that of air, with the result that the total flux in the armature does not enter by way of the teeth, as is assumed in calculating the apparent flux density; but a considerable proportion enters the armature by the parallel paths, of higher magnetic reluctance, offered by the slots, ventilating ducts, and insulation between adjacent armature laminations.

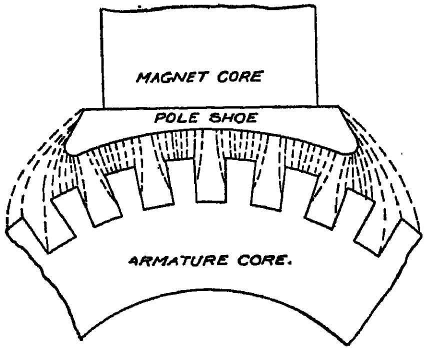


FIG. 186.—Distribution of flux in air-gap with slotted armature.

The relation between the total lines M_1 transmitted by a combined tooth and slot and those M_2 transmitted by a tooth alone, may be derived as follows:

Let a = mean width of a tooth.
b = width of slot.
l_n = net length of armature iron parallel to the shaft.
d = depth of slot.
η = net length of iron ÷ gross length of core.

The cross-section of iron per tooth $= a \times l_n$, and the cross-section of each slot $= \frac{b \times l_n}{\eta}$.

Since the flux also enters the core by way of ventilating ducts and insulation between the core plates, the area of cross-section of the latter per slot

$$= \frac{a \times l_n}{\eta} - a \times l_n = a l_n \left(\frac{1}{\eta} - 1\right).$$

The cross-section of the non-iron path per tooth therefore

$$=\frac{b\times l_n}{\eta}+al_n\left(\frac{1}{\eta}-1\right)$$

$$=\frac{l_n(b+a-\eta a)}{\eta}.$$

Let the flux M_1 entering the armature per tooth divide, so that M_2 lines enter by a tooth and (M_1-M_2) by air space and insulation, then if μ denote the permeability of the teeth

$$M_2\propto\frac{\text{area of cross section of teeth}\times\mu}{\text{depth of tooth}}$$

$$M_2\propto\frac{al_n\times\mu}{d}.$$

Similarly, for the flux in non-iron part per tooth

$$M_1-M_2\propto\frac{l_n(b+a-\eta a)}{d\eta}.$$

The ratio

$$\frac{M_2}{M_1-M_2}=\frac{al_n\mu\times\eta d}{dl_n(b+a-\eta a)}=\frac{a\eta\mu}{b+a-\eta a}$$

$$\text{or } M_1a\eta\mu=M_2(b+a-\eta a+a\eta\mu),$$

$$i.e.\ \frac{M_2}{M_1}=\frac{b+a-\eta a+a\eta\mu}{a\eta\mu}.$$

On an average the ratio of net length to gross length of core $=0.75$, so that substituting this value for η,

$$\frac{M_2}{M_1}=\frac{1.33b+0.33a+a\mu}{a\mu}.$$

If B_1 denote the real tooth density, and B_2 the apparent tooth density, then since $\frac{M_2}{M_1}=\frac{B_2}{B_1}$,

$$\frac{B_2}{B_1}=\frac{1.33b+0.33a+a\mu}{a\mu}.$$

The value of μ corresponding to any particular value of B_2 may be obtained from a permeability curve, and knowing *a* and *b* the corresponding value of B_1 can be computed.

The method is shown graphically in Figure 187, where the abscissæ represent apparent tooth density and the ordinates real or corrected values. The latter

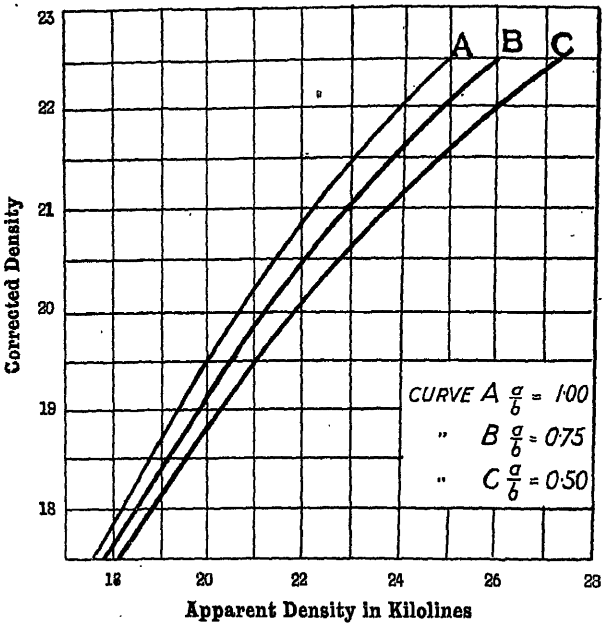


FIG. 187.—Curves of corrected tooth density.

depend upon the ratio of width of tooth to width of slot, and three curves are given where the value $\frac{a}{b}$ equals 1, 0.75, and 0.5. In practice the ratio $\frac{a}{b}$ lies between 1 and 0.5, and for intermediate values of $\frac{a}{b}$ the corrected tooth density can be estimated with an accuracy sufficient for all practical purposes.

Air-gap.—The flux density in the air-gap ranges from 6000 to 9000 lines per square centimetre, and the simplest method of determining it is to take the average density over the pole face, *i.e.* the total flux entering or leaving the armature per pole, divided by the surface area of the pole face. The actual area of exposed iron on the surface of the armature is less than that of the pole face due to the slots between the teeth, but on the other hand there is a spreading of the lines of force when passing from pole face to armature, and it is usual in practical calculations to assume that these two effects counteract each other.

The ampère-turns required to send the flux across an air-gap are obtained from the formula

$$H = 1.25 \frac{CT}{l}, \text{ i.e. } CT = 0.8Hl$$

where—

CT = Ampere-turns per pole.
H = Induction density in the air-gap.
l = Length of air-gap.

From this formula it will be seen that the greater the length of air-gap the larger the magneto-motive force required to send the flux through the air-gap, so that it would appear that the length of air-gap should be reduced to the mechanical limit. As will be shown later, the effects of armature reaction on the main field increase inversely as the length of air-gap. A compromise must therefore be made between these two considerations, and experience shows that air-gap lengths should not be less than half the width of a single tooth. In multipolar machines the length of air-gap ranges from 0.5 to 1.2 centimetres for machines in which the armature diameter is between 0.7 and 5.0 metres.

Magnet Cores and Pole Shoes.—Magnet cores are made of cast steel, wrought-iron forgings, or sheet iron or steel, and the induction density ranges from 14,000 to 16,000 lines per square centimetre. Figures 188 to 190 illustrate three types of magnet cores, all of which are extensively used. In Figure 188 the magnet core C and yoke Y are cast in one piece, while in Figure 189 the magnet core C, of circular cross-section, is cast separate, and after being machined is bolted to a seating

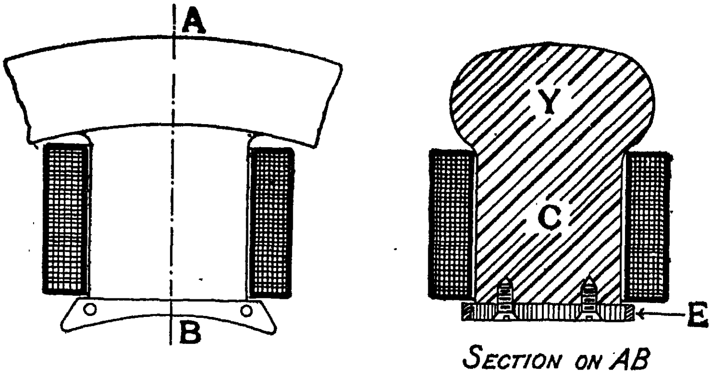


FIG. 188.—Magnet core and yoke cast solid.

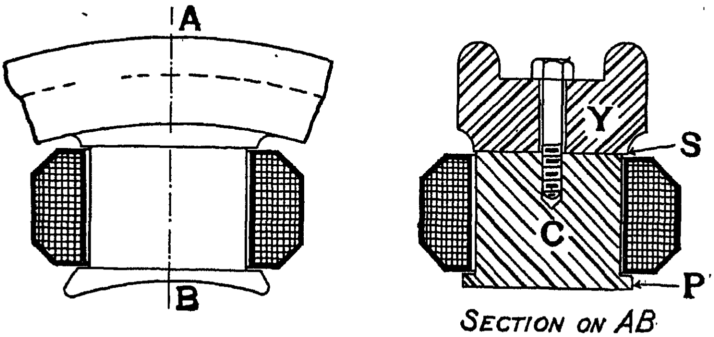


FIG. 189.—Magnet core bolted to yoke.

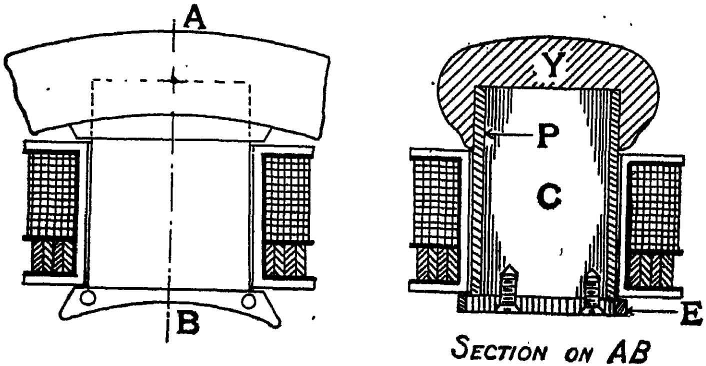


FIG. 190.—Magnet core laminated and cast into yoke.

S cast on the yoke. In the third type (Figure 190) the magnet core C is built up of sheet-iron laminations riveted together between two end plates P and cast into the yoke Y. The magnet core is of rectangular cross-section, and in some designs is left without polar extensions, in which case the flux density in the core must be reduced to about 12,000 lines per square centimetre, so as to prevent an uneconomically high air-gap density..

Pole shoes are usually of steel, and may be cast in one piece or built up of laminated sheets, preferably the latter in large machines, for reasons stated below. In the design shown in Figure 189 the pole shoe P can conveniently be cast in one piece with the magnet core, the field coils being placed over the core before it is bolted to the yoke. The pole shoe E in Figures 188 and 190 is laminated; the stampings are cut to the shape shown, and then riveted together between two stout end plates. The pole shoe is fixed to the magnet core by set-screws, the field coils being slipped on before the core is finally secured. The coils are held in position by the pole shoes.

Pole shoes are liable to have eddy currents induced in them due to unequal distribution of the flux, which extends in a marked degree to the pole face, as shown in Figure 186. As the armature teeth move under the pole face the tufts of flux change from one position to another, with the result that eddy currents are induced. In large machines the loss of energy and heating due to this cause may become so great that lamination of the pole shoes is necessary.

In machines of ordinary design the span of the pole face or the length of polar arc ranges from 0.68 to 0.72 of the polar pitch, and 0.7 may be taken as an average value. When, however, commutation poles (see *n s*, Figure 210) are fitted, considerations of magnetic leakage necessitate a shorter polar arc, which generally ranges from .55 to .65 of the polar pitch. The radial length of pole shoe varies from 2 to 4 centimetres, and in virtue of this length being so small, it is customary to include it in the length of the magnet core. Experience shows that this leads to no appreciable inaccuracy. In designing a dynamo the diameter of the armature is approximately determined in the manner described on page 447, and from this the area of pole face is settled, the gross length of the pole face being made equal to the gross length of the armature core. Then assuming a maximum flux density of from 8000 to 9000 lines per square centimetre in the air-gap, the total flux entering the armature from each pole face can be determined.

The axial length of the magnet core will depend upon the space necessary for the field winding, and a value is first assumed from experience with other machines. The dimensions of field coils must be such that the radiating surface is sufficient to prevent the temperature rise exceeding a pre-determined limit. The method of estimating the temperature rise is indicated in Chapter XII.

Yoke.—Magnet rings or yokes are generally of cast iron or steel, and the cross-sectional area is such that the flux density does not exceed 8500 or 15,000 respectively. From these figures it will be seen that with two machines having the same electrical and magnetic constants, but one having a cast-iron yoke and the other a cast-steel, the cross-sectional area of the latter will be approximately half of the former. In calculating the ampère-turns to send the flux through the yoke the magnetic length is the mean length of path, and it should be remembered that the flux in each section of the yoke in a multipolar machine is only one-half that of the flux per pole.

In calculating the number of ampère-turns per pole required to send a given flux through the magnetic circuit, use is made of such curves as are shown in Figure 15. The parts of the curves used in dynamo calculations for cast iron and cast steel are reproduced in Figure 191, along with the curve for armature laminations. A separate magnetisation curve for the high densities occurring in armature teeth is given in Figure 192.

Coefficient of Magnetic Leakage.—As explained in Chapter III. lines of force will pass between surfaces which are at different magnetic potentials: consequently in all dynamos the total flux in the magnet core does not pass through the armature, but a certain proportion leaks across from one pole to another by way of the air, as depicted in Figure 193. The ratio of the total flux generated to the useful flux in the armature has been called the coefficient of magnetic leakage, or the *dispersion coefficient*, as suggested by Prof. S. P. Thompson, and is given by the equation

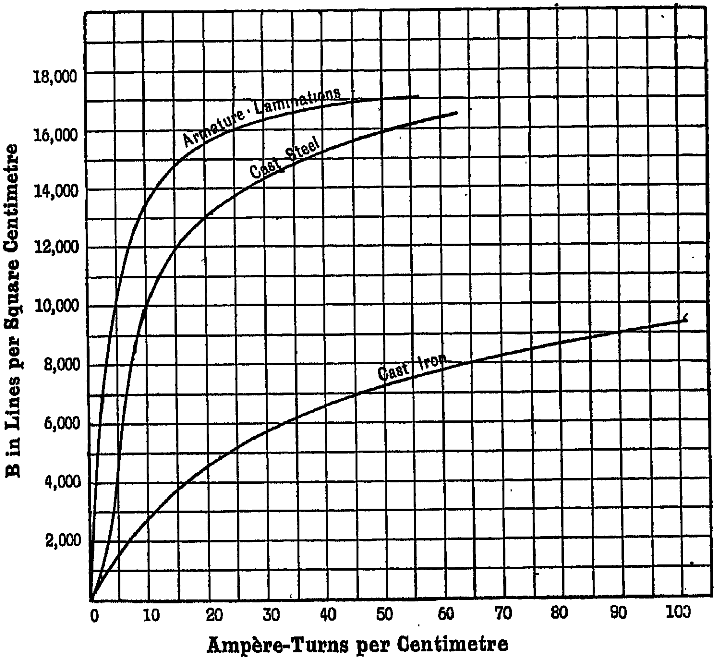


FIG. 191.—Magnetisation curves used in dynamo calculations.

$$v = N_m/N_a.$$

The flux set up in any one unit of the magnetic circuit is a maximum in the yoke, and gradually diminishes, until at the armature surface it is reduced to the useful flux that becomes linked with the inductors. In practical calculations it is convenient to assume that the useful flux passes through the armature core, teeth, and air-gap, and that all the leakage of lines of force takes place at the pole face, thus allowing the total flux generated to pass through the magnet core. This assumption considerably simplifies calculations relating to the magnetic circuit, and experience shows that the results obtained are of sufficient accuracy for ordinary work.

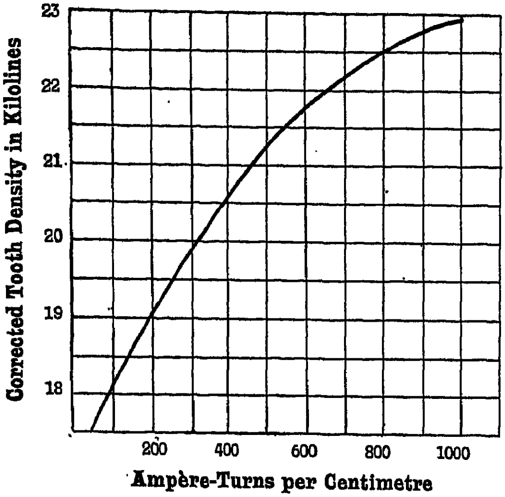


FIG. 192.—Ampère-turns for armature teeth.

Leakage lines of force necessitate an increased section of the yoke and magnet core beyond that actually necessary to carry the useful flux; and an increase in the section of the magnet core means that each turn of the field winding must be correspondingly increased. It therefore becomes important to design magnetic circuits with the minimum amount of leakage, so that a given output can be obtained with the minimum of materials.

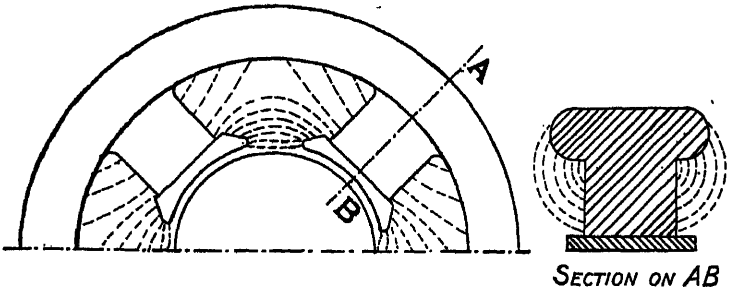


FIG. 193.—Magnetic leakage from multipolar magnet.

The extent of magnetic leakage depends upon—(1) The shape of the magnet core; (2) the length of air-gap; and (3) the induction density in the magnet core.

On account of the smaller surface area magnet cores of circular cross-section have less leakage than a rectangular pole having the same sectional area. The greater the length of air-gap the greater the reluctance of the magnetic circuit, and consequently also the greater the tendency for the flux to take alternative paths. By increasing the flux density in the magnet core the reluctance of this portion of the circuit increases while the leakage paths through the air remain constant, with the result that the leakage lines of force are increased.

The dispersion coefficients vary with different types of machines, and the actual values employed in practice are usually determined from experiment. The values of the dispersion coefficients for multipolar machines range from 1.3 in small machines and those in which the poles are close together, to 1.1 in very large machines.

Example of Calculation relating to the Magnetic Circuit.—An example will now be worked to illustrate the application of the foregoing principles. In Figure 194 is given a dimensioned drawing of the magnetic circuit of a 33-k.w. shunt-wound 4-pole dynamo, the armature details of which are given in the example on page 274.

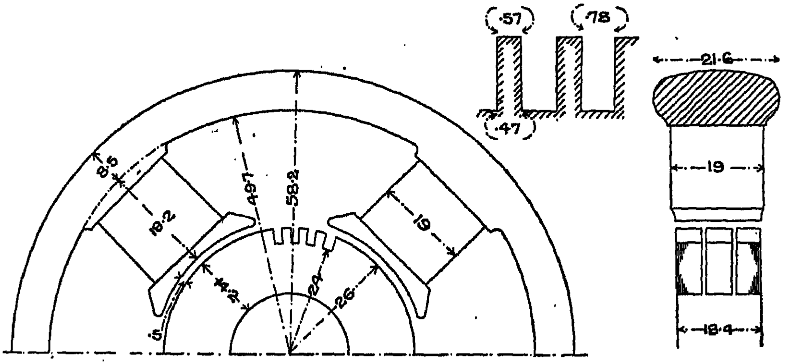


FIG. 194.—Magnetic circuit for example.

At no load the magnetic flux per pole passing through the armature equals 3.45 megalines. At full load the internal pressure drop due to armature and brush resistance is 22 volts, so that to maintain a constant terminal E.M.F. of 550 volts the flux per pole at full load is given by

$$M = \frac{E \times 10^8}{4 \times T \times N} = \frac{(550+22)10^8}{4 \times 181 \times 22} = 3.6 \text{ megalines.}$$

It is required to determine the ampère-turns per pole necessary to overcome the reluctance of the magnetic circuit (1) at no load, and (2) at full load. The method of procedure is as follows:

ARMATURE CORE

The area of cross-section of iron $= \{18.4 - (2 \times 1)\} \times 0.9 \times (24 - 11.8) = 180$ square centimetres.

The mean length of path taken by the flux as measured from drawing = 15 centimetres.

Flux density at no load $= \frac{3.45 \times 10^6}{2} \times \frac{1}{180} = 9.6$ kilolines per square centimetre.

From Figure 191 ampère-turns per centimetre of length = 4.5.

Ampère-turns at no load $= 4.5 \times 15 = 70$.

Flux density at full load $= \frac{3.6 \times 10^6}{2} \times \frac{1}{180} = 10.0$ kilolines per square centimetre.

Ampère-turns per centimetre of length = 5.4.

Ampère-turns at full load $= 15 \times 5.4 = 80$.

ARMATURE TEETH

Number of teeth per pole $= \frac{121}{4} = 30.3$.

In the machine under consideration the polar arc is 0.69 of the polar pitch, so that after allowing 10 per cent. for fringing the number of teeth under one pole $= 30.3 \times 0.69 \times 1.1 = 23$.

Mean area of cross-section of iron in one tooth $= \{18.4 - (2 \times 1)\} \times 0.9 \times 0.52 = 7$ square centimetres.

Area of cross-section of all the teeth under the polar arc $= 7 \times 23 = 161$ square centimetres.

Length of teeth = 2 centimetres.

$$\frac{\text{Mean width of tooth}}{\text{Width of slot}} = \frac{a}{b} = \frac{0.52}{0.78} = 0.67.$$

Apparent tooth density at no load $= \dfrac{3.45 \times 10^6}{161} = 21.5$ kilolines per square centimetre.

Corrected tooth density at no load (Figure 187) = 20.1 kilolines per square centimetre.

From Figure 192 the ampère-turns per centimetre of length = 320.

Ampère-turns at no load $= 320 \times 2 = 640$.

Apparent tooth density at full load $= \dfrac{3.6 \times 10^6}{161} = 22.4$ kilolines per square centimetre.

Corrected tooth density at full load = 20.7 kilolines per square centimetre.

Ampère-turns per centimetre of length = 425.

Ampère-turns at full load $= 425 \times 2 = 850$.

AIR-GAP

Circumference at air-gap $= \pi \times 2 \times 26 = 163$ centimetres.

Length of polar arc $= \dfrac{163}{4} \times 0.69 = 28.1$ centimetres.

Width of pole face = gross width of armature = 18.4 centimetres.

Area of cross-section of air-gap = area of pole face $= 18.4 \times 28.1 = 517$ square centimetres.

Length of air-gap = 0.5 centimetre.

Average flux density at no load $= \dfrac{3.45 \times 10^6}{517} = 6.7$ kilolines per square centimetre.

Ampère-turns at no load $= 0.8Hl = 0.8 \times 6700 \times 0.5 = 2680$.

Average flux density at full load $= \dfrac{3.6 \times 10^6}{517} = 7.0$ kilolines per square centimetre.

Ampère-turns at full load $= 0.8 \times 7000 \times 0.5 = 2800$.

MAGNET CORE

This is of circular cross-section and made of cast steel.

Area of cross-section $= \frac{\pi\ 19^2}{4} = 285$ square centimetres.

Length of core (including pole shoe) $= 19.2$ centimetres.

The dispersion coefficient for this machine was 1.2, though, strictly speaking, it would be greater at full load than at no load, due to an increase in the flux.

Flux density at no load $= \frac{3.45 \times 10^6}{285} \times 1.2 = 14.6$ kilolines per square centimetre.

From Figure 191 the ampère-turns per centimetre of length $= 33$.

Ampère-turns at no load $= 33 \times 19.2 = 630$.

Flux density at full load $= \frac{3.6 \times 10^6}{285} \times 1.2 = 15.2$ kilolines per square centimetre.

Ampère-turns per centimetre of length $= 43$.

Ampère-turns at full load $= 43 \times 19.2 = 820$.

YOKE

Also of cast steel.

Area of cross-section $= 21.6 \times 8.5 = 183$ square centimetres.

Mean length of path taken by flux as measured from drawing $= 44$ centimetres.

Flux density at no load $= \frac{3.45 \times 10^6}{2} \times 1.2 \times \frac{1}{183} = 11.3$ kilolines per square centimetre.

Ampère-turns per centimetre of length $= 13$.

Ampère-turns at no load $= 13 \times 44 = 570$.

Flux density at full load $= \frac{3.6 \times 10^6}{2} \times 1.2 \times \frac{1}{183} = 11.8$ kilolines per square centimetre.

Ampère-turns per centimetre of length $= 15$.

Ampère-turns at full load $= 15 \times 44 = 660$.

The ampère-turns per pole necessary for each component part of the magnetic circuit at no load and full load respectively are set forth in the following table:

Part.	No-load ampère-turns.	Full-load ampère-turns.
Armature core	70	80
Armature teeth	640	850
Air-gap	2680	2800
Magnet core	630	820
Yoke	570	660
Total	4590	5210

When the armature is on open circuit, 4590 ampère-turns are sufficient to provide a flux of 3.45 megalines through the armature and maintain a terminal E.M.F. of 550 volts; but as the load increases the excitation must also be increased in the same proportion, until at full load the ampère-turns necessary to provide a flux of 3.6 megalines, generate a total E.M.F. of 572 volts and maintain a constant terminal E.M.F. of 550 volts, are 5210. These 5210 ampère-turns provide the necessary flux at full load on the assumption that the flux is evenly distributed over the respective cross-sections, but such is only the case when the armature is on open circuit. As is explained later, the magnet coils require to give an additional number of ampère-turns in order to counteract the demagnetising and distorting effects of the armature when the latter supplies current. For this particular machine the field ampère-turns necessary to overcome armature reaction are 660 (as given on page 328), so that the total calculated ampère-turns per pole at full load $= (5210 + 660) = 5870$, as against 4590 at no load.

If the dynamo be shunt-wound, then in practice each shunt coil would be designed to give a maximum of 6000 ampère-turns, thus compensating for any slight increase in the reluctance of the magnetic circuit due to an increase in the length of air-gap or variation in the composition of the iron. Attention is here directed to the fact that of the total ampère-turns required at full load nearly 50 per cent. are utilised in overcoming the air-gap reluctance.

Calculation of Shunt Coils.—There are several useful methods available for calculating the size of wire required for the shunt field coils, two of which are given below:

Method 1.—To determine the size of wire required when the ampère-turns, volts per coil, and the mean length of one turn are known—

Let ρ denote the specific resistance of copper in ohms per centimetre cube, CT the ampère-turns per coil, V the available volts per coil, and L the mean length of one turn. The resistance of each coil is expressed by

$$R=\frac{V}{C}=\frac{\rho\times L\times T}{a}$$

where—

T = turns per coil.
a = area of cross-section of the wire.
C = current in coil in ampères.

Now $a=\frac{\pi d^2}{4}$, so that $\frac{V}{C}=\frac{\rho\times L\times T\times 4}{\pi d^2}$.

Therefore the diameter of the wire in centimetres is expressed by $d=\sqrt{\frac{4\times\rho\times L\times CT}{\pi V}}$.

Example.—Apply this formula to the above shunt winding, which has to give 6000 ampère-turns at full load. The bobbins on which the coils were wound were of such dimensions that the internal diameter of the winding was 20 centimetres, and the radial depth of winding 5 centimetres.

Mean diameter of winding $=20+5=25$ centimetres.

Mean length of one turn $=\pi\times 25=79$ centimetres.

Suppose that at full load 30 volts are absorbed by the shunt regulating resistance (R, Figure 182), then the available volts per coil $=\frac{550-30}{4}=130$.

The specific resistance of copper at 60° C. (*i.e.* the probable working temperature of the coil) equals 0.000002 ohms per centimetre cube.

The diameter of wire then

$$=d=\sqrt{\frac{4\times 0.000002\times 79\times 6000}{\pi\times 130}}=\sqrt{0.0093}$$
$$=0.096 \text{ centimetre.}$$

From Appendix I. it will be seen that No. 20 S.W.G., having a diameter of 0.0915 centimetre, is the nearest size of wire to that found by calculation. The wire in this case was triple-cotton covered, thus giving an overall diameter of 0.122 centimetre.

The shunt winding spools for this machine had an axial length of 17 centimetres, so that the turns per layer $=\frac{17}{0.122}=140$, and since the radial depth is 5 centimetres, the number of layers $=\frac{5}{0.122}=41$; say 43 after allowing for bedding.

The total turns on bobbin $=140\times 43=6000$.

Therefore the shunt current $=\frac{6000}{6000}=1$ ampère.

The resistance of a coil should now be calculated, and should approximately be equal to

$$\frac{V}{C}=\frac{130}{1}=130 \text{ ohms at } 60^\circ \text{ C.}$$

Calculated resistance

$$=R=\frac{\rho\times l\times 4}{\pi d^2};\ \text{but } l=L\times T=79\times 6000,$$

so that $R=\frac{0.000002\times 79\times 6000\times 4}{\pi\times 0.0915^2}=140$ ohms.

Had the resistance of the winding thus calculated been greater than that given by $\frac{V}{C}$ by say more than 8 per cent., the coil would have required to have been wound with a larger size of wire.

Method 2.—Another method largely used in practice is to make use of the formula

$$W=\frac{0.000176\times L^2\times(CT)^2}{K},$$

where—

W = Watts per shunt spool at 60° C.
L = mean length of one shunt turn in metres.
CT = ampère-turns per spool.
K = kilogrammes of copper per shunt spool.

This formula is derived as follows:

Let l = length of wire, in metres, forming the winding.
s = area of cross-section of wire in square centimetres.
ρ = specific resistance of copper at 60° C.,

then the resistance of the winding is expressed by

$$R = \frac{\rho \times l \times 100}{s} = \frac{0.000002 \times l \times 100}{s},$$

$$i.e.\ s = \frac{0.0002\ l}{R}.$$

Multiplying both sides the equation by l

$$sl = \frac{0.0002 \times l^2}{R}.$$

Further, let L = mean length of 1 turn in metres,
T = number of turns in the winding,
C = current in ampères,

then

$$l = LT,$$

$$\text{and } sl = \frac{0.0002 \times L^2 \times T^2}{R} = \frac{0.0002 \times {}^2L \times C^2 \times T^2}{C^2R}.$$

Now $sl \times 100$ = volume of copper in cubic centimetres, and $\frac{sl \times 100 \times 8.9}{1000}$ = mass per shunt spool in kilogrammes = K.

C^2R = watts per shunt spool at 60° C. = W.
CT = ampère-turns per shunt spool.

Therefore

$$K = \frac{sl \times 100 \times 8.9}{1000} = \frac{0.0002 \times L^2 \times (CT)^2 \times 100 \times 8.9}{1000\ W}$$

$$i.e.\ W = \frac{0.0002 \times L^2 \times (CT)^2 \times 100 \times 8.9}{1000 \times K}$$

$$= \frac{0.000178 \times L^2 \times (CT)^2}{K},$$

The procedure for calculating the size of wire for the field winding of the above example is as follows:

Cross-section of shunt spool winding =
radial depth × axial length $= 5 \times 17 = 85$ square centimetres.

Space factor of shunt spool = (say) 0.5.

Cross-section of copper in shunt spool $= 85 \times 0.5 = 42.5$.

Volume of copper in shunt spool $= 42.5 \times L = 42.5 \times 79 = 3350$ cubic centimetres.

Since 1 cubic centimetre of copper = 0.0089 kilogrammes, the kilogrammes of copper in shunt spool $= 3350 \times 0.0089 = 29.5$.

Watts per shunt spool

$$= \frac{0.000178 \times 0.79^2 \times 6000^2}{29.5} = 134 \text{ watts.}$$

Ampères per shunt spool (assuming as before that 30 volts are absorbed by the shunt-regulating resistance)

$$= \frac{\text{watts per spool}}{\text{volts per spool}} = \frac{134}{130} = 1 \text{ ampère.}$$

$$\text{Turns per shunt spool} = \frac{6000}{1} = 6000.$$

Sectional area of copper per turn

$$= \frac{\text{cross-section of copper on shunt spool winding}}{\text{turns per spool}}$$

$$= \frac{42.5}{6000} = 0.0071 \text{ square centimetre.}$$

Diameter of bare wire = 0.095 centimetre.

Thus the size of wire determined by this method is the same as that determined by the first.

ARMATURE REACTION

In order to get sparkless commutation the brushes require to be placed in such a position that the armature turns between the commutator segments under the brushes are approximately on a geometrical neutral axis, *i.e.* a radius of the armature which bisects the angle contained by the radial axes of two adjacent poles, as represented by the line *ob* (Figure 199). It is necessary to distinguish this geometrical neutral axis from the axis of commutation, which is a radius of the armature passing through points of contact of the brushes, as indicated by the line *COC* (Figure 199). In the foregoing diagrams, for the sake of simplicity of statement, the armature inductors are represented as being placed equidistantly apart upon the armature core and connected radially to their commutator segments. In actual drum armatures the connections to the commutator segments are made at the centre of the end connections (see Figures 147 and 148), so that the true axis of commutation, in a machine whose number of poles is $2p$, is displaced $\frac{90^\circ}{2p}$ relative to the brushes, but in both 2-pole and multipolar machines the axis of commutation refers to the position of the coils undergoing commutation, and not to the corresponding position of the brushes.

Figure 195 represents the armature of a 2-pole dynamo in which the axis of commutation is coincident with the geometrical neutral axis. When the armature supplies current the inductors on either side of the brushes will have currents flowing in them in opposite directions. The inductors marked with a dot are supposed to carry current flowing towards the reader, while those marked with a cross carry current in the opposite direction.

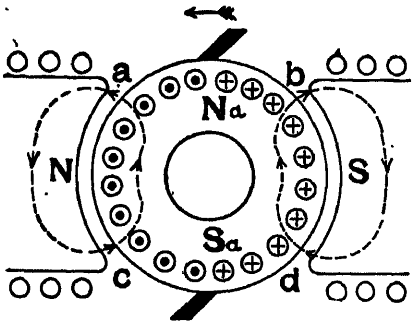


FIG. 195.—Magnetic field due to armature.

Each inductor may be imagined as connected across—in a plane at right angles to the plane of the geometrical neutral axis—to a corresponding inductor on the opposite side of the armature. The latter becomes an electro-magnet, the coils of which magnetise the core in a direction parallel to the plane of the geometrical neutral axis, as indicated by N*a* and S*a*, which extend along the entire length of the core. The lines of force set up by the armature pass through the core on either side of the neutral axis, and emanating from the upper part pass through the poles and enter again at the lower part, as shown in the figure.

Cross Ampère-turns.—The path taken by the magnetic flux set up by the field magnets is shown in Figure 196, from which it will be observed that at *a* and *d* the lines of force in the air-gap due to the armature current are in the opposite direction to the field magnet lines of force, while at *b* and *c* they are in the same direction. If the arrow indicates the direction of rotation of the armature, then the pole tips *a* and *d*, which are approached by the armature inductors as they move through the interpolar gap, are referred to as the *leading pole tips*, to distinguish them from the *trailing pole tips c* and *b* from which the inductors recede when entering the interpolar space. By superimposing the two magnetic fields, the combined field shown in Figure 197 is obtained, which represents the resultant distribution of flux as actually occurring in a dynamo supplying current.

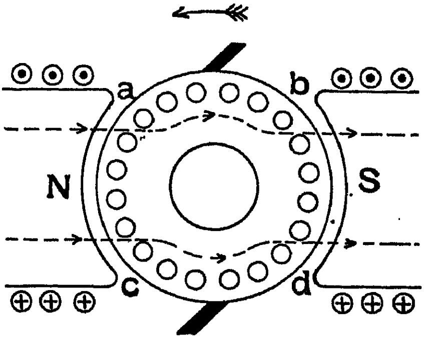


FIG. 196.—Magnetic field due to field magnets.

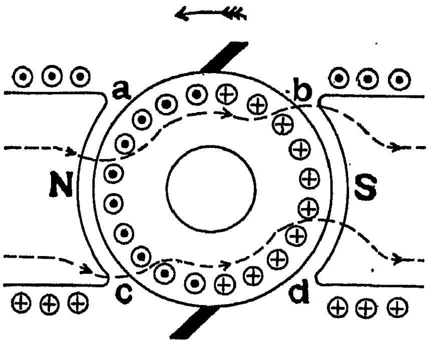


FIG. 197.—Magnetic field of a dynamo distorted by armature field.

The result of the reaction of the armature current is that the main flux is distorted in the direction of rotation, and the flux density in the air-gap under the trailing pole tips is increased, while the flux density in the air-gap under the leading pole tips is diminished. Since the flux produced by the armature current is in a direction at right angles to the main flux it is termed a *cross-flux*, and the ampère-turns which produce it are known as the cross ampère-turns of the armature. In Figure 198 the full-line curve shows the distribution of flux in the air-gap of a 4-pole machine when the armature current was zero, and the dotted curve shows the flux distribution for the same machine when the armature carried the full-load current, the ordinates of the curve representing the strength of the magnetic flux.

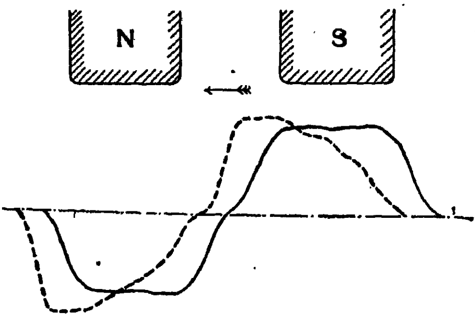


FIG. 198.—Distortional effect of armature.

From these curves it will be observed that the plane of neutral flux has become distorted, so that in order to obtain perfect commutation the brushes require to be moved forward through an angle θ (Figure 199) in the direction of rotation. The angle between the axis of commutation and the geometrical neutral axis is known as the *angle of lead* or *lag*, according as the angular displacement of the brushes is coincident with, or opposite to, the direction of rotation. In a dynamo the brushes are given lead, while in a motor they are given lag, or backward lead as it is sometimes called.

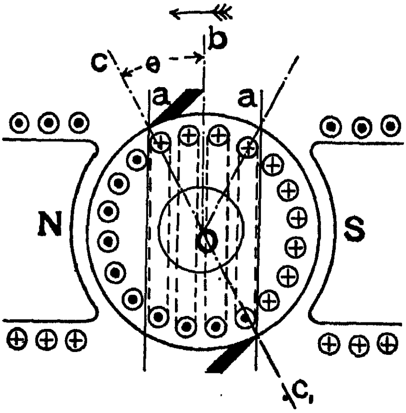


FIG. 199.—Demagnetising and cross ampère-turns of 2-pole armature.

Demagnetising Ampère-turns.—The effect of giving lead to the brushes of a dynamo can be studied from Figure 199, the direction of the currents in the armature and field coils being indicated by the crossed and dotted circles. The inductors can be divided into two groups by the lines *a a*, which are drawn perpendicular to the direction of the magnetic flux of the field magnets and pass through the forward tips of the two brushes. The angle *a o a* therefore equals twice the angle of lead, *i.e.* 2θ. Each group of inductors can be imagined as connected together across the ends of the armature so as to form two sets of windings, the imaginary end connections of the coils within the zone *a a* alone being shown.

The ampère-turns included between the lines *a a* act in the opposite direction to the ampère-turns on the field coils, and as they neutralise to a certain extent the action of the field-turns, they are known as the ***demagnetising*** or ***back ampère-turns*** of the armature. The second zone of ampère-turns, made up by the inductors outside the zone *a a*, have simply a cross magnetising effect, as is the case with all the ampère-turns when the neutral and commutation axes are coincident. Should the brushes be given an angle of lead of 90 degrees, the armature will be incapable of generating current; but if in this position current be sent through the armature, the effect of armature reaction would be purely demagnetising, there being no component tending to cause distortion.

Armature Reaction in Multipolar Machines.—Multipolar dynamos can similarly have their armature ampère-turns divided into groups, each group either having a distorting or demagnetising effect on the main field, as shown in Figure 200, where the brushes are at an angle θ in advance of the geometrical neutral axis OA.

The value of the demagnetising ampère-turns of the armature per pole is found by multiplying half the number of inductors within the angle $2\,\theta$ by the current flowing in each, and is expressed by

$$\mathrm{AT} = \frac{\mathrm{I}}{q} \times \frac{\mathrm{C}}{2} \times \frac{2\theta}{360},$$

where—

I = full-load armature current,
q = number of circuits through armature,
C = total number of armature inductors.

Similarly with an angle of lead θ, the value of the cross ampère-turns of the armature

$= \frac{I}{q} \times \frac{C}{2} \times \left(\frac{360^\circ}{2p} - 2\theta\right)$, where $2p$ is the number of poles.

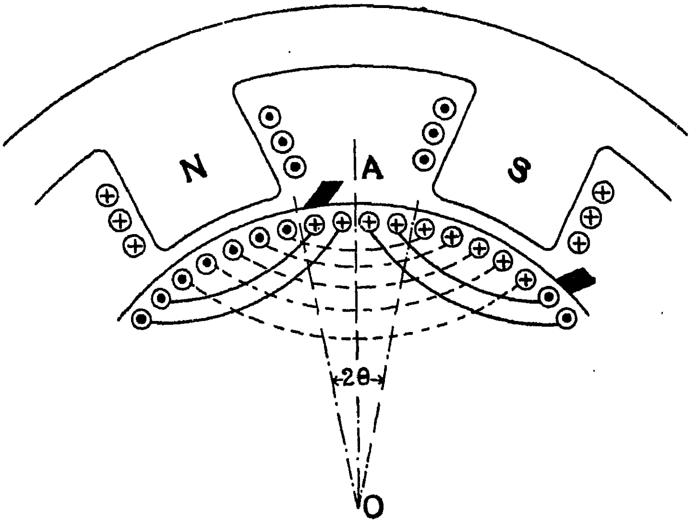


FIG. 200.—Demagnetising and cross ampère-turns of a multipolar armature.

Field Ampère-turns to overcome Armature Reaction —The demagnetising ampère-turns which are set up b the armature must be counteracted by an equal numbe of ampère-turns on the field coils.

In dynamos where the armature teeth are nearl saturated, the effect of distortion is to decrease the pe meability of the teeth, so that the main flux is diminishe and consequently the induced E.M.F. of the armatur The field ampère-turns must therefore be furthe increased to compensate for the diminution in th effective flux passing through the armature resultin from armature reaction. The increased field ampèr turns to overcome this distortion effect are rather difficult to calculate, but experiments have shown that they are a function of the total distorting ampère-turns and the ampère-turns for the teeth and air-gap.

Let D denote the total distorting ampère-turns per pole, F the field ampère-turns for overcoming the latter, and G the ampère-turns required for the teeth and air-gap. As the results of tests on various types of machines it has been found that, for the usual values of flux density in the armature teeth and air-gap, if the ratio F/D be plotted as a function of the ratio D/G, the points will lie approximately along the curve shown in Figure 201. This curve can therefore be employed for determining the value of F.

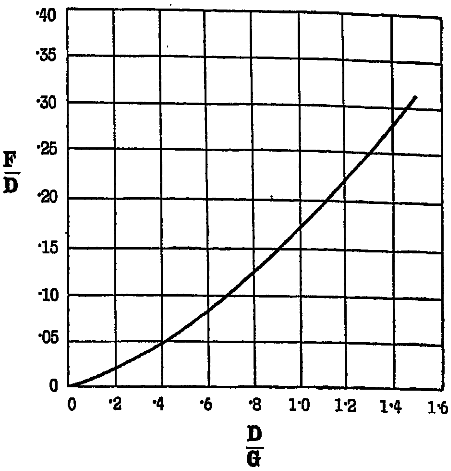


FIG. 201.—Curve for calculating the influence of distorting ampère-turns.

Example. — The brushes of the 33-k.w. dynamo previously referred to have a fixed lead of 7 degrees. It is required to find the number of ampère-turns per pole to counteract the effects of armature reaction.

Full-load current (I) = 60 ampères.

Number of circuits in parallel (q) = 2.

Total number of inductors (C) = 724.

The demagnetising ampère-turns per pole at full load

$$= \frac{60}{2} \times \frac{724}{2} \times \frac{14}{360} = 430.$$

Additional ampère-turns per pole at full load to overcome armature demagnetising action = 430.

The total ampère-turns per pole at full load

$$= \frac{60}{2} \times \frac{724}{2} \times \frac{1}{4} = 2700.$$

Hence the demagnetising ampère-turns of armature

$$= 430 \times \frac{100}{2700} = 16 \text{ per cent.}$$

Distorting ampère-turns per pole at full load
$= D = 2700 - 430 = 2270 = 84$ per cent. of total.
Ampère-turns required for teeth at full load
$= 800$ (*vide* p. 315).
Ampère-turns required for air-gap at full load
$= 2950$ (*vide* p. 315).
Ampère-turns required for teeth and air-gap
$= G = 800 + 2850 = 3650$.
The ratio $D/G = 2270/3650 = 0.63$.
From Figure 201 the ratio $\frac{F}{D} = 0.1$.

The field ampère-turns to overcome distortion
$= F = 0.1 \times D = 0.1 \times 2270 = 230$.

Therefore total field ampère-turns to overcome armature reaction $= 430 + 230 = 660$.

From this example it will be seen that the armature reaction increases the amount of copper required in the field-magnet coils, and also the amount of iron in the magnetic circuit, owing to the resultant increase in the length of the magnet core; this in turn tends to increase the amount of magnetic leakage, and therefore the cross-section of the magnetic circuit.

In modern dynamos it is required that they shall deliver, without undue sparking, any load up to a percentage considerably in excess of their full-rated output with the brushes in a fixed position. To satisfy this requirement the brushes are set with a permanent angle of lead varying, according to the output of the machine, from 6 to 15 per cent. of the angular distance between two successive geometrical neutral planes. The method of determining the necessary angle of lead of the brushes is discussed when dealing with commutation, and within the limits given above the demagnetising ampère-turns vary between 12 and 30 per cent. of the total. Until a few years ago it was looked upon as a matter of course that the brushes of any dynamo should be shifted forward in proportion to the load; but a better understanding of the theory of commutation has made it possible to design machines in which the above requirements as to fixed brush position can be adhered to.

Methods of reducing Armature Reaction.—The distortion of the magnetic field can be reduced by increasing the length of air-gap and by so dimensioning the teeth and slots that the former are highly saturated by the field flux. This procedure, besides increasing the reluctance in the path of the cross-magnetic flux of the armature, also increases the reluctance offered to the main flux, so that the main ampère-turns must be augmented. This will then make the field ampère-turns strong in comparison with those of the armature, with the result that the distortional effect of the armature is reduced. There is, however, a limit to the length of air-gap which is reached when the increased cost of copper on the field magnets outweighs the advantage of further reduction in the distortion effects.

In Figure 195 it was shown that the cross-flux due to the armature passes through the poles in a direction at right angles to the main flux; consequently a high reluctance may be introduced in the path of the cross-flux by forming a long narrow slot, parallel to the axial length of the magnet core, along its centre, as shown at A in Figure 202. This will have but a small effect on the main flux, while an additional air-gap is introduced into the path of the cross-flux.

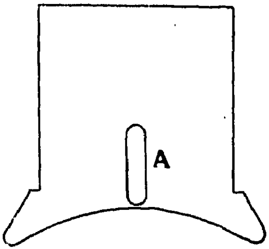


FIG. 202.

The best method of counteracting armature reaction is by means of a compensating winding on the main poles, which acts in the opposite direction to the cross ampère-turns of the armature. This is dealt with in the latter portion of this chapter.

Number of Poles required for a Machine of given output.—As a consequence of armature reaction it becomes not only desirable but necessary to limit the armature strength to such an amount at full load as shall not greatly interfere with the distribution and amount of the magnetic flux set up by the field magnets. With such values of flux density and air-gap length as have been previously recommended, experience shows that the best results are obtained from machines in which the armature strength at full load does not exceed from 5500 to 9000 ampère-turns per pole, when the output ranges from 100 k.w. or less, to 1500 k.w. respectively.

If T be the number of turns per circuit through the armature, and I the total current output of the machine, then in a lap-wound armature T equals the number of armature turns per pole, the number of circuits through the armature being the same as the number of poles. Further, if p denote the number of pairs of poles, the current flowing in each circuit of the armature equals $\frac{I}{2p}$, and therefore $T \times \frac{I}{2p}$ gives the number of ampère-turns per pole. Let AT_a denote the maximum permissible value of the armature ampère-turns at full load, then the number of pairs of poles required for a lap-wound armature is given by

$$T \cdot \frac{I}{2p} = AT_a.$$

From the E.M.F. equation $T = \dfrac{E \times 10^8}{4 \times \frac{Rp}{60} \times M}$, so that

$$\frac{E \times 10^8}{4 \times \frac{Rp}{60} \times M} \times \frac{I}{2p} = AT_a.$$

Let d and l denote the diameter and length of armature respectively, and r the ratio of the length of polar arc to polar pitch; then the total pole face area

$$= \pi d \times r \times l.$$

Further, if B_a denote the average flux density in the air-gap at full load, then M the flux per pole entering the armature is expressed by

$M = \frac{\pi d \times r \times l \times B_a}{2p}$. Substituting this value of M in the above equation

$$\frac{E \times 10^8 \times 2p \times 60}{4 \times R \times p \times \pi d \times r \times l \times B_a} \times \frac{I}{2p} = AT_a.$$

The number of pairs of poles is therefore expressed by

$$p = \frac{E \times I}{R \times d \times l} \times \frac{1}{B_a \times AT_a \times r} \times \frac{10^8 \times 60}{4\pi}.$$

In low and medium speed machines the flux density B_a in the air-gap does not usually exceed 8500 lines per square centimetre, and the ratio of polar arc to polar pitch is in most cases equal to 0.7, so that substituting these values of B_a and r in the above equation

$$p = \frac{E \times I}{R \times d \times l} \times \frac{1}{AT_a} \times \frac{10^8 \times 60}{8500 \times 0.7 \times 4\pi}$$
$$= \frac{E \times I \times 80000}{R \times d \times l \times AT_a}.$$

From this equation it will be observed that the number of poles is inversely proportional to $(d \times l)$, the component parts of which may be determined in a manner discussed on page 447.

In order to keep the armature iron losses in turbo-driven dynamos within reasonable limits, the flux density in the air-gap is limited to about 7000 lines per square centimetre, and as the average ratio of pole pitch to polar arc may be taken as 0.6, the number of pairs of poles is expressed by

$$p = \frac{E \times I \times 110000}{R \times d \times l \times AT_a}.$$

Example.—A dynamo has to be designed to give an output of 440 ampères at 500 volts (*i.e.* 220 k.w.). If the armature is 90 centimetres in diameter, 30 centimetres in length, and makes 380 revolutions per minute, determine the number of poles.

For this size of machine the armature ampère-turns per pole (AT_a) should be limited to about 6000, so that the number of pairs of poles is approximately given by

$$p = \frac{500 \times 440 \times 80000}{380 \times 90 \times 30 \times 6000} = 2.9.$$

Taking the nearest integer $p = 3$, *i.e.* the machine will have 6 poles.

COMMUTATION

Commutation is that process whereby the current in an armature coil, as it passes from one side of the axis of commutation to the other, is reduced to zero, and a reversed current built up equal in value to the current in the circuit of which it is about to become a part. This process must take place in such a manner that no appreciable sparking occurs between the brushes and commutator segments.

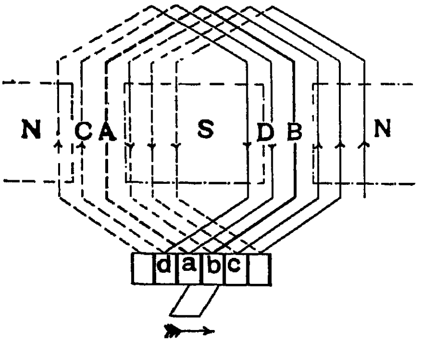


FIG. 203.

In Figure 203 is shown a portion of a lap-winding. For the sake of simplicity it is assumed that the width of contact of the brushes on the circumference of the commutator is equal to the width of a segment. Thus each brush only short circuits one armature coil at a time. In an actual machine the brushes have an arc of contact sufficient to bridge over three or four segments. The armature rotates from left to right as indicated by the arrow, so that when the commutator segments *a* and *b* are on the left-hand side of the brush the current in the coil AB is flowing through it towards the brush, as indicated by the arrow-heads on the coil CD. Once in every revolution the coil AB will be short-circuited under the brush, and when the segment B passes from underneath it the coil thus short-circuited will be carrying a current in the reverse direction, but still flowing towards the brush. During the small interval of time in which a coil is under the brush the previous existing current should diminish to zero, and a reversed current of equal strength be established. This change in the direction of the current is depicted in Figure 204; the width of the shaded part represents the time during which the coil is short-circuited under the brush, and the ordinate 2A the change in the value of the current.

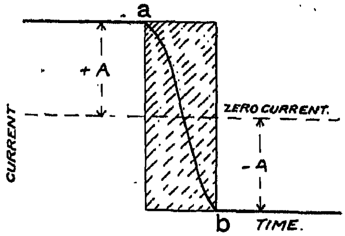


FIG. 204.—Diagram of perfect commutation.

Reactance Voltage.—When the current flowing in a circuit is suddenly altered a change is produced in the number of lines of force linked with that circuit, and any change in the number of lines of force is accompanied by an induced E.M.F. This induced E.M.F. tends to oppose the change in the value of the current, and is known as the E.M.F of self-inductance. The phenomenon as a whole is called self-induction. The number of lines of force linked with a circuit when carrying unit current is called the *coefficient of self-inductance.* The henry or practical unit of self-inductance equals 10^9 absolute units.*

From this it will be seen that the reversal of the current in the short-circuited coil is accompanied by a change in the magnetic flux linked with the coil, and consequently an electromotive force is induced in it which tends to oppose the reversal of the current. This counter E.M.F. is termed the reactance voltage, and is expressed by the formula

$$V = 2\pi n L C$$

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* Those wishing to further study the subject of self-induction are referred to text-books on Alternating Currents.

where—

n = periodicity of reversal (*i.e.* the number of complete cycles per second).

L = inductance (in henrys) per coil or commutator segment.

C = current per circuit.

This formula is based upon the assumption that the current in the short-circuited coil is a sine function, *i.e.* the curve $a\ b$ (Figure 204) is supposed to represent half a sine curve, and the current is treated as reversing like an ordinary alternating current. Such an assumption is probably far from correct, but nevertheless it affords a basis upon which all machines may be similarly calculated, and its value has been found to be a sufficiently good criterion for the quality of commutation. The shape of the current and time curve is influenced by such considerations as shaping of the pole tips and armature interference, but up to the present there is no reliable experimental data on this subject.

For determining the value of n, let s denote the peripheral speed of the commutator in metres per second, l the length of the arc of brush contact in centimetres: then the time during which any one coil is short-circuited by the brush $= \frac{l}{100\,s}$ seconds. During this time the current in the coil goes through one-half of a complete cycle (see Figure 204), so that the average periodicity of reversal in complete cycles per second is expressed by

$$n = \frac{100\,s}{2l}.$$

The inductance (L) per coil is given by the product of the number of lines of force (M), per ampère of current, linked with each turn, and the number of turns (t) squared, *i.e.*

$$L = M\ .\ t^2/10^8 \text{ henrys.}$$

With a view to forming a basis for estimating the value of L, Hobart* performed a series of experiments, and with present-day types of machines, where the depth of a slot is about three times the width, has shown that there will be set up on an average (1) 4 lines of force per ampère per centimetre of *embedded length*, and (2) 0.8 lines of force per ampère per centimetre of *free length*. The embedded length is that portion of the coil lying in a slot, and the free lengths are the end connections, and included with them should be those portions of the inductors not strictly iron-clad, *i.e.* the length of inductor corresponding to ventilating ducts and insulation between laminations.

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* *Journal of Institution of Electrical Engineers*, vol. xxxi. pp. 170–217 (1901).

If there be m turns short-circuited under one brush,

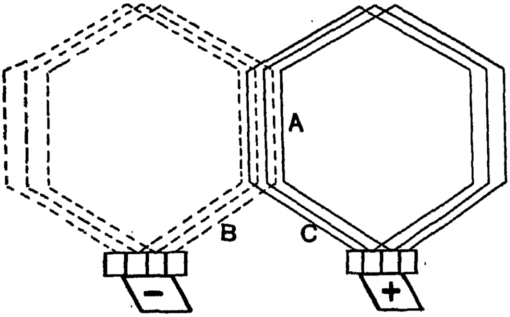


FIG. 205.

then the lines of force per centimetre of embedded length, linked with any one of the short-circuited coils, will be due to $2m$ turns; whereas the lines of force per centimetre of free length are set up by m turns. That this is so, will be evident from Figure 205. In the embedded portion, the inductors short-circuited under the positive brush lie near to or in the same slot as those inductors simultaneously short-circuited by the negative brush, as shown at A; consequently there will be a mutually inductive action. The two corresponding groups of end connections B and C are located separately. The method of calculating L is set forth in the following example.

Example.—Calculate the reactance voltage at full load of an armature having the following constants:

Diameter of commutator = 0.45 metres.
Speed = 660 R.P.M.
Number of commutator segments = 357.
Length of arc of brush contact = 0.65 centimetre.
Mean length of one turn = 152 centimetres.
Effective length of core = 14.8 centimetres.
Turns per commutator segment = 1.
Full load current = 60 ampères.
Number of armature circuits = 2.

Peripheral speed of commutator (s)

$$= \pi \times 0.45 \times \frac{660}{60} = 15.5 \text{ metres per second.}$$

Periodicity of reversal (n)

$$= \frac{100 \times 15.5}{2 \times 0.65} = 1190.$$

Embedded length per turn
= 2 × effective length of core $= 2 \times 14.8 = 29.6$ centimetres.

Free length per turn
= mean length of one turn $- 29.6 = 152 - 29.6 = 122.4$.

Lines per ampère-turn of embedded length
$= 29.6 \times 4 = 118$.

Lines per ampère-turn of free length
$= 122.4 \times 0.8 = 98$.

Maximum number of coils short-circuited under one brush

$$= \frac{\text{Length of arc of contact}}{\text{Width of segment} + \text{insulation}}$$

$$= \frac{0.65}{\frac{\pi \times 45}{357}} = 1.65.$$

Number of turns short-circuited per brush
$= 1.65 \times 1 = 1.65$ (taking nearest integer) = 2.

Lines per ampère for embedded length
$= 118 \times 2 \times 2 = 472$.

Lines per ampère for free length
$= 98 \times 2 = 196$.

Total lines linked with the short-circuited coil per ampère $= 472 + 196 = 668$.

Inductance per segment (L)

$$= \frac{668}{10^8} = 0.00000668.$$

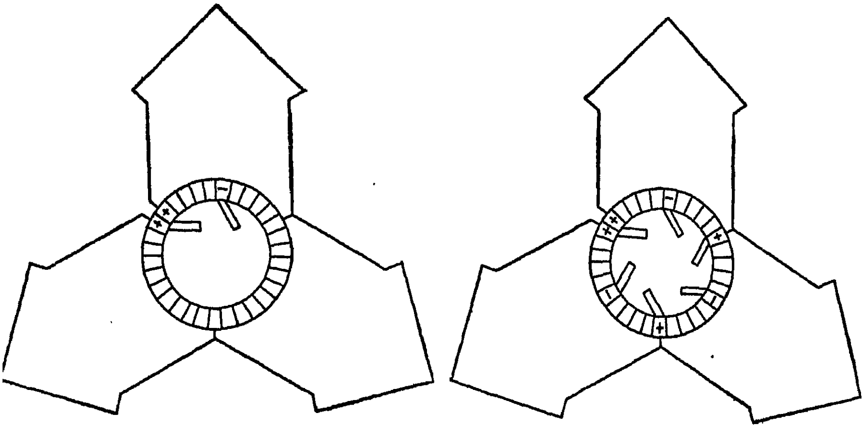
Current per circuit (C)

$$= \frac{60}{2} = 30 \text{ ampères.}$$

Reactance voltage

$$= V = 2\,\pi\,n\,CL$$
$$= 2\,\pi \times 1190 \times 0.00000668 \times 30$$
$$= 1.5 \text{ volts.}$$

Reactance Voltage of a 2-Circuit Winding.—The value of the reactance voltage of a 2-circuit winding depends upon the number of sets of brushes employed. To illustrate the case of a 6-pole armature, consider Figure 206, which is a diagram showing those inductors, of the winding represented in Figure 147, temporarily undergoing short-circuit at the positive brush. The coil short-circuited by the positive brush consists of 3 turns (*i.e.* 6 inductors) in series, and if V denote the reactance voltage per turn the reactance voltage per coil equals 3V.



FIGS. 206, 207.—Reactance voltage of 2-circuit windings.

Next suppose that the commutator is fitted with as many sets of brushes as there are poles, *i.e.* 6. Under such conditions each pair of inductors is short-circuited by the cable which connects brushes of similar polarity, as shown in Figure 207. There are now three independent circuits, each consisting of one turn, and the reactance voltage per commutator segment is reduced to V; consequently the commutation will be much better than that obtained when only two sets of brushes are fitted.

Reversal of the Current by a Reversing E.M.F.—In order to effect sparkless commutation the brushes may be set in such a position that during the time the coil AB (Figure 203) is short-circuited it is cutting a magnetic flux in such a direction that a reversed E.M.F. is induced in it: this latter E.M.F. should be sufficient to first reduce to zero the current previously existing in the coil, and then—still in opposition to the reactance voltage—to induce a current in the coil in the opposite direction, which at the instant the short circuit is removed shall have attained a strength equal to that of the current in the circuit which the coil is about to enter.

At no load there will be no current to reverse, and hence no reactance voltage to overcome, so that perfect commutation is obtained when the short-circuited coil is in neutral field, *i.e.* when the axis of commutation is coincident with the geometrical neutral axis. But when the armature supplies current it is necessary to move the brushes forward by an amount depending on the load, so that the short-circuited coil shall be cutting the fringe of the flux in the air-gap under the leading pole. (In Figure 203 the coil AB is shown short-circuited when nearer to the leading pole than to the trailing pole, *i.e.* the short-circuited coil is entering the fringe of the former's field.)

It will now be clear that as the armature current increases a stronger field will be necessary to commutate this stronger current and overcome the correspondingly greater reactance voltage. But, as has already been explained, this stronger current flowing through the armature distorts the neutral magnetic field into a position in advance of the brushes, so that in order to bring the short-circuited coil into a sufficiently strong reversing field the brushes require to be given a greater angle of lead than that which would be necessary if there were no distortion. This increased angle of lead will in turn increase the demagnetising effect of the armature, so that finally a load will be reached at which sparkless commutation becomes impossible, owing to the absence of a sufficiently strong reversing field to commutate the current.

Position of the Brushes.—With machines of present-day design, the practice of permitting different brush positions for different loads has been abandoned, and there must be a fixed brush position at all loads. With *electro-magnetic* commutation described above, perfect commutation can only occur at some intermediate value of the load. The usual practice is to set the brushes with as much forward lead as is possible, without incurring sparking at no load, and to limit the output of the machine, if thermal considerations permit, to that load which does not produce sparking when the brushes are in this position.

At no load, half load, and full load the variation of current in the short-circuited coil will be somewhat as shown in Figure 208. At no load the reversing field will build up an induced current to the strength represented by $-C$, with the result that as the segment leaves the brush the current falls abruptly to zero. This sudden cessation of current in an inductive circuit sets up an E.M.F. between the brush and commutator segment, which tends to produce a spark at the point of separation. This latter E.M.F. may be called the *sparking E.M.F.*, and should not be confounded with the reactance voltage already referred to.

At half load the tendency to spark is eliminated, as the electromotive force set up by the reversing field is of just sufficient strength to reverse the direction of the current and overcome the reactance voltage.

At full load the coil is in a weak reversing field—the latter having been distorted due to the increased armature reaction—so that as the segment is about to leave the brush the reversed current has not attained a strength equal to that of the circuit which the coil is about to enter. Hence, at the moment of leaving the brush there is an abrupt change in the current from a value $-B$ to a value $-2A$, which in turn gives rise to a sparking E.M.F., and the former should be of such a strength that the latter does not produce appreciable sparking.

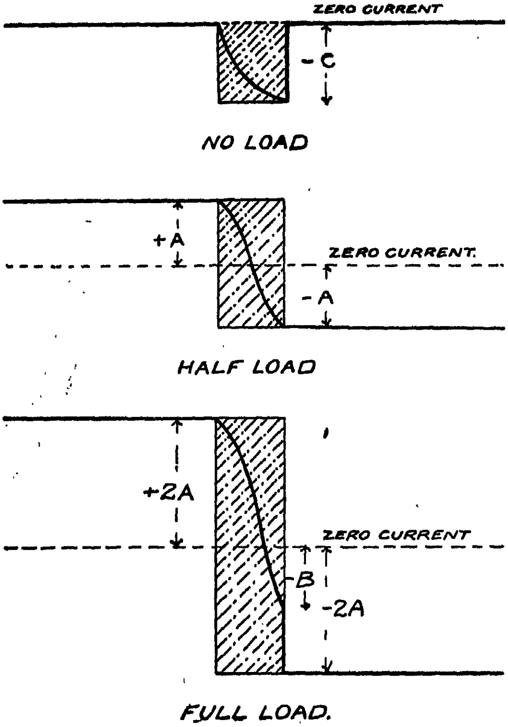


FIG. 208.—Diagram of commutation.

It must be understood that the variation of current in the short-circuited coil may not be exactly as shown, but, in the absence of more definite data, these diagrams may convey to the student an idea of what is likely to occur.

From this reasoning it will be clear that the less the inductance of each coil the greater can be the change in the strength of the short-circuited current at the instant the segment passes from underneath the brush, and consequently the greater will be the limiting output of the machine in so far as sparking is concerned. Now the inductance is proportional to the square of the number of turns, so that for low-reactance voltages it is necessary to have few turns per commutator segment. For moderate-sized machines one or two turns per commutator segment is considered good practice.

An examination of the various quantities involved in the reactance voltage formula will show the desirability of large diameters and small core lengths from the commutation standpoint. This is due to a combination of several features. With a large diameter a larger number of commutator segments is possible, and, consequently, for a given number of armature inductors a smaller number of turns per segment can be employed, giving a small inductance per coil. Shorter core lengths also tend to a reduction in inductance by reason of the shorter lengths of conductors embedded in the iron.

Since the frequency of commutation is inversely proportional to the length of brush arc, it will be evident that the width of brush should be as large as practicable. This, however, is limited by the number of segments it is permissible to cover. If the brush can only bridge across two segments at a time, then it short-circuits one coil, and the lines linked with the coil are due only to the current flowing in its own turns. If, on the other hand, the brush can bridge over three segments, it short-circuits two coils, and the lines linked with one coil are due to the current flowing in the turns of both coils. Thus the width of a brush is limited in that it is not desirable to cover more than three segments in order to keep down the inductance of the short-circuited turns. In ordinary cases the frequency of commutation ranges from 200 to 800, though in some machines it may be much higher than this, as was the case in the numerical example considered.

When carbon brushes are used, the limiting values of reactance voltage for sparkless commutation may be taken as ranging from 1.5 volts in small machines to 4 volts in machines of 500 k.w. or more; but the liability of a machine to spark will depend greatly upon the type of brush employed. Again, machines of most recent design are fitted with commutation poles, and they may permit of the armature having a reactance voltage as high as 20.

Advantage of a High-Resistance Brush.—Figure 209 is a diagram of connections showing three armature coils A, B, and C, connected by commutator lugs to segments *a*, *b*, *c*, and *d*. The coil B is shown under the brush D, and the direction of the current in the coils on each side of the brush is as indicated by the arrow-heads. Suppose the armature to revolve from right to left. At the instant the coil B is short-circuited it is carrying a current of the value, say, +I, and flowing in the direction shown in the coil C. This current must first be reduced to zero, and the change can be accelerated by short-circuiting the coil with a brush of high-contact resistance. The building up of a reversed current to a value −I is also accelerated by a high-resistance brush in the following manner:

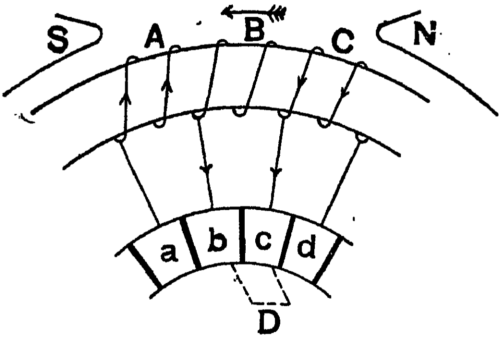


FIG. 209.—Effect of high-resistance brush in forcing commutation.

When the commutator has moved round so that the brush is in the position shown, the area of contact of the latter with the trailing segment *b* is reduced, consequently the increased resistance thereby introduced between the coil A and the brush tends to divert the current in the lug of segment *b* to the alternative path formed by the coil B and segment *c*, *i.e.* the high resistance offered by the diminished area of brush contact tends to reverse the current in the short-circuited coil, or it *forces commutation.*

Owing to the greater contact resistance of carbon over metal, brushes of hard graphitic carbon are now invariably used, although metal brushes are often preferred for high-speed turbine-driven dynamos. A more complete discussion of the relative advantages of metal and carbon brushes is given in Chapter XII., under the heading of Commutator Losses.

With electro-magnetic commutation, where the brushes are adjusted to give perfect commutation at somewhere near half load, carbon brushes tend to counteract the possibility of sparking at no load and full load, due to the excess and deficiency respectively in the strength of the reversed current.

Forced Commutation.—Another procedure is to design a machine to have a low-reactance voltage and resort to forced commutation, *i.e.* where the current does not die down to zero by the action of a reversing E.M.F. in the short-circuited coil, but is merely throttled by the decreasing area of contact between segment and brush, as described above. Carbon brushes must necessarily be used, and they are fixed in positions at the geometrical neutral axes. For forced commutation it is preferable to have as small a leakage field as possible in the interpolar space, so as to diminish the strength of the circulating currents in the short-circuited coils. This can be best obtained by having a narrow air-gap and a polar arc of about 65 per cent. of polar pitch.

At all loads the current will not have reversed and risen to a strength equal to that of the circuit which it is about to enter when the short-circuited coil passes from underneath the brush. The consequence is that an abrupt change in the current takes place, and in all such cases there will be a tendency to produce a spark. The sparking E.M.F., resulting from this sudden change in the current, is, however, curtailed by designing the armature to have a low reactance, and a useful criterion is that the reactance voltage, as determined in the manner previously described, should not exceed from 2 to 2.5 volts.

Commutation Pole Machines

In large dynamos for steam turbine speeds the reactance voltage per segment cannot be maintained within practical limits by the ordinary methods of design. Thus in all such machines where the output was limited by the question of sparking, and not the consideration of temperature, it soon became obvious that to obtain satisfactory commutation, recourse had to be made to some form of compensating winding, either on the armature or field coils, or to auxiliary magnetic poles placed between the main poles.

W. B. Sayers in 1892 proposed the introduction of compensating coils in the armature winding: the coils between any two segments to be so placed that they moved under a main pole, while the armature coil between those segments was undergoing commutation, and had induced in them a voltage equal and opposite to the reactance voltage. Owing, however, to the large amount of inactive copper involved and the somewhat complicated nature of the winding, this method was not adopted to any considerable extent.

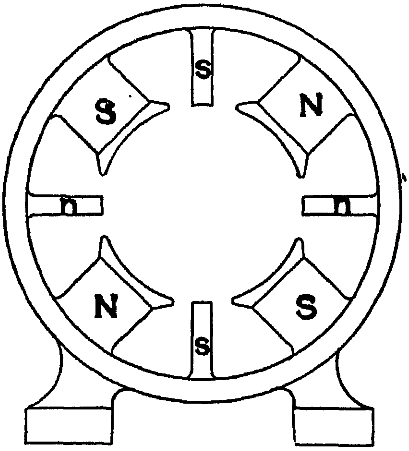


FIG. 210.—Field-magnet frame with commutation poles.

The employment of auxiliary poles has since proved to afford a more satisfactory solution of the commutation problem, and has now a widely extending field of application. The auxiliary poles *n s*, generally called *commutation poles*, are fixed midway between the main poles NS, as shown in Figure 210, and their exciting coils connected in series with the armature, which will rotate in a counter-clock direction. In special cases, where it is not at all times desirable to pass the full main current through the auxiliary coils, the latter are shunted by means of an adjustable low resistance commonly called a *diverter*.

Dimensions of Commutation Poles.—The commutation poles may be either of cast steel or built up of sheet-steel laminations bolted on to a machined seating on the yoke exactly in the centre of the interpolar space, as shown in Figure 210.

Polar Arc.—The length of the polar arc is such that during the whole period a coil is short-circuited by the brush it cuts the flux of the auxiliary pole, and as the short-circuited coil is about to pass from under the brush, it moves in a sufficiently strong reversing field to set up a current equal in strength to the current in the circuit it is about to become a part of.

A machine having a narrow commutation pole arc has the advantage over a machine with a larger pole arc, in that it will have a comparatively small leakage coefficient, and, in addition, there will be a larger space between the auxiliary and main field coils, thus, owing to improved air circulation, a lower temperature rise is ensured. So far as the commutation of the current is concerned, it is essential that the pole arc be not too narrow, otherwise rapid fluctuations in the strength of the auxiliary field will result. That this is the case will be made clear from Figure 211, where the auxiliary pole arc is represented as being equal to the slot width. The magnetic reluctance of the path of the auxiliary flux will vary considerably as the armature teeth T pass under the pole P, being considerably greater in position A than in position B. In the former position the value of the flux will be much less than in the case

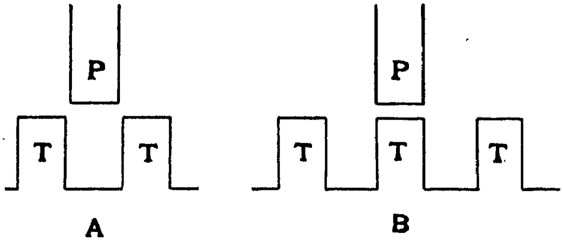


FIG. 211.

of the latter, and part of the coils which are undergoing commutation will lie outside the influence of the auxiliary field. Under such conditions it will be impossible to obtain fair results, particularly with high-speed machines.

These bad features are improved or made worse as the radial depth of air-gap is increased or decreased, the flux variation being less, and the fringing effect greater, the longer the air-gap. As a rule, machines having a very narrow commutation pole arc are liable to hunt or flash over at the brushes, due to the consequent increase in the maximum voltage between adjacent commutator segments which occurs when there is a sudden variation in the load. Even at constant loads commutation will be accompanied by undue sparking, resulting in every third segment or so, according to the number of segments per slot, being blackened. Usually two or three segments are simultaneously under the brush, and the polar arc should cover as many slots as are carrying short-circuited inductors. The best practice is to have a 30 to 40 per cent. larger polar arc than this, and so allow for any distortion of the auxiliary flux which is liable to occur with varying loads.

Professor Arnold recommends a polar arc equal to at least twice a slot pitch. The arc of the commutation pole must not, however, occupy too great a percentage of the interpolar gap, if magnetic leakage, and, consequently, the copper on the auxiliary poles, has to be kept within reasonable limits. On an average the arc of the auxiliary pole occupies between 35 and 40 per cent. of the interpolar arc. The higher the reactance voltage the wider should be the auxiliary pole arc, and the greater the number of teeth covered by the latter.

Pole Shoe Length.—The axial length of the commutation pole shoe will depend upon the value of the reactance voltage, and hence also upon the flux density employed in the air-gap. It is better to reduce the axial length as much as possible, and thereby reduce magnetic leakage. In practice the axial length of the pole shoe ranges from 50 to 75 per cent. of the axial length of the armature core. When the average value of the flux per centimetre of armature periphery has been determined, the length of pole shoe is settled by dividing the value obtained by the flux density most suitable for the conditions. The average flux density in the air-gap under the commutation pole generally ranges from 5000 to 8000 lines per square centimetre.

Area of Cross-section of Pole Core.—From a knowledge of the width of polar arc, axial length of pole shoe, permissible flux density (B) in the air-gap, and the dispersion coefficient for the auxiliary poles, the value of the flux in the pole core can be estimated. Since the reactance voltage increases directly with the armature current, the reversing field due to the auxiliary pole should also increase at the same rate. In order to effect this it is essential that the auxiliary poles do not reach their saturation value before full load is attained, otherwise there will be insufficient reversing E.M.F. and sparking will ensue. The cross-section of the pole core should therefore be dimensioned with a view to fulfilling this condition. To observe this latter consideration the flux density in the pole core at full load should not exceed 14,000 lines per square centimetre. In order to reduce the surface area from which leakage can take place, and also the length of copper per mean turn, it is advisable to have pole cores of circular cross-section. This is, however, not always possible owing to the lack of space.

Flux entering the Air-gap from Commutation Pole.—Let M_1 denote the flux in air-gap per centimetre of armature periphery and S the peripheral speed of the armature in centimetres per second, then the E.M.F. induced in an inductor as it moves under the auxiliary pole $= M_1 \times S \times 10^{-8}$ volts. If the coil short-circuited have t turns in series, then the E.M.F. induced in each short-circuited coil due to the auxiliary flux $= M_1 \times S \times 2t \times 10^{-8}$ volts.

This induced E.M.F. must be sufficient to neutralise the reactance voltage, the mean value of which is expressed by

$$e = \text{reactance voltage} \times \frac{2}{\pi}.$$

(If an alternating E.M.F. varies as a sine function, then its mean or average value = maximum value $\times \frac{2}{\pi}$. For proof the student is referred to a text-book on Alternating Currents.)

The commutation field must be of such a strength that the following equation is satisfied:—

$$e = \mathrm{M}_1 \times \mathrm{S} \times 2t \times 10^{-8},$$
$$i.e.\ \mathrm{M}_1 = \frac{e \times 10^{-8}}{2t \times \mathrm{S}}.$$

The flux leaving the face of each auxiliary pole and entering the air-gap = M = M_1 × width of pole arc.

The flux required to be generated in the pole core = M × dispersion coefficient.

By assigning a suitable value to B, the flux density in the air-gap, the length of *embedded inductor* lying under the auxiliary pole is expressed by $l_1 = \frac{\mathrm{M}_1}{\mathrm{B}}$. The axial length of auxiliary pole shoe = *gross length of inductor* cutting the commutation flux

$$= l = l_1 \times \frac{\text{gross length of armature core}}{\text{effective length of armature core}} \times \frac{1}{1.1}.$$

The constant 1.1 is introduced to allow for fringing at pole shoe ends.

Ampère-turns for Commutation Poles.—The ampère-turns on these poles should be such that they provide at all loads (1) a magneto-motive force of sufficient strength to counteract the armature magneto-motive force causing distortion; and (2) a reversing field of sufficient strength to neutralise the reactance voltage of the coils undergoing commutation, the brushes being fixed in the geometrical neutral axis. The total ampère-turns required for the auxiliary poles is equal to the armature ampère-turns per pole plus the ampère-turns necessary to send the flux through the auxiliary magnetic circuit. The latter consist of the pole core, air-gap, and teeth, directly under the pole, the ampère-turns for which are calculated by the ordinary method. In practice it is customary to allow 30 to 40 per cent. more ampère-turns on the auxiliary winding than the armature ampère-turns per pole, and to experimentally adjust the auxiliary winding on test to give the best commutation results.

In calculating the magnetic circuits of commutation pole machines, it must be remembered that the coefficient of magnetic leakage of the main poles is considerably increased, the reluctance of the leakage paths being greatly reduced, due to the presence of the auxiliary poles; hence the main flux will have a dispersion coefficient of about 1.35 as compared with 1.2 for machines of ordinary design. To reduce this leakage to a minimum, the polar arc is made from 5 to 7 per cent. less. The leakage from the auxiliary poles is even greater than this, and a dispersion coefficient of 1.4 is usually allowed for.

Commutation poles, although essentially the outcome of the high-speed practice which marked the advent of the steam turbine, have been extensively adopted for both high- and low-speed machines of various outputs, and in some cases quite indiscriminately and needlessly. Dynamos should, as a general rule, be fitted with commutation poles when the reactance voltage is greater than 6 volts per commutator segment, and particularly in cases of high peripheral commutator speed, where there is a tendency to spark due to possible vibration of the brushes.

For correctly designed machines of moderate speed and output, having satisfactory commutation constants within the thermal limits of output, there is nothing to be gained by the use of auxiliary poles, which in such cases only add needlessly to the cost, and actually decrease the efficiency.

Example.—From the following data relating to a 1000-k.w. turbo-dynamo, determine the dimensions of the auxiliary pole, and also the flux generated in the latter at full load.

the load and therefore with the cross magneto-motive force of the armature. The current flows through the compensating winding in such a direction that its magneto-motive force opposes the cross ampère-turns of the armature, as will be seen by the crossed and dotted circles in the figure, which shows a section through the poles and armature in a plane at right angles to the axis of the shaft. The ampère-turns of the compensating winding range from 1.2 to 1.3 times the cross ampère-turns of the armature.

Since this compensating winding is arranged opposite the armature surface and distributed over almost the entire air-gap, the cross ampère-turns of the armature are completely neutralised, with the result that there is no distortion of the main magnetic flux. This principle has been applied to turbo-dynamos by several manufacturers.

FLASH-OVER LIMIT IN TURBO-DYNAMOS

The output of slow- and medium-speed dynamos is limited by considerations of temperature rise and commutation, but in turbine-driven machines there is another limitation, namely, the liability of a machine to "flash-over" at the commutator from one set of brushes to another. Troubles due to flashing over have become very serious in the case of high-voltage machines. The flash-over limit is quite distinct from that of sparking, as not a few turbo-generators which had excellent commutation properties have proved to be failures on account of their tendency to arc round the whole commutator. At one time these occurrences were attributed to insufficient insulation between the shrinking rings and the commutator segments. This may in some cases have been the cause, but the more important factor now appears to be an excessive P.D. between adjacent commutator segments.

In slow-speed machines the flash-over limit is about 60 volts per segment, but owing to other considerations the maximum voltage between commutator segments seldom exceeds 50 per cent. of this value. In the case of high-speed turbo-dynamos experience dictates 40 volts to be taken as the safe limit, though in some machines this value has been slightly exceeded. Owing to the small diameter of commutator employed in high-speed machines the number of commutator segments is small, and the safe voltage per segment required by flash-over considerations may quite easily be exceeded. The extent to which the flash-over limit affects the dimensions of the armature, and the value of the flux in the air-gap will now be examined.

When an inductor of length l centimetres moves through a field of intensity B lines per square centimetre at a velocity of v centimetres per second, the E.M.F. induced in that inductor $= Blv10^{-8}$ volts. If l_a denote the length of armature core, $B_{g\,max.}$ the maximum flux density in the air-gap, and v the peripheral speed of the armature in centimetres per second, then the maximum E.M.F. induced in each armature inductor

$$= B_{g\,max.} \times l_a \times v \times 10^{-8} \text{ volts.}$$

In turbo-dynamos there is, as a rule, 1 turn (*i.e.* 2 inductors) per commutator segment, in which case the maximum E.M.F. between two adjacent commutator segments is expressed by

$$e_{max.} = B_{g\,max.} \times l_a \times 2 \times v \times 10^{-8} \text{ volts.}$$

In turbo-driven dynamos the peripheral speed of the armature is, for mechanical considerations, limited to about 75 metres per second. Since the maximum voltage between two adjacent commutator segments must not exceed 40 volts, a speed of 75 metres per second makes

$$B_{g\,max.} \times l_a = \frac{40 \times 10^8}{2 \times 75 \times 100} = 267{,}000.$$

CASE 1.—*Machines with Commutation Poles.*—In the case of machines with commutation poles it may be assumed that, on an average, armature distortion at full load causes $B_{g\,max.}$ to exceed $B_{g\,mean}$ by about 15 per cent., in which case $B_{g\,mean} \times l_a = 267{,}000 \times \frac{100}{115} = 230{,}000.$

7000 lines per square centimetre may be taken as an average value of $B_{g\,mean}$, so that the greatest permissible length of armature, independent of its diameter

$$=\frac{230{,}000}{7000}=33 \text{ centimetres.}$$

In the case of machines with commutation poles, the ratio of pole arc to pole pitch (*i.e.* r) may be taken as 0.6, so that the total number of lines entering or leaving the armature

$$=2pM=\pi d_a \times 0.6 \times B_{g\,mean} \times l_a = 0.43 d_a \times 10^6,$$

where—

p = number of pairs of poles,
M = flux per pole,
d_a = diameter of armature.

It will thus be seen that the flash-over limit restricts the number of lines of force entering or leaving the armature, of a given diameter, to $0.43 d_a \times 10^6$.

Case 2.—*Machines with a Compensating Winding.*—In machines fitted with a compensating winding (for instance, those described on pp. 358 to 361) the armature distorting force is almost neutralised at all loads, with the result that $B_{g\,max.} \simeq B_{g\,mean}$. In such machines, assuming $B_{g\,mean} = 7000$, the permissible length of armature attains a value of $\frac{267{,}000}{7000} = 38$ centimetres. Since the value of r in such machines is about 0.7, the total number of lines entering or leaving the armature is restricted to

$$\pi \times d_a \times 0.7 \times 267{,}000 = 0.58\, d_a \times 10^6.$$

In machines of the same output and voltage, and having the same values of l_a, d_a, and $2pM$, the liability to flash-over depends solely upon the amount of armature distortion. In machines fitted with a compensating winding there is practically no armature distortion, and the designers of these machines state that no flash-over difficulties are experienced.

In dynamos with commutation poles armature reaction is not so neutralised, and flash-over, if occurring at all, generally occurs when there is a sudden overload thrown on the machine. The resulting increase in the armature current causes $B_{g\,max.}$ to increase, and therefore also the value of $e_{max.}$ When flash-over occurs, the latter has, of course, exceeded the safe limit.

The above limits of armature core length and flux entering or leaving the armature hold good only for peripheral speeds of 75 metres per second. The limit values of l_a and $2p$M for a range of peripheral speeds between 50 and 75 metres per second are given in Table XIV. It will be observed that the lower the speed the greater can be the values of l_a and $2p$M.

TABLE XIV

LIMIT VALUES OF l_a AND $2p$M FOR TURBO-DYNAMOS

Peripheral speed in metres per second.	Machines with commutation poles.		Machines with compensating windings.	
	l_a.	$2p$M.	l_a.	$2p$M.
75	33	$0.43d_a \times 10^6$	38	$0.58d_a \times 10^6$
70	36	$0.47d^a \times 10^6$	41	$0.63d_a \times 10^6$
65	39	$0.51d_a \times 10^6$	44	$0.68d_a \times 10^6$
60	42	$0.55d_a \times 10^6$	48	$0.74d_a \times 10^6$
55	45	$0.59d_a \times 10^6$	52	$0.80d_a \times 10^6$
50	50	$0.65d_a \times 10^6$	57	$0.88d_a \times 10^6$

EXAMPLES OF COMPLETE MACHINES

Slow-speed Dynamo.—Figure 213 shows the mechanical construction of a 220-k.w. 530/440-volt 6-pole shunt-wound generator, the armature of which is driven at a speed of 380 R.P.M. The magnet yoke is fitted with commutation poles, and the design may be taken as typical for slow- and medium-speed machines giving an output of from 150 to 500 k.w.

The armature is built up on a six-armed spider, the boss of which is keyed to the shaft. One end of the boss terminates in a coupling by means of which the armature is bolted to the prime mover; this, as already stated, relieves the shaft of the driving stresses. The armature laminations are mounted on the arms of the spider and held between two end flanges, to each of which is cast a skeleton ring for supporting the end connections of the winding. The left-hand end flange abuts against lugs projecting from the arms of the spider, while the other is held in position by keys driven into each arm. The core is provided with four ventilating ducts. The armature is lap-wound, and those parts of the end flanges on which the end connections rest are insulated with press-spahn as indicated.

The commutator spider is in two parts, one of which is bolted to the arms of the armature spider. The V-shaped rings, between which the commutator is retained, are cast in one with each portion of the spider. The two members of the spider are held together by bolts and insulated from the commutator by the ordinary V-shaped mica rings. Connection is made to the ends of the armature windings by lugs projecting from the segments. Directly below the supporting ring at the commutator end of the armature are 10 equalising rings, each of which is connected to those commutator lugs which should be at the same potential.

The yoke of the field magnets is of cast steel, and to the lower half of the ring are cast feet by means of which the yoke is bolted to the bedplate. The yoke, at each face, is cast with overhanging rims, the one at the commutator end being turned to receive a skeleton cast-iron ring for supporting the brush gear. The brush spindles are insulated from their supporting rings by vulcanite bushes and washers, spindles of the same polarity being connected to one of two connectors, from which leads go to the machine terminals.

The main poles are built of sheet-iron laminations, and bolted to machined seatings on the yoke. The laminations are stamped with projections at their lower end, so that when assembled the latter form polar extensions, and serve to support the winding spools. The auxiliary poles are of cast steel, and are also bolted to machined seatings on the yoke.

High-speed Dynamos.—Type 1.—Figure 214 shows the mechanical construction of a 1000-k.w. turbine-driven 4-pole dynamo. The armature laminations, pierced with numerous ventilating tunnels, are mounted direct on the shaft. The end flanges are of brass, and designed so as to support the end connections of the winding. The end flange at the commutator end abuts against a turned collar on the shaft, and the other one is forced home by a nut screwed on to the shaft, thus making it compress the laminations together. The latter end flange is keyed to the shaft, and the nut is prevented from turning by fixing it to the end flange with a set pin. To the end flanges are attached blades, which act as a fan in driving air through the interior of the armature.

The armature winding is placed on the surface of the core, and prevented from moving relative to the latter by brass drivers fastened into the core at each end, as shown in Figure 142. The entire winding is held against centrifugal force by binding wire. The placing of the winding on the surface of the core ensures a lower reactance voltage than would be the case with a winding enclosed in slots. This will of course permit of .better commutation. The commutator is mounted direct on the shaft, and its construction is identical with that shown in Figure 170.

The magnet core is of cast steel, and divided along a horizontal diameter, the upper and lower halves being bolted together. The lower half of the yoke is provided with feet for bolting to the bedplate. The steel magnet cores and their polar extensions are cast solid and bolted to the yoke. Each exciting coil is wound in three sections of different overall dimensions. This, as will be seen from the drawing, considerably increases the cooling surface of each coil.

The machine is provided with a compensating winding fixed to the main poles and connected in series with the armature. For holding this winding the pole shoes are provided with slots: the teeth are generally made separate, and bolted to the polar extensions. After the winding has been put in place, the tops of the teeth are fitted with small shoes, which overlap the slots as shown. This increases the area of iron by which the flux enters or leaves the air-gap, and also retains the coils in position. The end connections of the compensating winding are supported by brass bridge-pieces, which in turn are bolted between adjacent pole shoes. The bearings are supported on an extension of the yoke, and the armature and field magnets are partially enclosed by thin brass end covers.

Type 2.—Figure 215 shows sectional drawings of a 250-k.w. 500/600-volt 4-pole turbo-dynamo, the armature of which is driven at a speed of 2500 R.P.M. The drawings are dimensioned in millimetres.

The armature laminations are mounted direct on the shaft and held between end flanges in the manner shown. The inductor portions of the winding are placed in slots on the armature periphery and held in place against the action of centrifugal force by wooden wedges driven into key-ways formed near to the tops of the teeth. The end connections are retained against centrifugal force by metal covers which are bolted to the supporting flanges. The commutator is of the same construction as that shown in Figure 170.

The field magnet is constructed on a different principle to that generally adopted for direct-current machines. The yoke is built up of sheet-iron laminations fitted to radial arms projecting inwardly from a cast-iron frame, and, after being assembled, are clamped between end flanges or rings. The laminations forming the yoke are stamped with inward projections of short radial length, so that, when assembled, the latter form the main poles, as shown in Figure 216. In order to assist ventilation, the field-magnet system has a number of radial air-ducts through which air can be driven by the fanning action of the armature.

At the centre of each interpolar space a tooth, formed with overhanging ends, projects from the yoke, and this, along with the sides of adjacent poles, forms wide slots into which are placed the coils exciting the main poles. The latter are formed with a series of almost totally enclosed slots, in which is wound a compensating winding. Slots of this type are used so as to increase the section of iron by which the main flux enters or leaves the pole face.

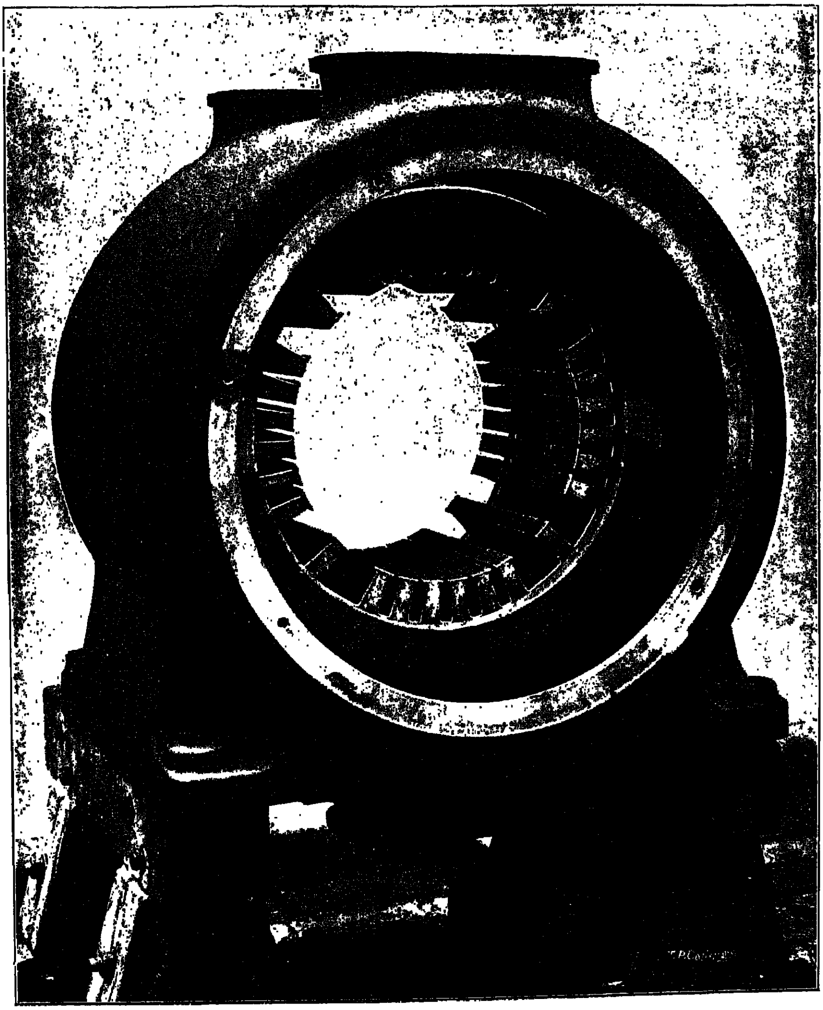


FIG. 216.—Field-magnet frame of turbo-dynamo fitted with a Dèri winding.

The compensating winding, often referred to as a Dèri winding, after the name of its inventor, is connected

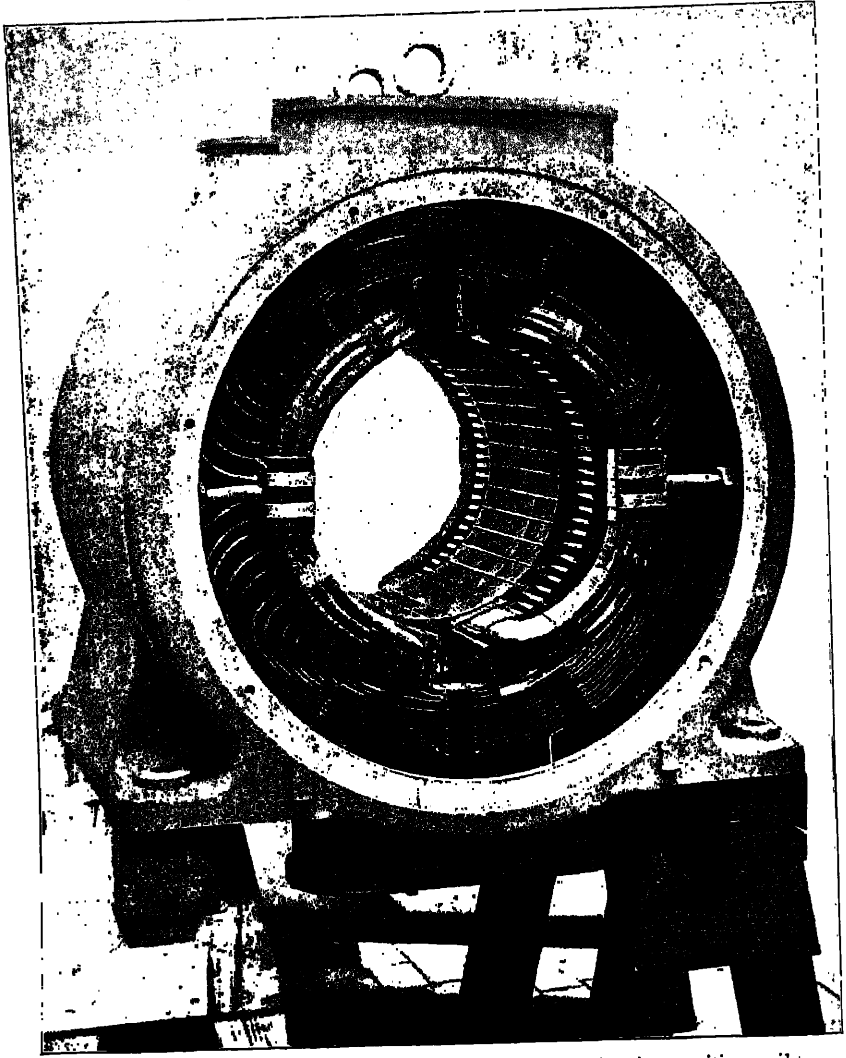


FIG. 217.—Field-magnet frame of turbo-dynamo, showing exciting coils and Dèri winding.

in series with the armature, and is an application of the principle illustrated in Figure 212. Figure 217 is a photograph of the field-magnet frame of a 2-pole dynamo, showing the main field coils and Dèri winding in position.

In machines of this type the number of compensating ampère-turns is relatively large, and should a dynamo, operating in parallel with others, be driven as a motor, owing to a reverse current flowing, a reversal of the main field might also take place, and if the reversed field be weak there is a possibility of the machine attaining a dangerous speed. In order to prevent the latter contingency, separate direct-coupled exciters are supplied for providing the excitation current, thus ensuring a constant direction of field. In addition to preventing a reversal of the main field, the strength of the latter increases with increase of speed; hence the additional loss thrown on the machine has a strong tendency to prevent excessive speed being attained should any accident occur.

The armature is fitted with a number of vanes so that the revolving armature acts as a fan. Air, drawn from the inlet opening at each end of the machine, is propelled through the ducts in the armature core and field-magnet system into the annular space which surrounds the field-magnet core, and is finally ejected into the atmosphere through the ventilating openings shown in the top of the field frame. By this arrangement every part of the machine which is the seat of the generation of heat is reached by the cooling air, thus resulting in a moderate and uniform temperature rise.

CHAPTER X

THE DYNAMO—CHARACTERISTIC CURVES, PARALLEL WORKING, AND BOOSTERS

CHARACTERISTIC CURVES

Magnetisation Curve.—It has been shown in the example, page 317, that for the magnetic circuit of the 33-k.w. dynamo (Figure 194) 4590 and 5210 field ampère-turns are necessary to maintain a magnetic flux of 3.45 and 3.6 megalines respectively through the armature. By similar calculations the ampère-turns required to force any magnetic flux across the air-gap and through the iron parts can be determined. The ampère-turns necessary to maintain five values of the magnetic flux ranging from 1.25 to 3.6 megalines are given in the accompanying table. In the third column is the total E.M.F. induced in the armature at a constant speed of 660 R.P.M., corresponding to each value of the magnetic flux.

Magnetic Flux in Megalines.	Ampère-turns.	Induced E.M.F.
1.25	1000	200
1.90	1650	300
2.80	3100	450
3.45	4590	550
3.60	5210	572

In Figure 218 the values of the calculated ampère-turns are plotted against the corresponding values of the magnetic flux, and the curve obtained is known as the magnetisation or saturation curve of the dynamo under consideration. If the speed remain constant the total E.M.F. induced in the armature will be directly proportional to the magnetic flux, and is given by $E = 4\ TNM\ 10^{-8} = KM$, where K is a constant. The curve showing the variation of the induced E.M.F. with the field ampère-turns is known as the *internal characteristic* of the machine, and may be obtained from the magnetisation curve by marking off along the ordinate reference axis the corresponding values of induced E.M.F., as is shown in Figure 218.

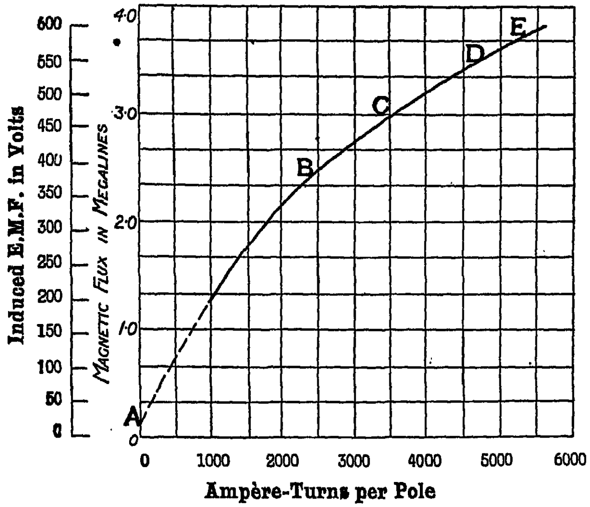


FIG. 218.—Magnetisation curve for a shunt dynamo.

The magnetisation curve of a dynamo is obtained experimentally by running the machine on open circuit at normal speed and separately exciting the field coils. In series with the latter a regulating resistance should be connected, so that the exciting current can be adjusted to a number of suitable values, and the corresponding values of induced E.M.F. read off on a voltmeter connected across the armature terminals.

It will invariably be found that the magnetisation curve does not pass through the origin, but when the exciting current is zero there will be a small E.M.F. induced in the armature, due to residual magnetism in the field magnets. This effect is shown in the figure, the ordinate oA representing the E.M.F. due to residual magnetism.

That part of the magnetisation curve between A and B is practically a straight line, as the ampère-turns required to send the flux across the air-gap form almost the whole of the excitation; the magneto-motive force required for the iron parts of the magnetic circuit being only a very small percentage of the total. If the proportions and magnetic densities in the component parts of the magnetic circuit have the values recommended in the previous chapter, it will be found that machines are worked on a portion of the magnetisation curve well over the knee. For instance in the machine under consideration the flux through the armature ranges from 3.45 to 3.6 megalines between no load and full load, so that the working portion of the curve is that between the points D and E. Should a dynamo be worked on a portion of the curve below the knee the machine will be very unstable, and a slight variation in speed will cause a large variation in the terminal E.M.F., a result which is always undesirable.

For example, if in the case of a shunt dynamo the speed slightly increases, the resulting increase in the E.M.F. will of course increase the current through the shunt coils: now, below the knee of the curve a small increase in exciting current produces a large increase in E.M.F., so that the E.M.F. will be still further increased. It is therefore of great importance to design dynamos which are excited by shunt coils, so that they work fairly high up on the magnetisation curves, and the best portions to work on are within the limits marked by CE in the figure. Another advantage of working high up on the curve is that the difference of the ampère-turns of the shunt winding when hot and when cold then produces but a small effect on the E.M.F. induced in the armature owing to the iron parts of the magnetic circuit being nearly saturated.

Critical Resistance of a Shunt Dynamo.—The general form of the internal characteristic of any shunt dynamo is shown in Figure 219, where values of shunt current are plotted as abscissæ and the corresponding values of induced E.M.F. as ordinates. Take any point P on the curve and join P to the origin O. At P the exciting current is given by the abscissæ OC, and the corresponding value of induced E.M.F. by the ordinate OE. Now the resistance of the shunt circuit equals

$$\frac{\text{E.M.F. across shunt winding (including rheostat)}}{\text{shunt current}},$$

so that the resistance of the latter corresponding to the point P, is given by

$$R = \frac{OE}{OC} = \frac{PM}{PE} = \tan POC = \tan \theta,$$

i.e. the resistance is given by the slope of the line OP. Expressed in words: The resistance of the shunt circuit corresponding to any point on the characteristic is given by the tangent of the angle made by the line joining that point to the origin.

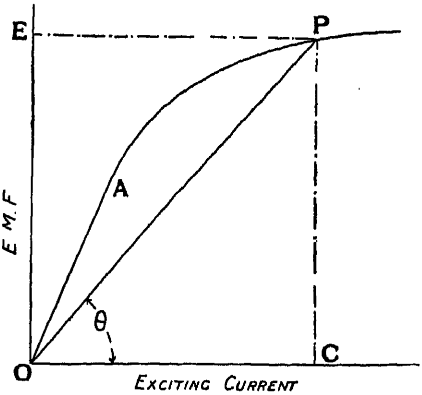


FIG. 219.—Critical resistance of a shunt dynamo.

If the resistance of the shunt circuit be gradually increased from that represented by tan POC, the point P will move along the curve towards the origin, the induced E.M.F. and exciting current decreasing while the slope of the line OP increases. The resistance may be increased until OP practically forms a tangent to the part OA which is approximately a straight line. Any further increase in the shunt resistance beyond that represented by the slope OA will have so reduced the exciting current that it is not sufficient to adequately magnetise the field magnets, with the result that the dynamo loses its magnetism. The slope of the line OA, is known as the *critical resistance* of a shunt dynamo, *i.e.* the maximum resistance of the shunt circuit which will permit of the dynamo being adequately magnetised.

External Characteristic of a Series Dynamo.—The external characteristic of a dynamo is a curve showing the relation between the electromotive force at the armature terminals and the current output of the machine when the latter is varied; the values of E.M.F. being plotted as ordinates, and those of current as abscissæ. Such a curve can be experimentally determined by running the machine at constant speed, varying the current by means of an adjustable resistance in the external circuit, and observing the corresponding values of current output and terminal voltage.

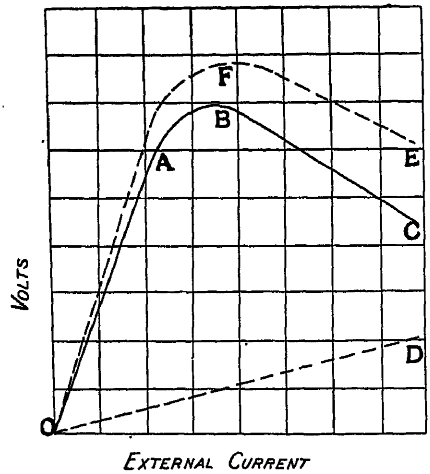


FIG. 220.—Characteristic curves of a series dynamo.

In Figure 220 the curve OC shows the form of the external characteristic of a series dynamo, and since the external current is also the magnetising current it resembles to some extent the curve shown in the previous figure. As the external current is increased from zero the E.M.F. on the armature terminals rapidly increases, as shown by the portion OA. If the current be further increased the curve takes a decided bend and reaches a maximum at B; but beyond this any further increase in the current is accompanied by a decrease in terminal E.M.F. as shown.

This fall of E.M.F. when the current is increased beyond a certain value, is due to (1) the pressure drop over the resistance of the armature and series coils, and (2) the demagnetising effect of the armature ampère-turns. The former is proportional to the current and is expressed by $V = C\ (R_a + R_s)$, where C is the armature current and R_a and R_s the resistance of the armature and series coils respectively. If $C\ (R_a + R_s)$ be known, the voltage drop in the armature and series coils can be calculated for various values of current, and when the values of $C\ (R_a + R_s)$ are plotted the straight line OD will be obtained, which shows the variation of pressure drop with the load.

By adding to the ordinates of the curve OC the corresponding ordinates of the curve OD, a third curve OE is obtained, which shows the variation of the total E.M.F. induced in the armature with the external current. If it were not for the demagnetising action of the armature ampère-turns the curve OE (*i.e.* the total characteristic) of a series dynamo would be identical with its magnetisation curve.

When the field magnets are approaching saturation any increase in the external current is not accompanied by a corresponding increase in the magnetic flux, the result being that the rate of increase of the induced E.M.F. diminishes as the current increases. Thus, if the armature demagnetising effect be zero, the total characteristic curve would tend to rise continuously with an increase of current: but the demagnetising action of the armature current also increases in proportion to the external current, and tends to reduce the terminal E.M.F. in the following manner:

As already stated, the armature core is worked at a much lower induction than the field magnets, so that when the latter are nearly saturated the armature core will still be considerably below saturation. When the current is increased beyond that corresponding to the point B, the magnetic flux due to the field coils remains approximately constant, whereas the demagnetising flux of the armature will at this stage increase in proportion to the current, with the result that the total flux linked with the armature inductors decreases, and therefore, also, the induced E.M.F., as is shown by the part FE in the curve OE. When the field magnets are saturated any increase in the magneto-motive force increases the coefficient of magnetic leakage so that a smaller percentage of the flux generated in the magnets enters the armature; this further helps to decrease the induced E.M.F.

External Characteristic of a Shunt Dynamo.—The external characteristic curve of a shunt dynamo takes the form shown in Figure 221. The E.M.F. across the armature terminals is a maximum on open circuit. When the external circuit is closed and the armature current increased from the no load value, the E.M.F. on the armature terminals gradually decreases in the manner shown, the resistance of the shunt circuit being maintained constant. This fall of E.M.F. at the armature terminals is attributable to—(1) The drop in pressure—due to armature resistance—expressed by CR_a, where C is the armature current and R_a the resistance. (2) The demagnetising action of the armature current, which increases with the load and can be calculated in the manner shown on page 327. (3) The shunt coils being connected across the armature terminals, any decrease in the terminal E.M.F. due to (1) and (2) also decreases the strength of the exciting current, and thereby further reduces the terminal E.M.F.

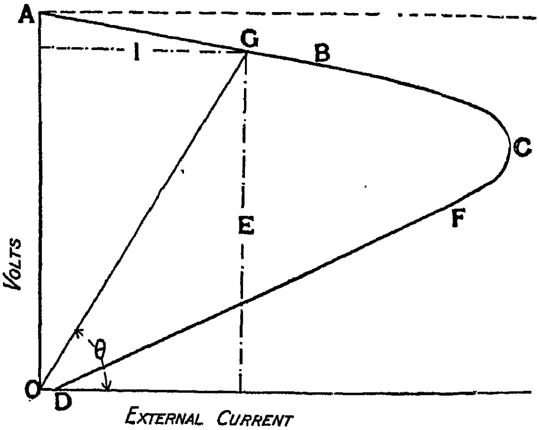


FIG. 221.—External characteristic of a shunt dynamo.

Owing, then, to the combined effects of the increase in the demagnetising ampère-turns of the armature and the pressure drop due to armature resistance, the voltage of a shunt dynamo continuously decreases, as the output is increased, up to a point indicated by C, where the current attains its maximum. At this point the flux set up by the demagnetising ampère-turns of the armature is equal to the flux set up by the field magnets, and consequently the voltage decreases to zero. The curve bends back upon itself as shown by the part CD, which cuts the axis of abscissæ a little to the right of the origin as a result of the pressure generated in the armature due to the residual magnetism of the field magnets. In a shunt dynamo there will thus be a maximum value of the external current beyond which it is impossible to go: but owing to thermal considerations it will generally be impossible to allow the armature to carry this maximum current for any appreciable time. In a well-designed dynamo the full-load current is about 50 per cent. of the maximum. The part AB of the characteristic which shows the change in E.M.F. between no load and full load is known as the *regulation* or *load curve.* If the armature be of low resistance the drop in volts for any current within the working limits will be small.

From the external characteristic it will be seen that for every value of the current, except the maximum, there are two values of the terminal E.M.F., and which one of these two E.M.Fs. is obtained will depend upon the resistance of the external circuit. The latter $= \frac{\text{terminal E.M.F.}}{\text{external current}} = \frac{E}{I} = \tan\theta$, so that the resistance of the external circuit is given by the slope of the line OG, where G is any point on the characteristic at which the current and terminal E.M.F. are expressed by E and I respectively. There is a critical portion of the curve just before the maximum current is reached, and if the resistance of the external circuit be diminished there will be a comparatively large drop in pressure at the armature terminals. After rounding the point of maximum current the curve is almost a straight line to the point D, so that very small changes in the external resistance will cause great changes in pressure and current, because the voltage across the shunt is insufficient to adequately magnetise the iron. The slope of the curve between F and D defines the *critical resistance* of the external circuit, within which the magnetic flux of the field magnets is unstable, and the machine fails to excite.

This is of great practical importance, for if a shunt dynamo be accidentally short-circuited when supplying current the armature is not liable to destruction by the passage of an excessive current, for when the maximum current is attained the field magnets are demagnetised and the armature current immediately falls to zero. In some cases when a shunt dynamo is suddenly short-circuited the field magnets, through certain secondary reactions, become slightly magnetised in the reverse direction, so that when again self-excited the polarity of the terminals is reversed.

Voltage Control.—(1) *Shunt Regulation.*—In order to regulate the voltage of a shunt dynamo a variable resistance or rheostat R (Figure 182) is connected in series with the field-magnet winding. The rheostat is so designed that at no load the machine gives its full terminal voltage when the whole of the resistance is in series with the field coils. As the load increases the shunt current must be increased, in order to compensate for the pressure drop due to armature resistance and reaction. This is accomplished by manipulating the field rheostat, so that the resistance in series with the field coils is diminished, until at full load nearly the whole of the resistance is cut out.

The function of the shunt-regulating resistance for a dynamo will be best illustrated by an example. In the calculation of the shunt coils for the 33-k.w. dynamo the following was determined (*vide* page 318):

Ampère-turns at no load = 4590.
Ampère-turns at full load = 6000.
Number of turns per shunt coil = 6000.
Exciting current at full load = 1 ampère.
Constant voltage across shunt + rheostat = 550.
Resistance of each shunt coil (60° C.) = 130 ohms.
Total resistance of shunt winding (60° C.) = 130 × 4
= 520 ohms.

It is usual to design the shunt rheostat so that when full load is attained the entire regulating resistance does not require to be cut out of circuit, thus reserving a possible increment of exciting current sufficient to maintain a constant terminal E.M.F. for a reasonable percentage of overload. The value of the shunt-regulating resistance is such that at full load from 5 to 10 per cent. of the terminal pressure is absorbed, and in the example under consideration 30 volts were so absorbed.

Since the full-load shunt current $= 1$ ampère, the value of the regulating resistance in circuit at full load $= 30$ ohms. In order that the field ampère-turns at no load may be 4590, the exciting current must be reduced to $\frac{4590}{6000} = 0.76$ of an ampère. The total resistance of the shunt circuit will then $= \frac{550}{0.76} = 725$ ohms.

Since the field coils have a permanent resistance of 520 ohms the total value of the shunt-regulating resistance $= 725 - 520 = 205$ ohms, of which 30 ohms are in circuit at full load. (At no load the watts absorbed by the shunt rheostat $= C^2R = 0.76^2 \times 205 = 120$ watts.)

The number of steps into which a shunt-regulating resistance should be divided will depend upon the permissible percentage variation in the total induced E.M.F. when one section of the resistance is cut out of or put into circuit. In practice the variation allowed per step is usually about 4 per cent. of the total range of induced E.M.F. between no load and full load. In the case under consideration there is a range of 22 (*i.e.* $572 - 550$) volts. Allowing for a 4 per cent. variation, the change in the induced E.M.F. at any one step $= 4 \times \frac{22}{100} = 0.88$ volt, so that the number of steps $= \frac{22}{0.88} = 25$. From Figure 218 it will be seen that between no load and full load the magnetisation curve is practically a straight line, so that each section of the resistance will be of the same value, namely, $\frac{205}{25} = 8.2$ ohms. In the majority of dynamos the working part of the characteristic is practically a straight line, so that in practice, except in special cases, dynamo shunt rheostats are divided into sections, each of which has the same resistance.

Design of Shunt Rheostat.—The resistance coils of shunt rheostats are usually made from one of the alloys given in Table XV., these alloys having a high specific resistance and low temperature coefficient. Iron is also given, but owing to its high temperature coefficient it is never employed for shunt rheostats. It is, however, often used in the construction of motor starting resistances, which are described in the next chapter.

TABLE XV

ALLOYS USED FOR RHEOSTATS

Material.	Specific resistance in microhms per centimetre cube at 0° C.	Temperature coefficient per ° C.	Specific gravity.	Specific heat.
Beacon	74	0.00070	8.1	0.1
Constantan	43	zero.	8.8	0.1
Eureka	47	0.000005	8.8	0.1
German silver	30	0.000273	8.5	0.098
Iron (pure)	9	0.00625	7.9	0.104
Kruppin	84	0.00077	8.7	0.13
Nickelin	33	0.00030	9.0	0.08
Platinoid	40	0.00031	8.6	0.098
Rheostan	52	0.00041	8.6	0.097
Resista	75	...	...	0.1

The size of wire forming the coils of a regulating rheostat should be such that the steady temperature which the wire attains when carrying continuously the maximum current, should not exceed a safe limit. For shunt resistances the permissible temperature rise is generally about 100° C. A steady temperature is reached when the rate of loss of heat by radiation from the surface of the wire is equal to the rate of generation of heat by the current. The heat generated per second

$$= C^2R \times 0.24 \text{ calories} = C^2 \times \frac{\rho \times l \times 4}{\pi d^2} \times 0.24 \text{ calories}$$

$$= C^2 \times \frac{\rho \times l}{\pi d^2} \times 0.96$$

where—

ρ = specific resistance of wire.
l = length of wire in centimetres.
d = diameter of wire in centimetres.

Let σ denote the emissivity of the wire, *i.e.* the rate of loss of heat in calories by radiation per unit area of the wire per ° C. difference in temperature between the wire and the surrounding air. The emissivity is not actually a constant, being higher for black surfaces than for bright, but on an average may be taken at 0.0005. The rate of loss of heat by radiation will therefore = surface area of wire × σ × temperature rise above surrounding air

$$= \pi d \times l \times 0.0005 \times (T_2 - T_1) \text{ calories per second,}$$

where—

T_2 = temperature of wire.
T_1 = temperature of surrounding air.

In practice the coils are usually proportioned so that the temperature rise will not exceed 100° C., in which case the loss of heat by radiation

$$= \pi d \times l \times 0.0005 \times 100 = 0.05 \times \pi d \times l \text{ calories per second.}$$

When a steady temperature has been attained

$$C^2 \times \frac{\rho \times l}{\pi d^2} \times 0.96 = 0.05 \times \pi d \times l$$

$$i.e.\ d^3 = C^2 \times \rho \times \frac{0.96}{\pi^2 \times 0.05} = 2\ C^2\rho$$

and diameter of wire is expressed by

$$d = \sqrt[3]{2\ C^2\rho.}$$

Having determined the diameter of wire, the length required to give any particular value of resistance is obtained from the formula $R = \frac{\rho \times l \times 4}{\pi d^2}$

$$i.e.\ l = \frac{\pi d^2 \times R}{\rho \times 4}.$$

Example.—In the shunt rheostat for the 33-k.w. dynamo, the resistance per section has to be 8.2 ohms and the maximum field current 1 ampère. If the resistance coils be of platinoid, determine the diameter of wire required and the length of wire for each section.

$$d = \sqrt[3]{2 \times 1^2 \times 0.00004}$$
$$= 0.043 \text{ centimetre.}$$

The nearest standard size to this is No. 26 S.W.G., which has a diameter of 0.0457 centimetres.

Length of wire per section

$$= \frac{\pi \times 0.0457^2 \times 8.2}{0.00004 \times 4} = 340 \text{ centimetres.}$$

Resistance Box.—Resistance wires are usually wound either on porcelain tubes or in the form of open spirals,

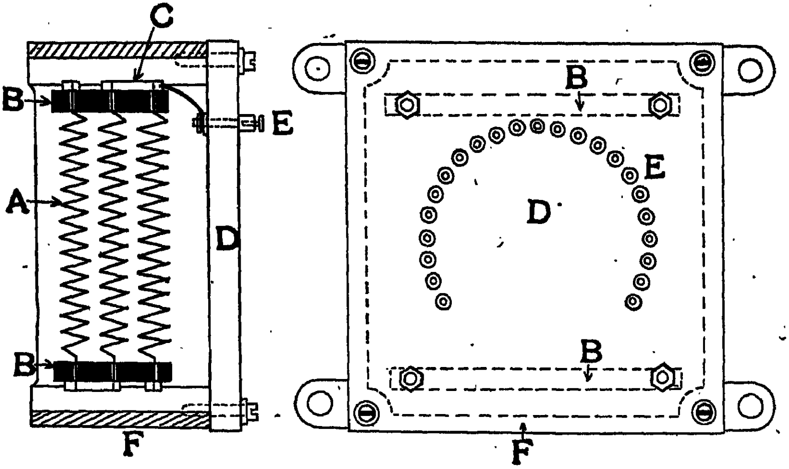


FIG. 222.—Resistance box.

and enclosed in ventilated cast-iron boxes arranged so that the heated air rises and cool air is drawn in from below into the box. Figure 222 shows a design of resistance box suitable for arranging the wires in open spirals. The resistance coils A are stretched between two slate supports BB, which are braced together. The coils A are arranged in tiers with two or more in each tier, and are held in position by fixing each end of a coil into the terminal blocks C; these also serve to connect the coils in series. The supports BB are bolted to a slate slab D on which are mounted the terminals E. The latter are, at the back, connected to the various points of the resistance, and from the front connection is made to a multiple contact switch (described on page 478), which is usually mounted on a switchboard. The resistance coils are enclosed inside the ventilated cast-iron box F, the cover of which is provided by the slate slab D on which the terminals are fixed.

(2) *Compounding.*—The best method of maintaining a constant terminal E.M.F. is by winding the field magnets with both shunt and series coils. The shunt winding should be sufficient to give the full terminal pressure at no load, and as the load is increased the current flowing in the series coils should augment the field ampère-turns in such a proportion that the terminal E.M.F. is maintained constant.

In a compound dynamo the shunt simply provides a certain initial flux and voltage, and since the excitation of the series turns varies directly with the external current, the series ampère-turns increase the field flux in such a proportion that they compensate for the drop in pressure due to armature resistance and reaction.

The external characteristic of a correctly compounded dynamo is therefore approximately a straight line throughout the entire working range. Thus in the case of the 33-k.w. dynamo, for which the ampère-turns per pole at no load and full load are 4590 and 6000 respectively, the shunt winding would be designed to give the 4590 ampère-turns at no load, and the additional ampère-turns necessary as the load increased would be provided by the series coils, which at full load would provide a magneto-motive force of $6000 - 4590 = 1410$ ampère-turns.

The full-load current of this machine is 60 ampères, so that the number of series turns per pole $= \frac{1410}{60} = 24$.

When the load happens to be situated at the end of a long transmission line, it is often required that the voltage at the point of distribution shall remain constant at all loads. In such a case the generator must be *over-compounded*, *i.e.* the voltage at the dynamo terminals must increase with the load, thus compensating for the additional CR drop in the transmission line. For instance, generators used for traction work are usually over-compounded, to give a terminal E.M.F. of 500 and 550 volts at no load and full load respectively, the 50 volts increase in terminal E.M.F. at full load compensating for the fall of potential along the transmission line.

PARALLELING OF DYNAMOS

The prime mover and dynamo of a generating set are invariably designed to give their maximum efficiency at or near full load. The output of a generating station usually varies within wide limits at different times of the day, so that it is best to employ two or three smaller units rather than one unit large enough to supply the maximum demand. With this arrangement the number of dynamos coupled in parallel can be increased with the load, and disconnected one after the other as the load diminishes. The individual generators can thereby be maintained at nearly full load, and consequently work near to their maximum efficiency, whereas if one large unit be used, its output during the greater part of the day might be considerably less than half its full-load capacity, with the result that it would have a very low efficiency. The method of connecting shunt or compound dynamos in parallel will now be considered.

Shunt Dynamos in parallel.—Figure 223 shows the connections between two shunt-wound dynamos, D_1 and D_2, which are to run in parallel. The positive and negative terminals of each dynamo are connected through double pole switches S_1 and S_2 to the common conductors $B+$ and $B-$ respectively. The latter are known as *bus-bars*, and from them feeders are run to the network of mains. In series with the positive main of each dynamo are connected the ammeters A_1

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still lower, but for every volt it falls coil C will add one volt to the booster.

Suppose, next, that the load falls to 100 ampères. Since the generator field will be adjusted so that its output current remains approximately constant at 500 ampères, 400 ampères must go to charge the battery. This will cause the battery E.M.F. to rise to about 540 volts, and the current in the coil C to reverse, so that the booster voltage equals -40. The battery and booster volts will therefore be $540 - 40 = 500$, the bus-bar voltage.

This example shows how the generator may be maintained constantly at full load, and all variations in output taken up entirely by the battery.

When the average output of the plant is greater than 300 ampères, the series winding of the booster may be shunted with a low-resistance, adjustable diverter, so that the current in the coil S_E is a definite percentage of the total output current.

Exciter Control or Highfield Booster.—Figure 229 illustrates the principle of a second form of reversible booster designed by J. S. Highfield. The booster armature A is driven by a motor M as before. The field magnets of the booster are wound with two coils S_H and S_E. The series coil S_E carries either the main current or a definite percentage of it. The shunt or fine wire coil S_H is connected, through a small shunt-excited

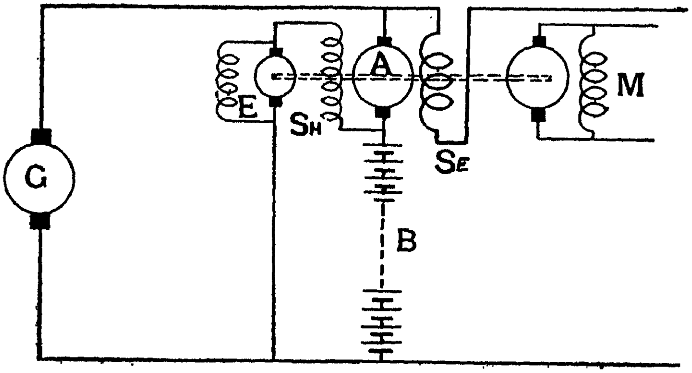

FIG. 229.—Highfield's exciter control booster.

shown in Figure 228. The current in the coil C will thus be proportional to the difference between the battery and bus-bar E.M.Fs. With this arrangement the shunt coil S_H is connected in parallel with the battery.

When the battery volts are equal to the bus-bar volts no current flows in C. If the battery pressure be lower than that of the bus-bars a current flows through C in one direction, and in the reverse direction when the battery pressure is the greater. The winding of the coil C is such that for every volt fall of the battery the booster gives one volt positive and *vice versâ.*

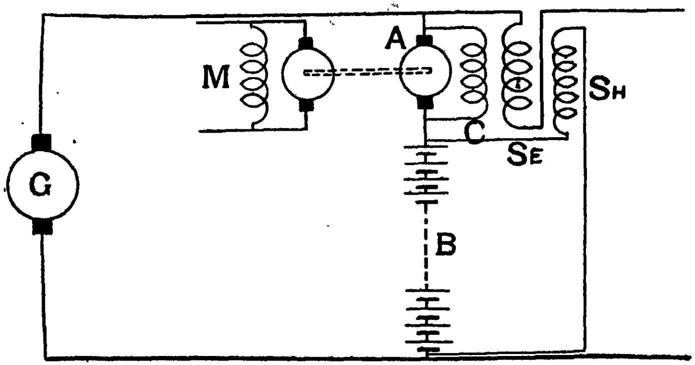

FIG. 228.—Differential reversible booster.

The action of the booster will be best illustrated by a concrete example. Suppose the bus-bar voltage and average output of the plant be 500 volts and 500 ampères respectively. When the feeder current is at the normal value of 500 ampères the coils S_H and S_E will neutralise each other so that the booster E.M.F. is zero. Now suppose the feeder current rises to 900 ampères, 400 of which would be supplied by the battery; this will cause the E.M.F. of the latter to fall to say 460 volts. Under these conditions a current of such a value will flow in the coil C that it will give exactly 40 volts positive to the booster, so that the battery and booster volts will be $460+40=500=$ bus-bar volts. As the discharge continues the battery pressure will fall is zero, and the battery B will be neither charged nor discharged. When the line current is less than the normal output the shunt winding predominates, and the booster generates an E.M.F. in such a direction that it adds to the pressure of the generator G and so charges the battery. With a current greater than normal the series winding predominates, and therefore changes the polarity of the booster terminals, thus adding to the pressure of the battery circuit and helping to discharge the battery.

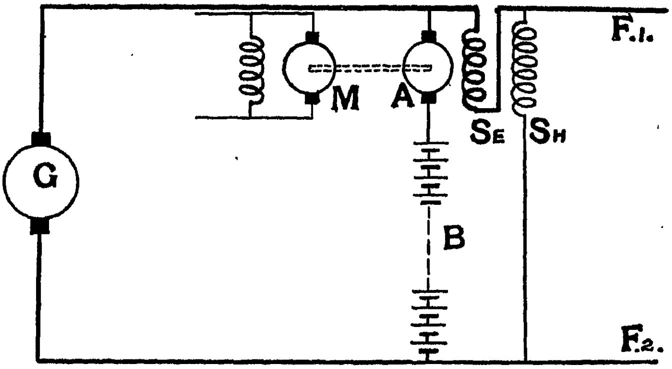

FIG. 227.—Reversible differential booster.

In the case of this simple differential booster, the current supplied to, or taken from, the battery is dependent upon the E.M.F. of the latter as well as upon the line current. Suppose, for instance, that when the battery is fully charged its E.M.F. is equal to that of the bus-bars, then at normal load the booster will be neutral and the battery current zero. But if the battery be *not* fully charged and the load is again normal, the booster will still be neutral, and since the battery voltage will be less than that of the bus-bars a current will flow from the generators to charge the battery. The current supplied to, or taken from, the battery is thus dependent upon the E.M.F. of the latter.

In order that the battery output will be dependent only upon the load, the simple differential booster requires to be fitted with an additional exciting coil C, which is connected across the booster terminals, as it; on the other hand, should the load be greater than normal the battery discharges, owing to an increase in its E.M.F. Under such conditions the share of the load which falls upon the generators remains approximately constant, the battery taking in all the surplus current from the generators when the load falls below normal, and supplying the excess current when the load is above normal.

If the generators were compounded for a constant E.M.F. at all loads, then a battery alone is of little use; for when current is taken out of it, its voltage will fall and the discharge cease; and when a current is put into it, the voltage will rise and the charge cease. In most traction systems in Britain and America the generators are over-compounded, in which case a battery connected direct across the bus-bars would simply increase the fluctuation of load; at heaviest loads the increased pressure of the generators would send the maximum charging current through the cells.

It will now be evident that a battery used to maintain a constant load on the generators must have some auxiliary machine connected in series with it in order to overcome the above-mentioned difficulties. Traction batteries are therefore worked in conjunction with *Automatic Reversible Boosters*, and the combined battery and booster is often referred to as a *floating battery*. The advantages of this are—

(1) Saving in initial capital outlay on boilers, engines, and generators. The actual advantage under this heading is, however, considerably reduced owing to the large capital outlay on battery and booster.

(2) High working efficiency owing to the constant operation of the plant at full load.

Differential Boosters.—A diagram of connections for a reversible differential booster is given in Figure 227. The armature A of the booster is driven by the motor M and the field magnets are excited by shunt and series coils S_H and S_E respectively. S_H is connected in parallel with the mains F_1 and F_2, and S_E in series with F_1, the two windings acting in opposition. At normal load the field windings neutralise each other, so that the booster pressure

Irreversible Booster.—Figure 226 shows the method of connecting a booster to charge a battery B, which when discharging is connected in parallel with the generator G. The booster armature A is driven by the motor M, and the field coils F are connected across the bus-bars. The pressure generated by the armature can be varied by a rheostat R in series with the field. The armature is connected to the contacts *a a* of a double-pole throw-over switch S. When the latter is at *b b* the battery is connected in parallel with the generator. To charge the battery the switch S is thrown over to *a a*, thus connecting the armature of the booster in series with the battery. The rheostat in the field is then adjusted so that the requisite charging current flows from the bus-bars through the cells, the charging voltage being equal to that of the bus-bars plus that of the booster. As the charge proceeds the latter can be increased by adjusting the rheostat R until, when the cells are nearly charged, the voltage will be about 2.5 per cell. After the cells have been charged the switch S is thrown over to *b b*, thus cutting the booster armature out of circuit.

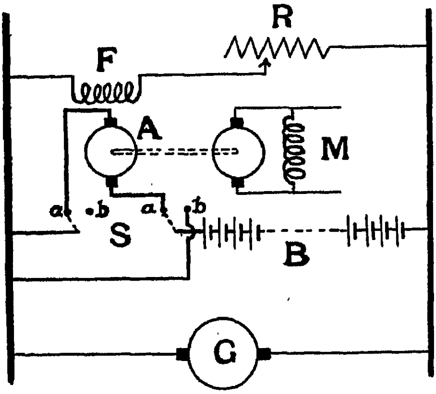

FIG. 226.—Shunt booster for battery charging.

Automatic Reversible Boosters for Traction generating Plants.—In all traction generating stations the load undergoes violent fluctuations, and in order to equalise the load on the generators a secondary battery is, as a rule, connected in parallel with the bus-bars. The connections are such that when the load is equal to the normal output of the generators, the bus-bar and battery E.M.Fs. are equal. If the load be less than normal, the battery E.M.F. is lowered and a current flows from the generators through the battery, thereby charging

Feeder Booster.—Figure 225 shows two feeders F_1 and F_2 connected to central station bus-bars B + and B –, the pressure of the latter being maintained constant at say 460 volts. Now suppose that the pressure at the distributing end A of the feeders is also required to be maintained constant at 460 volts, and that the fall of potential along the feeder at full load is 20 volts. To ensure this, the pressure at the generator end of the feeder is regulated by means of a booster B driven by a motor M. The armature and field windings of B are connected in series with the feeder F_1. Since the field of the booster is excited by the line current the excitation and consequently the value of the boosting E.M.F. will vary with the feeder current. The fall of potential in the feeder will be directly proportional to the current transmitted, and in order that the booster may truly compensate for any fall of potential it should be designed so that the E.M.F. generated is directly proportional to the exciting current, *i.e.* the load current. To effect this boosters are worked on the straight line portion of their magnetisation curve, *i.e.* the part AB, Figure 218. Should they be designed to work on or above the knee of the curve, the saturation of the field magnets would prevent the induced E.M.F. varying in direct proportion to the exciting current.

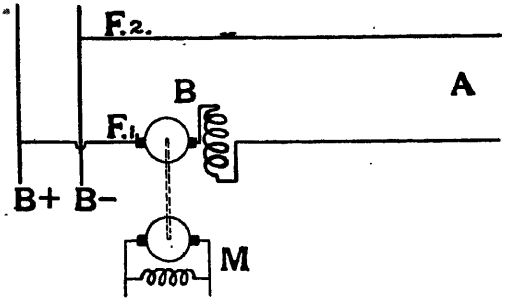

FIG. 225.—Diagram of connections for feeder booster.

Battery boosters may be divided into two classes—

1. Irreversible.
2. Reversible.

If it be required to unparallel machine D_2, its shunt excitation should be slightly reduced until it supplies a very small current, after which S_2 and S_4 are opened. The opening and closing of the equalising and main switches in the required order may be conveniently effected by a 3-pole switch S_1 (Figure 287), in which the equalising connection is made first on closing the switch, and broken last on opening it.

Should the speed of one of a number of dynamos connected in parallel slow down to such an extent that the voltage generated is less than that of the bus-bars, then current would flow into it from the other machines, *i.e.* the faulty dynamo would be driven as a motor. To prevent this an automatic circuit-breaker (described on page 493) is usually connected between each dynamo and the bus-bars, so that a dynamo may be disconnected from the latter, when either its current falls to zero or a predetermined reverse current flows.

BOOSTERS

A booster is a dynamo which is used to alter the voltage of a circuit in which an E.M.F. already exists. The operation is known as *boosting*, and was first suggested by Professor Perry. A booster may be driven by an electric motor, a steam engine, or any other suitable prime mover. The usual practice, however, is to drive it at constant speed by a shunt motor. The booster armature is connected in series with the circuit in which it is desired to alter the voltage, while the field magnets may be excited in various ways, depending upon the manner in which the pressure has to be altered.

Boosters are principally employed for—(1) Raising the voltage of feeders, which are of much greater length than others connected to the same bus-bars: in such cases the increased voltage due to the booster compensates for the fall of potential in the feeders resulting from ohmic resistance. (2) Altering the voltage of a battery circuit, so that current may flow from the generators to the battery and so charge it, or *vice versâ.*

distribution of current supplied by the two machines increases more and more until the load on D_2 becomes excessive, while the current supplied by D_1 is reduced to zero. The latter is finally driven as a motor taking current from D_2.

To eliminate this instability and to ensure the proper distribution of load among two or more compound dynamos connected in parallel, the terminal of each dynamo at which the armature and field coils are connected in series is joined through switches S_3 and S_4 to a common conductor, known as an equalising bus-bar, and shown in Figure 224. The series coils of the dynamos D_1 and D_2 are thus connected in parallel with each other. The equalising bus-bar must be of negligible resistance as compared with the series coils.

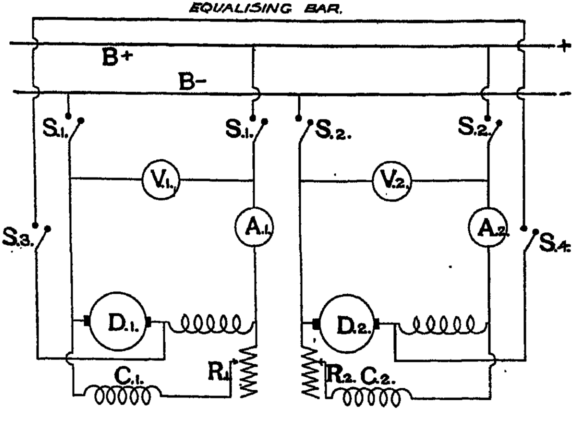

FIG. 224.—Compound dynamos in parallel.

To connect machine D_2 in parallel with machine D_1, when the latter is supplying current, D_2 is run up to speed and excited so as to give the same voltage as D_1, the excitation being provided by the shunt winding only. Switch S_4 is then closed. This excites the series field to its proper amount, and the shunt current is adjusted until the voltage at the terminals of D_2 is the same as D_1. The switch S_2 is closed, and the machine may be made to supply current by adjusting the field rheostat.

resistance of either machine from its total internal E.M.F., the remainder or terminal voltage must be alike in both cases. Thus if two similar machines, similarly excited and run at the same speed, be connected in parallel, each will take half the total current, and the proportion of the current which each machine supplies can be regulated by the shunt rheostat.

Assuming the two machines D_1 and D_2 to be equally loaded, then if there be a slight momentary decrease in the speed of D_1, the E.M.F. generated by it will decrease, thus shifting a larger portion of the current on to D_2. The decreased loss over the armature resistance of D_1 and the decreased armature reaction along with the resulting slight increase in the speed of the prime mover, combine to increase both the internal and terminal E.M.F. of D_1. At the same time the increased current through D_1 tends to reduce the terminal E.M.F., with the result that the electrical inter-reactions of the two machines exert an inherent tendency to equalise their speeds and loads.

In order to disconnect one of a number of dynamos in parallel, its load is shifted over to the other dynamos by gradually reducing its field current and increasing the field current of the other generators. When the current output of the machine to be disconnected is reduced to almost zero the main switch is opened.

Compound Dynamos in parallel.—If two compound-wound dynamos are paralleled by connecting their main terminals to common bus-bars, as shown in Figure 223, they will be unstable. Suppose that two machines D_1 and D_2 are connected in parallel and have their excitation adjusted so that each supplies one-half of the total current delivered to the mains. If D_1 runs momentarily at a slightly decreased speed a momentary decrease in the induced E.M.F. is effected, which in turn diminishes the current supplied by D_1 and increases the current of D_2. The decreased current supplied by D_1 decreases the strength of the series excitation, so that the induced E.M.F. and current output will be still further reduced. When once started, this unequal and A_2, while the voltmeters V_1 and V_2 indicate the pressure generated by the respective dynamos. The shunt coils C_2 C_2 are joined in series with the regulating resistances R_1 and R_2 respectively.

Suppose the machine D_1 is supplying the total current output, and that the latter has attained such a value as to necessitate the second machine D_2 being connected to the bus-bars. The method of paralleling the two dynamos is as follows:

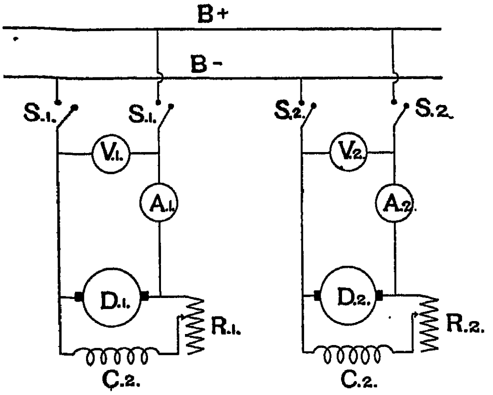

FIG. 223.—Shunt dynamos in parallel.

The incoming dynamo D_2 is run up to its normal speed, and by adjusting the field rheostat it is allowed to excite itself to approximately the same voltage as that of D_1. When the voltmeters V_1 and V_2 indicate the same, the switch S_2 is closed, thus connecting D_1 and D_2 in parallel. If at the instant of paralleling the two machines have the same terminal pressure, then the switch S_2 may be closed without any current flowing either into or out of D_2. After paralleling, if the field current of D_2 be increased, the voltage generated is also increased, so that D_2 will begin to take some of the load while the load on D_1 will diminish. The exact proportion in which the total current divides between the two machines will depend on their respective internal E.M.Fs. and armature resistances. The condition which determines this division of current is, that after deducting the volts drop due to the armature